1.010 Uncertainty in Engineering
Fall 2008

1.010 – Mini-Quiz #5
(45 min – open books and notes)

Problem 1 (60 Points)

Two concrete beams have strengths X_1 and X_2 with joint normal distribution:

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \sim N\left(\begin{bmatrix} 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 & 0.75 \\ 0.75 & 1 \end{bmatrix}\right) \quad \text{(in some appropriate units)}$$

You intend to use beam 1 for construction. For structural competency, its strength must exceed 2 with probability at least 0.99, i.e. $P[X_1 > 2] \geq 0.99$.

a. Find $P[X_1 > 2]$. If this value is less than 0.99, beam 1 cannot be used, unless more information is obtained on its strength.
b. To get more information on X_1, you test the second beam.
c. What is the probability that, after testing the second beam, the first beam is found to be acceptable? Hint: First find x_2^* such that $P[X_1 > 2 \mid X_2 = x_2^*] = 0.99$. Then calculate $P[X_2 = x_2^*]$.

Problem 2 (40 Points)

The strength X of a concrete batch has normal distribution $X \sim N(m, \sigma^2)$ where σ is known (σ =1000 psi (pounds per square inch)) but m is uncertain with normal distribution $m \sim N(5000\text{psi}, (800\text{psi})^2)$. To better constrain m, you test 4 concrete cylinders in the lab from which you obtain the sample average $\overline{X} = \frac{1}{4}\sum_i^4 X_i$. Notice that $\overline{X}$ has distribution $\overline{X} \sim N(m, \frac{\sigma^2}{4})$ and that it can be thought of as a noisy measurement of m, $\overline{X} = m + \varepsilon$, where $\varepsilon \sim N(0, \frac{\sigma^2}{4})$.

Suppose that you measure $\overline{X} = 6000\text{psi}$. Find the mean value and variance of $(m \mid \overline{X} = 6000\text{psi})$.