1.010 Uncertainty in Engineering
Fall 2008

## Application Example 2

### (Total Probability Theorem)

# EVALUATION OF NATURAL AND MAN-MADE RISKS

An important application area of probability and statistics is the assessment of natural and man-made risks. For example, one may need to evaluate the safety of an engineering facility against extreme environmental actions, such as earthquakes, strong winds, extreme floods, ocean waves, etc. Since environmental loads vary in time, one usually expresses reliability through the probability that some undesirable "failure" event (severe structural damage or collapse, levee breach, dam overtopping, ship hull buckling, etc.) occurs at least once during a reference period of time T, for example 50 or 100 years.

In order to quantify **risk**, one needs to combine two elements:

1. A description of the severity of the environment, in terms of the probability $P[L_T > l]$ with which the maximum environmental load in T years, $L_T$, exceeds various levels $l$. Evaluating $P[L_T > l]$ is often referred to as **hazard assessment**;

2. A description of the resistance of the facility in terms of the dependence of the probability of system failure $P_f$ on the magnitude $l$ of the environmental load. This function, $P_f(l)$, is often referred to as the **fragility function**.

Once quantified, the hazard and fragility functions are combined to produce the probability of (at least one) failure in T. This is done by using the **Total Probability Theorem**, which says that, if $\{B_1,..., B_n\}$ is a set of mutually exclusive and collectively exhaustive events and A is any other event, then the probability of A can be calculated as

$$P[A] = \sum_{i=1}^{n} P[A \mid B_i] P[B_i] \quad (1)$$

To use this result for risk assessment, the environmental load $L_T$ is discretized into n distinct levels, say $l_1, ..., l_n$ and the events A and $B_i$ in Eq. 1 are taken as

$$A = \text{"the facility fails at least once in T"}$$
$$B_i = \text{"}L_T = l_i\text{"} \quad (2)$$

It is often reasonable to assume that the facility of interest survives in T if it does not fail under the most intense load $L_T$ experienced during that period. Under such simplifying assumption and with the notation in Eq. 2, the probability of failure in T is obtained from Eq. 1 as

$$P[\text{at least one failure in T}] = \Sigma_{i=1,n} P[\text{failure}|L_T = l_i]\ P[L_T = l_i] \quad (3)$$

Eq. 3 shows how the hazard (probabilities $P[L_T = l_i]$) and the fragility (the probabilities $P[\text{failure}|L_T = l_i]$) are combined in the assessment of risk.

In practical applications, it is typical for the hazard and the fragility to be quantified by different experts. For example, in the case of earthquake risk, a seismologist is usually responsible for assessing the hazard (the frequency with which various levels of ground shaking occur at a given site), while an engineer quantifies the fragility of the system (the performance of the facility under various levels of ground shaking).

Example. In a recent study, the seismic hazard in Boston has been assessed as follows in terms of Modified Mercalli Intensity or MMI (MMI is a discrete scale of ground motion intensity, with integer values from 1 to 12). Over a period of 100 years, the probability that the maximum MMI value equals I is

| I | P[max MMI in 100 years = I] |
|---|---|
| 6 | 0.3 |
| 7 | 0.1 |
| 8 | 0.03 |
| 9 | 0.01 |
| 10 | 0.003 |
| 11 | 0.001 |
| 12 | 0.0003 |

*Note*: these probabilities do not add to 1 because there is a significant probability that the maximum MMI is Boston in 100 years is less than 6. Values of I smaller than 6 do not usually pose significant threat to engineering facilities and are therefore neglected.

In a separate study, the seismic fragility of various types of structures was assessed by a group of engineers. Some of their results are reproduced below in the form of values of the probability of failure for different MMI.

| MMI, I | Pf of bridge | Pf of reinforced concrete building | Pf of brick building |
|---|---|---|---|
| **6** | 0.00 | 0.00 | 0.00 |
| **7** | 0.01 | 0.00 | 0.02 |
| **8** | 0.03 | 0.01 | 0.08 |
| **9** | 0.10 | 0.03 | 0.20 |
| **10** | 0.20 | 0.10 | 0.40 |
| **11** | 0.50 | 0.30 | 0.80 |
| **12** | 0.90 | 0.60 | 1.00 |

***Problem 2.1***

*(a) Use the above hazard and fragility assessments to determine the seismic risk in 100 year for different structural systems in Boston.*

*(b) If seismic risk is judged to be excessive, corrective action may be taken by strengthening the structures that are most at risk. This operation, called seismic retrofitting, has the effect of modifying the fragility of the structure, not the seismic hazard at the site. Suppose that a certain retrofitting technique would strengthen the structures "by one MMI unit", meaning that the probability of failure for MMI = I after retrofitting is the same as the probability of failure under MMI = I-1 before retrofitting. Re-evaluate the seismic risk of various structural types in Boston after such retrofitting operation.*

In some cases, one is not interested in the physical damage to a facility, but in the consequences that such damage might have on the exposed population or the environment. For example, in the case of a nuclear reactor damaged by an earthquake, the consequences may range widely depending on the amount of radioactive release caused by the event, the weather conditions at the time of the earthquake, etc. The risk should in this case be measured through the probability that a consequence C (for example, C = number of fatalities) exceeds a certain level c* in T years. In a simplified model, one may again ignore all seismic events in T except the one with highest intensity in T. Under this simplifying assumption, the probability that C > c* at least once in T years can be evaluated through a second application of the total probability theorem, as follows:

$$P[C > c^*] = \sum_{j=1}^{m} P[C > c^* \mid D = d_j]\, P[D = d_j] \qquad (4)$$

where $d_1,\ldots, d_m$ are m discretized levels of damage D. The probabilities $P[D = d_j]$ are evaluated through repeated application of Eq. 3, each time defining "failure" as the event $D = d_j$, whereas the probabilities $P[C > c^*|D = d_j]$ are the result of a **consequence model**. Consequence models are usually developed by yet another group of experts.

***Problem 2.2*** *Suppose that a chemical plant in the Boston area has the following fragility characteristics:*

| **MMI,I** | **P[D=1]** | **P[D=2]** | **P[D=3]** |
|---|---|---|---|
| **6** | 0.10 | 0.00 | 0.00 |
| **7** | 0.30 | 0.10 | 0.00 |
| **8** | 0.40 | 0.20 | 0.10 |
| **9** | 0.30 | 0.30 | 0.20 |
| **10** | 0.10 | 0.50 | 0.30 |
| **11** | 0.00 | 0.40 | 0.60 |
| **12** | 0.00 | 0.20 | 0.80 |

*where D = 1, 2, 3 are three levels of damage, in increasing order of severity. Further suppose that the following consequence model applies:*

| **D** | **P[Nf = 0]** | **P[1 ≤ Nf < 10]** | **P[Nf ≥ 10]** |
|---|---|---|---|
| **1** | 0.99 | 0.01 | 0.00 |
| **2** | 0.90 | 0.08 | 0.02 |
| **3** | 0.50 | 0.40 | 0.10 |

*where Nf = number of fatalities. Consider the event with maximum MMI in the next 100 years (this maximum MMI is random; the probability of different values are given on Page 3). What is the probability that, as a result of that maximum event, Nf =0, 1 ≤ Nf < 10, and Nf ≥ 10?*