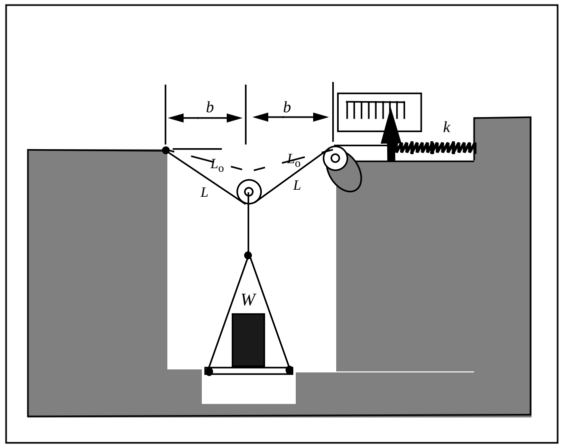
Reflections: Design Exercise 1
1.050 Solid Mechanics
Fall 2004

Some reflections on Design Exercise #1, a non-linear scale for weighing baggage.

Consider the initial state of the system, before loading the baggage onto the platform but with the platform, pulleys, cables and spring all assembled and in place. If the weight of the platform (and center pulley) is significant with respect to the anticipated weight of the baggage, then there will be a significant deflection of the system in its initial state. θ_o will not be, can not be, taken as zero.



Say the weight of the platform is 50 lbs. We can estimate θ_o using the equilibrium, compatibility, and force-deflection relationships we have already derived, taking the deflection prior to the addition of the platform and pulley as zero.

From equilibrium, letting *T* be the tension in the cable (not shown) we have $T = \frac{W}{2\sin\theta}$

Compatibility yields $\frac{\Delta}{L_o} = 2 \cdot \left(\frac{1}{\cos\theta} - 1\right)$. where Δ is the deflection of the spring, positive to the left in the figure[1].

To tie the two together, we have the spring, force-deformation relationship $T = k\Delta$ where *k* is either 100 or 500 lbs/inch. With this, we can eliminate *T* from equilibrium and write

$$\frac{W}{2kL_o} = \sin\theta \cdot \frac{\Delta}{L_o}$$

Taking *W = 50 lbs, k = 100 lbs/in,* and L_o *= 48 in.* we produce the following equation for determine θ_o:

$$0.0052 = 2\sin\theta_0 \cdot \left(\frac{1}{\cos\theta_0} - 1\right)$$

This does not solve explicitly for θ_o but we note that the left hand side is a small number so we look at the right hand side and explore what would make it small. Well, if θ_o is small, then the cosine is near 1, so the bracket is small and furthermore, the sine will be small. Thus we conclude that θ_o will be close to zero (which we might have inferred from the “physics” of the situation but note that this inference is possible only if we know that the spring is of sufficient stiffness to hold θ_o small with a platform weight of 50 lbs. The mathematical analysis stands on its own).

Taking advantage of this, we expand the sine and the cosine in the vicinity of zero. Going back to my calculus book, or deep within memory, I write

$$\sin\theta_0 \cong \theta_0 - \frac{\theta_0^3}{3!}$$

$$\cos\theta_0 \cong 1 - \frac{\theta_0^2}{2!} \qquad so \qquad \frac{1}{\cos\theta_0} \cong 1 + \frac{\theta_0^2}{2!}$$

which gives

$$0.0052 = 2\left(\theta_0 - \frac{\theta_0^3}{3!}\right) \cdot \left(1 + \frac{\theta_0^2}{2!} - 1\right)$$

1. It’s safe to assume here that without the platform and pulley, the “initial” θ_o is zero. Hence the 1 in the first term within the bracket.

or retaining only the leading order terms in θ_o, we have

$$0.0052 = 2(\theta_0)\cdot\left(\frac{\theta_0^2}{2!}\right) \qquad so \qquad \theta_0 = 0.138 radians \qquad i.e., 7.9\ deg.$$

So one strategy in designing the scale is to assume the cable is level (or nearly so) prior to assembling and fixing the pulley and platform in place, estimating the weight of this assembly, then deducing a θ_o from above.

Alternatively, one can adjust the cable length, with the platform assembly in place to give a θ_o of one's choice but this must be greater than that obtained above.

There is, however, another way to control and fix θ_o; We can, prior to loading the system with the pulley and platform weight, *pre-tension the spring*. Physically, we stretch the spring to some desired tension, then attach the end of the cable. In this case we write our force/deformation relationship $T = k\Delta + T_0$ The equilibrium and compatibility relations remain the same. (Δ is the deflection of the spring from the level cable, but pre-tensioned state.

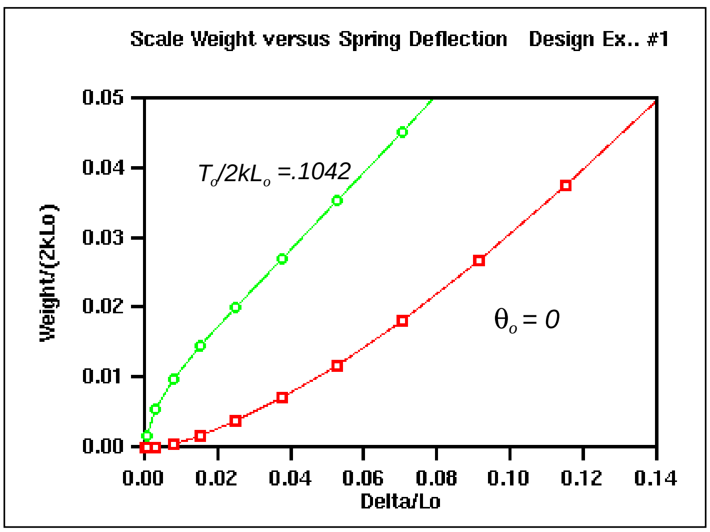
Proceeding as before we have now:

$$\frac{W}{2kL_o} = 2\sin\theta\cdot\left(\frac{1}{\cos\theta}-1\right) + \sin\theta\cdot\frac{T_0}{2kL_o}$$

To illustrate the effect of pretension, we take T_0 = *1000 lbs*, and the same values for *k* and L_0 as above. We have, then, the following two relationships for determining how the scale displaces with increasing weight.

$$\frac{W}{2kL_o} = 2\sin\theta\cdot\left(\frac{1}{\cos\theta}-1\right) + (0.1042)\cdot\sin\theta \qquad and \qquad \frac{\Delta}{L_o} = 2\cdot\left(\frac{1}{\cos\theta}-1\right)$$

A spread sheet plot is shown below:



The effect of the pretension is two - fold: A bigger weight is required to move the pointer to the same position (as that taken up by a system with $\theta_o = 0$ and no pre-tension) and the graph appears to become linear earlier on, at a lower weight.

Both of these changes can be advantageous: Pre-tensioning the spring provides an additional parameter for us to work with in the design.