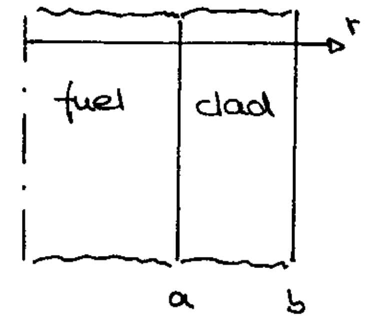
Solution - Problem Set No. 6

Geometry:



clad:

inner radius: $a = 4.9$ mm

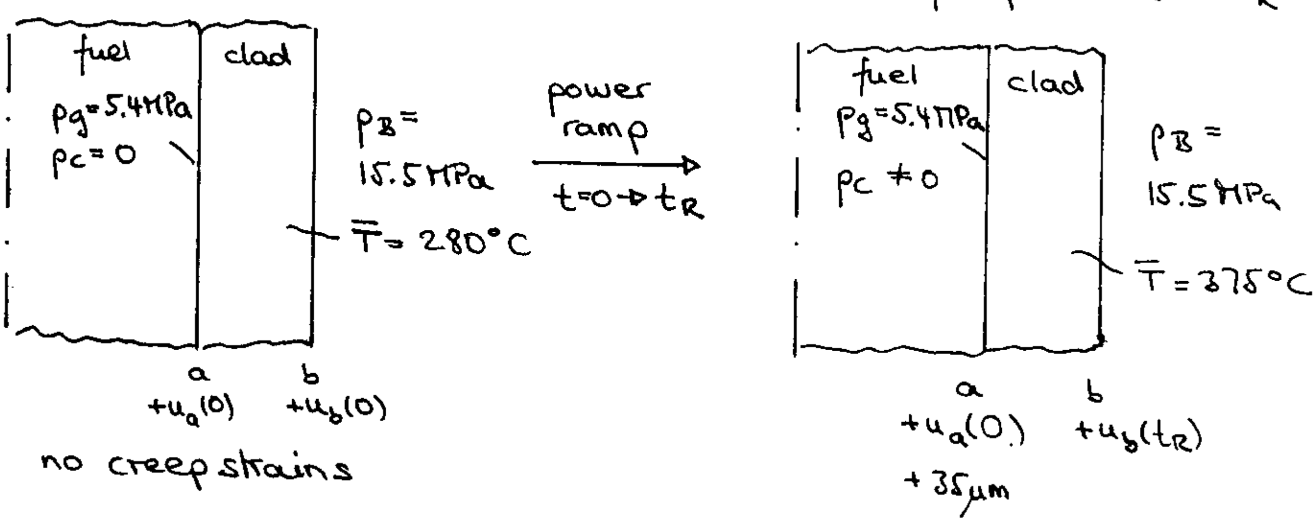
outer radius: $b = 5.6$ mm

wall thickness: $b - a = 0.7$ mm

Operating Conditions:

hot zero power ($t=0$) — time

hot full power ($t = t_R$)



Calculational Basis

- Finite-Element-treatment for clad: single ring
- $(u_a - u_a(0)) \propto t$ for each power ramp
- $(\overline{T} - \overline{T}_{HZP}) \propto t$ for each power ramp
- no axial force from fuel clad contact ($F_{zc} = 0$)

From the class-notes, we get the following equations when treating the clad as a single ring:

$$\begin{pmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \end{pmatrix} = \begin{pmatrix} \frac{-b}{a+b} & \frac{-a}{a+b} & 0 \\ -\frac{b}{b-a} & \frac{a}{b-a} & 0 \\ 0 & 0 & \frac{1}{\pi(b^2-a^2)} \end{pmatrix} \begin{pmatrix} p_b \\ p_a \\ F_z \end{pmatrix} \tag{1}$$

with

$$p_b = p_B \tag{2}$$

$$p_a = p_g + p_c \tag{3}$$

$$F_z = \pi a^2 p_g - \pi b^2 p_B \tag{4}$$

The strains and the displacements are related by

$$\begin{pmatrix} \varepsilon_r \\ \varepsilon_\theta \\ \varepsilon_z \end{pmatrix} = \begin{pmatrix} \frac{1}{b-a} & -\frac{1}{b-a} & 0 \\ \frac{1}{a+b} & \frac{1}{a+b} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_b \\ u_a \\ \varepsilon_z \end{pmatrix} \tag{5}$$

Otherwise, the strains can be calculated from the stresses, the thermal strain ε_T and the strains due to plasticity and creep ε_{mr}, $\varepsilon_{m\theta}$, ε_{mz}.

$$\begin{pmatrix} \varepsilon_r \\ \varepsilon_\theta \\ \varepsilon_z \end{pmatrix} = \underbrace{\frac{1}{E} \begin{pmatrix} 1 & -\nu & -\nu \\ -\nu & 1 & -\nu \\ -\nu & -\nu & 1 \end{pmatrix} \begin{pmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \end{pmatrix}}_{\text{elastic effects}} + \underbrace{\varepsilon_T \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}_{\text{thermal}} + \underbrace{\begin{pmatrix} \varepsilon_{mr} \\ \varepsilon_{m\theta} \\ \varepsilon_{mz} \end{pmatrix}}_{\text{plasticity and creep}} \tag{6}$$

Furthermore, for the strains due to creep we have the equations

$$\begin{pmatrix} \dot\varepsilon_{mr} \\ \dot\varepsilon_{m\theta} \\ \dot\varepsilon_{mz} \end{pmatrix} = \frac{\dot\varepsilon_g}{\sigma_g} \begin{pmatrix} 1 & -1/2 & -1/2 \\ -1/2 & 1 & -1/2 \\ -1/2 & -1/2 & 1 \end{pmatrix} \begin{pmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \end{pmatrix} \tag{7}$$

with the equivalent von-Mises stress σ_g and

$\dot\varepsilon_g = 10^{-9} \underbrace{\sigma_g}_{\text{in MPa}}$, so that eq. (7) is independent of σ_g.

In order to use eq. (7) we have to derive :

- eq. (1)

$$\begin{pmatrix} \dot{\sigma}_r \\ \dot{\sigma}_\theta \\ \dot{\sigma}_z \end{pmatrix} = \begin{pmatrix} \frac{-b}{a+b} & \frac{-a}{a+b} & 0 \\ \frac{-b}{b-a} & \frac{a}{b-a} & 0 \\ 0 & 0 & \frac{1}{\pi(b^2-a^2)} \end{pmatrix} \begin{pmatrix} \dot{p}_b \\ \dot{p}_a \\ \dot{F}_z \end{pmatrix}$$

and from eq. (2) to (4), with

$p_g =$ const., $p_B =$ const.

we get

$$\dot{p}_b = 0$$
$$\dot{p}_a = \dot{p}_c$$
$$\dot{F}_z = 0$$

and therefore

$$\begin{pmatrix} \dot{\sigma}_r \\ \dot{\sigma}_\theta \\ \dot{\sigma}_z \end{pmatrix} = \begin{pmatrix} -\frac{a}{a+b} \\ \frac{a}{b-a} \\ 0 \end{pmatrix} \dot{p}_c \tag{8}$$

- eq. (5)

$$\begin{pmatrix} \dot{\varepsilon}_r \\ \dot{\varepsilon}_\theta \\ \dot{\varepsilon}_z \end{pmatrix} = \begin{pmatrix} \frac{1}{b-a} & \frac{-1}{b-a} & 0 \\ \frac{1}{a+b} & \frac{1}{a+b} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \dot{u}_b \\ \dot{u}_a \\ \dot{\varepsilon}_z \end{pmatrix} \tag{9}$$

- eq. (6)

$$\begin{pmatrix} \dot{\varepsilon}_r \\ \dot{\varepsilon}_\theta \\ \dot{\varepsilon}_z \end{pmatrix} = \frac{1}{E} \begin{pmatrix} 1 & -\nu & -\nu \\ -\nu & 1 & -\nu \\ -\nu & -\nu & 1 \end{pmatrix} \begin{pmatrix} \dot{\sigma}_r \\ \dot{\sigma}_\theta \\ \dot{\sigma}_z \end{pmatrix} + \dot{\varepsilon}_T \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} \dot{\varepsilon}_{mr} \\ \dot{\varepsilon}_{m\theta} \\ \dot{\varepsilon}_{mz} \end{pmatrix} \tag{10}$$

Plugging eq. (7), (8) and (9) in eq. (10) yields:

$$\begin{pmatrix} \frac{1}{b-a} & \frac{-1}{b-a} & 0 \\ \frac{1}{a+b} & \frac{1}{a+b} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \dot{u}_b \\ \dot{u}_a \\ \dot{\varepsilon}_z \end{pmatrix} = \frac{1}{E} \begin{pmatrix} 1 & -\nu & -\nu \\ -\nu & 1 & -\nu \\ -\nu & -\nu & 1 \end{pmatrix} \begin{pmatrix} \frac{-a}{a+b} \\ \frac{a}{b-a} \\ 0 \end{pmatrix} \dot{p}_c + \dot{\varepsilon}_T \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad (11)$$

$$+ \beta \begin{pmatrix} 1 & -1/2 & -1/2 \\ -1/2 & 1 & -1/2 \\ -1/2 & -1/2 & 1 \end{pmatrix} \begin{pmatrix} \frac{-b}{a+b} & \frac{-a}{a+b} & 0 \\ \frac{-b}{b-a} & \frac{a}{b-a} & 0 \\ 0 & 0 & \frac{1}{\pi(b^2-a^2)} \end{pmatrix} \begin{pmatrix} p_B \\ p_g+p_c \\ F_z \end{pmatrix}$$

with $$\beta = \frac{\dot{\varepsilon}_g}{\sigma_g} = 10^{-9} \frac{1}{5\,\mathrm{MPa}} . \qquad (12)$$

Writing eq. (11) in components gives:

$$\frac{1}{b-a}(\dot{u}_b - \dot{u}_a) = -\frac{\dot{p}_c}{E} a \left(\frac{1}{a+b} + \frac{\nu}{b-a}\right) + \dot{\varepsilon}_T + \frac{p_B}{2} \beta b \left(\frac{1}{b-a} - \frac{2}{a+b}\right)$$
$$+ \frac{p_c+p_g}{2} \beta a \left(-\frac{1}{b-a} - \frac{2}{a+b}\right) - \beta \frac{F_z}{2} \frac{1}{\pi(b^2-a^2)} \qquad (13)$$

$$\frac{1}{a+b}(\dot{u}_b + \dot{u}_a) = \frac{\dot{p}_c}{E} a \left(\frac{\nu}{a+b} + \frac{1}{b-a}\right) + \dot{\varepsilon}_T + \frac{p_B}{2} \beta b \left(\frac{1}{a+b} - \frac{2}{b-a}\right)$$
$$+ \frac{p_c+p_g}{2} \beta a \left(\frac{1}{a+b} + \frac{2}{b-a}\right) - \beta \frac{F_z}{2} \frac{1}{\pi(b^2-a^2)} \qquad (14)$$

$$\dot{\varepsilon}_z = \frac{\dot{p}_c}{E} a \left(\frac{\nu}{a+b} - \frac{\nu}{b-a}\right) + \dot{\varepsilon}_T + \beta \frac{p_B}{2} b \left(\frac{1}{a+b} + \frac{1}{b-a}\right)$$
$$+ \frac{p_c+p_g}{2} \beta a \left(\frac{1}{a+b} - \frac{1}{b-a}\right) + \beta F_z \frac{1}{\pi(b^2-a^2)} \qquad (15)$$

Now, we have 3 equations (eq (13) to (15)) and 5 unknowns:

$\dot{u}_b$, $\dot{u}_a$, p_c and $\dot{p}_c$, $\dot{\varepsilon}_T$, $\dot{\varepsilon}_z$.

We need two more equations to calculate these unknowns. (we only want to calculate $p_c(t)$, so we can reduce the problem to 2 equations (eq. (13) and (14)) with 4 unknowns $\dot{u}_b$, $\dot{u}_a$, $\dot{p}_c$ and p_c $\dot{\varepsilon}_T$)

From the calculational basis we know, that

- $[u_a - u_a(t \overset{\text{HZP}}{=} 0)] \propto t$

and thereby $u_a(t) = u_a(t=0) + \dfrac{u_{a\,HFP} - u_a(t \overset{\text{HZP}}{=} 0)}{t_R}\, t$

with $u_{a\,HFP} - u_{a\,HZP} = 35\,\mu m$.

Therefore $$\dot{u}_a = \frac{u_{a\,HFP} - u_{a\,HZP}}{t_R} \qquad (16)$$

- $(\bar{T} - \bar{T}_{HZP}) \propto t$

and thereby with $\varepsilon_T = \alpha\,(\bar{T}(t) - T_0)$ cold condition

$$\varepsilon_T(t) = \alpha\left[(\bar{T}_{HZP} - T_0) + \gamma t\right] \qquad (17)$$

with

$$\gamma = \frac{\bar{T}_{HFP} - \bar{T}_{HZP}}{t_R} \qquad (18)$$

and

$$\dot{\varepsilon}_T = \alpha\gamma \qquad (19)$$

Using eq. (16) and (19), eq. (13) and (14) reduce to two equations with two unknowns, $\dot{u}_b$ and $p_c(t)$. We can now eliminate $\dot{u}_b$ from eq. (13) and (14) and get

$$2\dot{u}_a = \frac{1}{b^2-a^2}\Big[\frac{\dot{p}_c}{E}\,2a\,\big(b^2(1+\nu) + a^2(1-\nu)\big) + 2a\,\dot{\varepsilon}_T\,(b^2-a^2)$$
$$- p_B\,4ab^2 + (p_c + p_g)\,\beta a\,(3b^2 + a^2) - \frac{F_z}{\pi}\,\beta a\Big] \qquad (20)$$

Now, write eq. (20) in the form

$$\dot{p}_c + \varphi\, p_c = \psi \qquad (21)$$

with

$$\zeta = \frac{E\,(b^2-a^2)}{2a\left[b^2(1+\nu)+a^2(1-\nu)\right]}$$

$$\varphi = \frac{\beta a}{b^2-a^2}\,(3b^2+a^2)\,\zeta$$

(22)

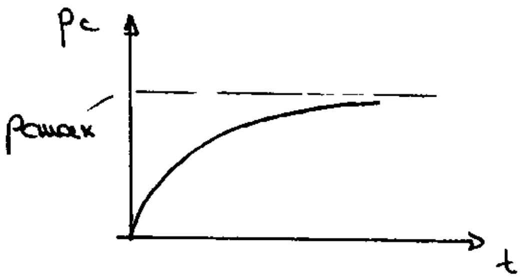
$$\psi = \zeta\left[\frac{1}{b^2-a^2}\left(\frac{\beta F_z a}{\pi} + 4ab^2\beta p_B - \beta a(3b^2+a^2)\,p_g\right) + 2\,(\dot{u}_a - a\,\dot{\varepsilon}_T)\right]$$

Equation (21) can be easily solved and has the solution

$$p_c(t) = c\,e^{-\varphi t} + \frac{\psi}{\varphi} \qquad (23)$$

Applying the condition, that $p_c(t=0)=0$ the constant c can be determined and eq. (23) becomes

$$p_c(t) = \frac{\psi}{\varphi}\left(1-e^{-\varphi t}\right) \qquad (24)$$



$$p_{cmax} = \frac{\psi}{\varphi}$$

Now, with $p_c(t=t_R) = p_{cR}$ from eq. (24) the clad stresses can be calculated using eq. (1)

$$\begin{pmatrix}\sigma_r\\ \sigma_\theta\\ \sigma_z\end{pmatrix}_{t=t_R} = \begin{pmatrix} -\frac{b}{a+b} & -\frac{a}{a+b} & 0\\ -\frac{b}{b-a} & \frac{a}{b-a} & 0\\ 0 & 0 & \frac{1}{\pi(b^2-a^2)}\end{pmatrix}\begin{pmatrix} p_B\\ p_g+p_{cR}\\ F_z\end{pmatrix} \qquad (25)$$

In order to obtain the clad strains from eq. (6) we need to calculate the strains due to thermal and plasticity and creep effects.

The thermal strains at hot full power case (also from eq. (17))

$$\varepsilon_{T_R} = \varepsilon_T(t = t_R) = \alpha\left(\bar{T}_{HFP} - \overset{\text{cold, shutdown}}{T_0}\right) \tag{26}$$

The plasticity and creep strains are determined by eq. (1) and (7):

$$\begin{pmatrix} \varepsilon_{mr} \\ \varepsilon_{m\theta} \\ \varepsilon_{mz} \end{pmatrix} = \int_0^{t_R} \beta \begin{pmatrix} 1 & -1/2 & -1/2 \\ -1/2 & 1 & -1/2 \\ -1/2 & -1/2 & 1 \end{pmatrix} \begin{pmatrix} \frac{-b}{a+b} & \frac{-a}{a+b} & 0 \\ \frac{-b}{b-a} & \frac{a}{b-a} & 0 \\ 0 & 0 & \frac{1}{\pi(b^2-a^2)} \end{pmatrix} \begin{pmatrix} pB \\ pg + \frac{\psi}{\varphi}\left(1 - e^{-\varphi t}\right) \\ F_z \end{pmatrix} dt$$

$$= \frac{\beta}{2} \begin{pmatrix} -b^2+3ab & a^2-3ab & -\frac{1}{\pi} \\ -b^2-3ab & a^2+3ab & -\frac{1}{\pi} \\ 2b^2 & -2a^2 & +\frac{2}{\pi} \end{pmatrix} \frac{1}{b^2-a^2} \begin{pmatrix} pB\,t_R \\ pg\,t_R + \frac{\psi}{\varphi}\left(t_R - \frac{1}{\varphi}\left(1 - e^{-\varphi t_R}\right)\right) \\ F_z\,t_R \end{pmatrix} \tag{27}$$

Applying eq. (25), (26) and (27) to eq. (6) yields the clad strains.

The radial displacement at the outer surface can be calculated by using eq. (5)

$$\begin{pmatrix} u_b \\ u_a \\ \varepsilon_z \end{pmatrix}_{t=t_R} = \begin{pmatrix} \frac{1}{b-a} & -\frac{1}{b-a} & 0 \\ \frac{1}{a+b} & \frac{1}{a+b} & 0 \\ 0 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} \varepsilon_r \\ \varepsilon_\theta \\ \varepsilon_z \end{pmatrix}_{t=t_R}$$

or, easier from

$$\left.\begin{aligned} \varepsilon_r &= \frac{u_b - u_a}{b-a} \\ \varepsilon_\theta &= \frac{u_b + u_a}{a+b} \end{aligned}\right\} \quad u_b = \frac{1}{2}\left(\varepsilon_r(b-a) + \varepsilon_\theta(a+b)\right) \tag{28}$$

Results:

- $t_R = 30\,\text{min}$:

$$\begin{pmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \end{pmatrix} = \begin{pmatrix} -38.9 \\ 336.0 \\ -48.5 \end{pmatrix} \text{MPa}$$

$$\begin{pmatrix} \varepsilon_r \\ \varepsilon_\theta \\ \varepsilon_z \end{pmatrix} = \begin{pmatrix} 0.8 \\ 7.4 \\ 0.6 \end{pmatrix} \times 10^{-3}$$

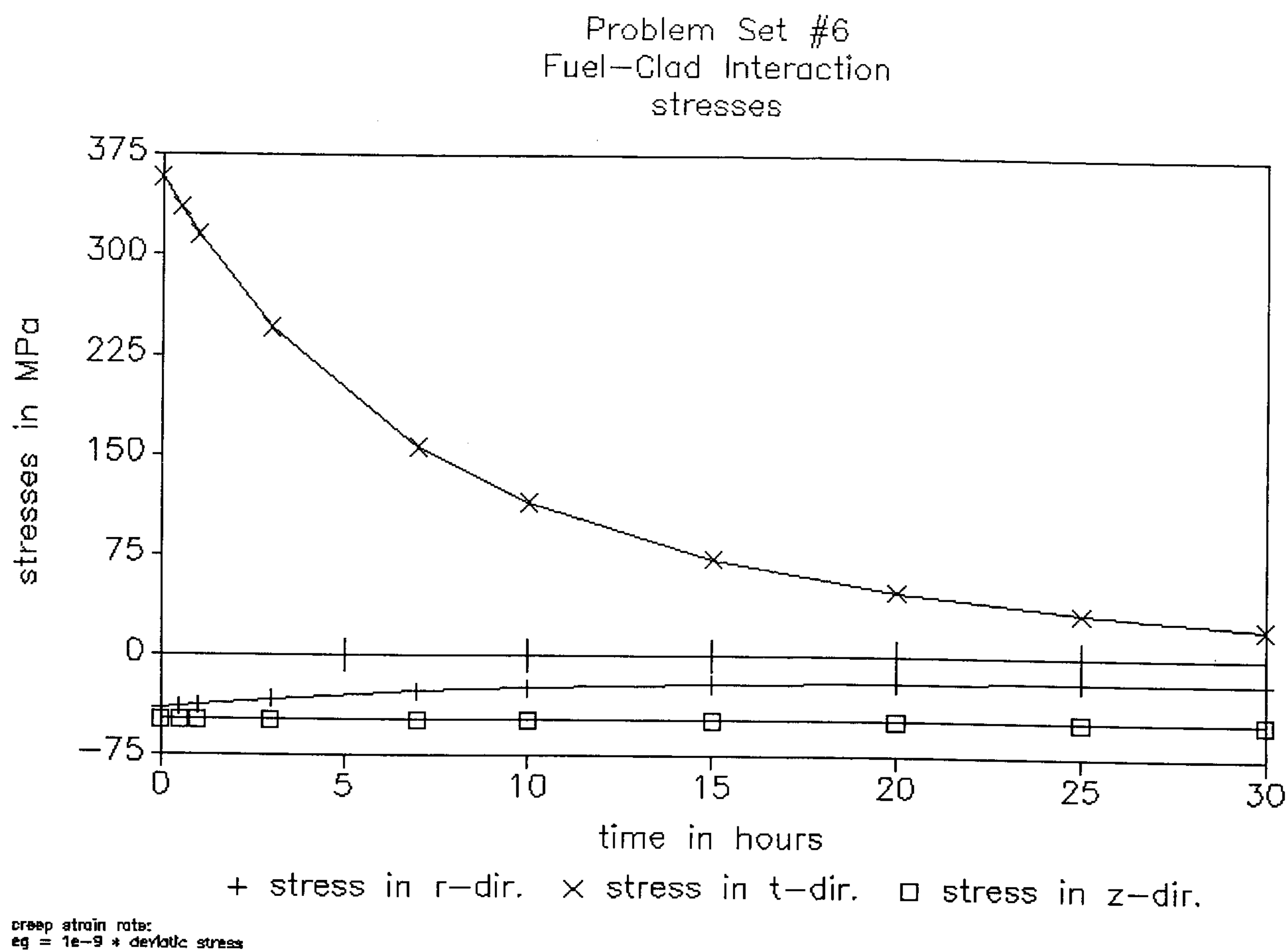
$$u_b = 3.91 \times 10^{-5}\,\text{m}$$

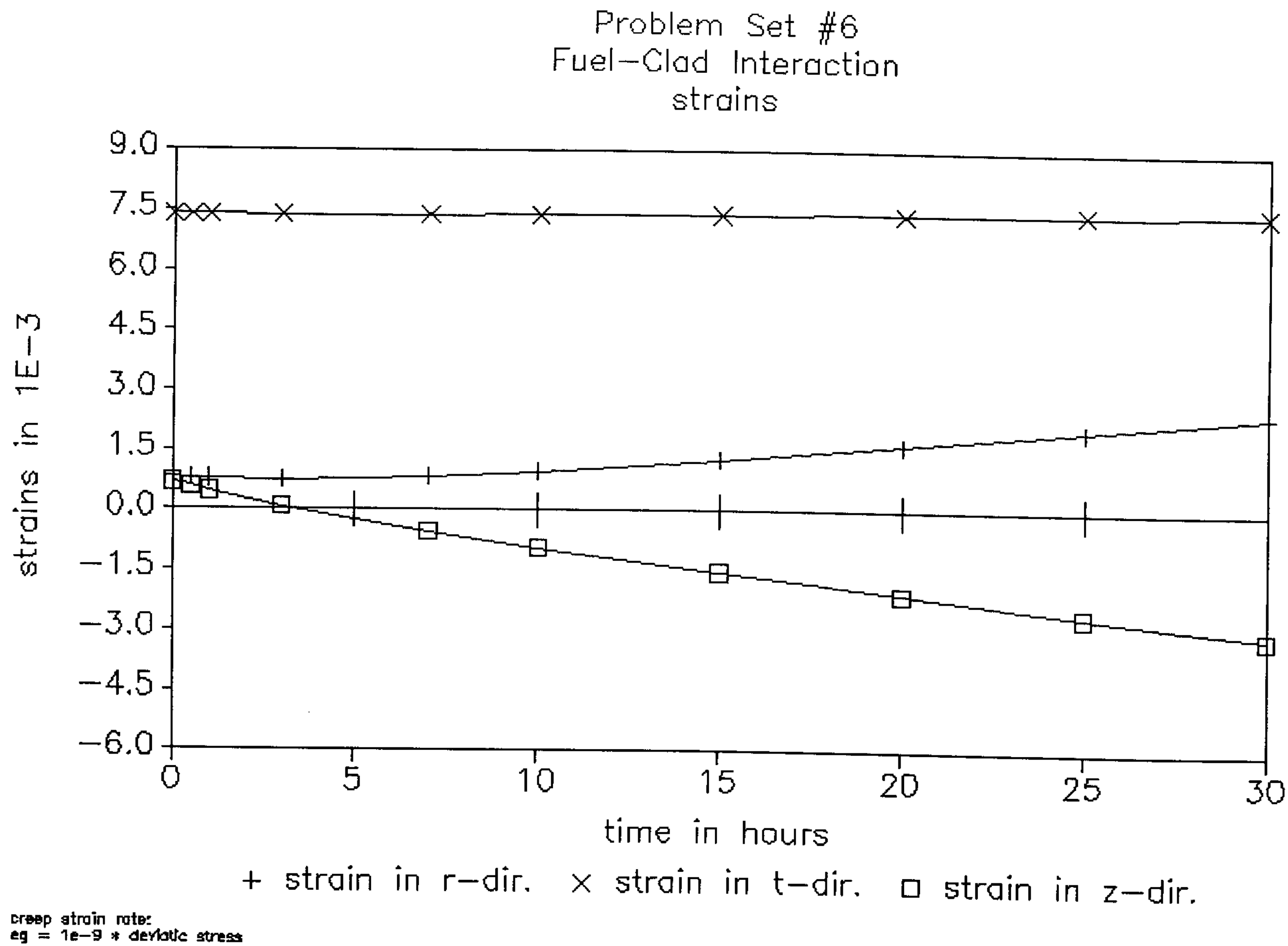
- $t_R = 30$ hours

$$\begin{pmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \end{pmatrix} = \begin{pmatrix} -18.0 \\ 22.6 \\ -48.5 \end{pmatrix} \text{MPa}$$

$$\begin{pmatrix} \varepsilon_r \\ \varepsilon_\theta \\ \varepsilon_z \end{pmatrix} = \begin{pmatrix} 2.5 \\ 7.5 \\ -4.8 \end{pmatrix} \times 10^{-3}$$

$$u_b = 4.02 \times 10^{-5}\,\text{m}$$


Problem Set #6
Fuel-Clad Interaction
stresses
stresses in MPa
375
300
225
150
75
0
-75
0
5
10
15
20
25
30
time in hours
+ stress in r-dir.
× stress in t-dir.
□ stress in z-dir.
creep strain rate:
eg = 1e-9 * deviatic stress



Problem Set #6
Fuel-Clad Interaction
strains
strains in 1E-3
9.0
7.5
6.0
4.5
3.0
1.5
0.0
-1.5
-3.0
-4.5
-6.0
0
5
10
15
20
25
30
time in hours
+ strain in r-dir. × strain in t-dir. □ strain in z-dir.
creep strain rate:
eg = 1e-9 * deviatic stress