

**1.033/1.57**

**Mechanics of Material Systems**  
(Mechanics and Durability of Solids I)

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*Lecture: MWF1 // Recitation: F3:00-4:30*

# Part II: Momentum Balance, Stresses and Stress States

## 4. Stress States / Failure Criteria

# Content 1.033/1.57

## Part I. **Deformation and Strain**

- 1 Description of Finite Deformation
- 2 Infinitesimal Deformation

## Part II. **Momentum Balance and Stresses**

- 3 Momentum Balance
-  4 Stress States / Failure Criterion

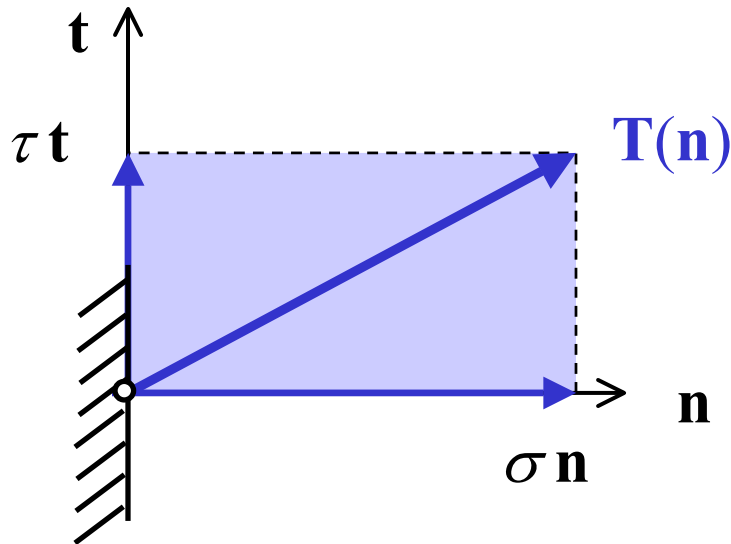
## Part III. **Elasticity and Elasticity Bounds**

- 5 Thermoelasticity,
- 6 Variational Methods

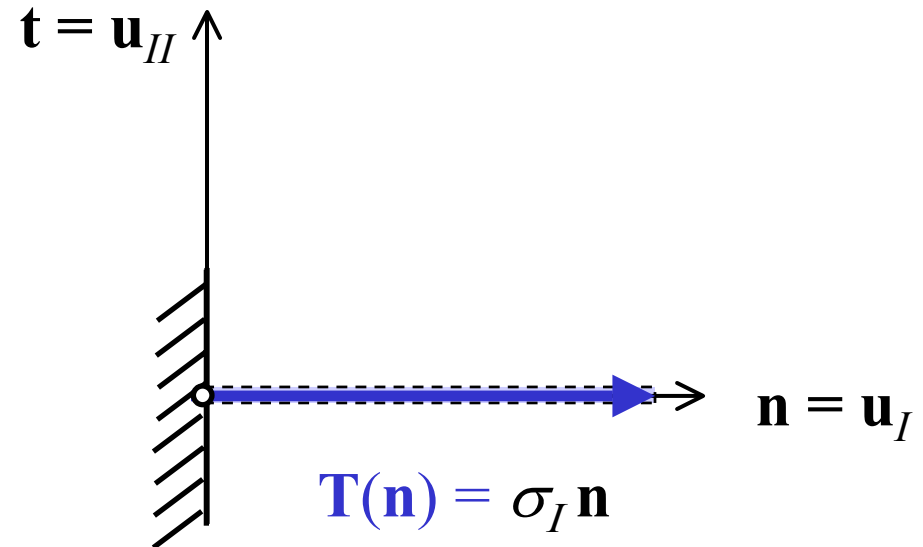
## Part IV. **Plasticity and Yield Design**

- 7 1D-Plasticity – An Energy Approach
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# Stress Vector and Stress Components



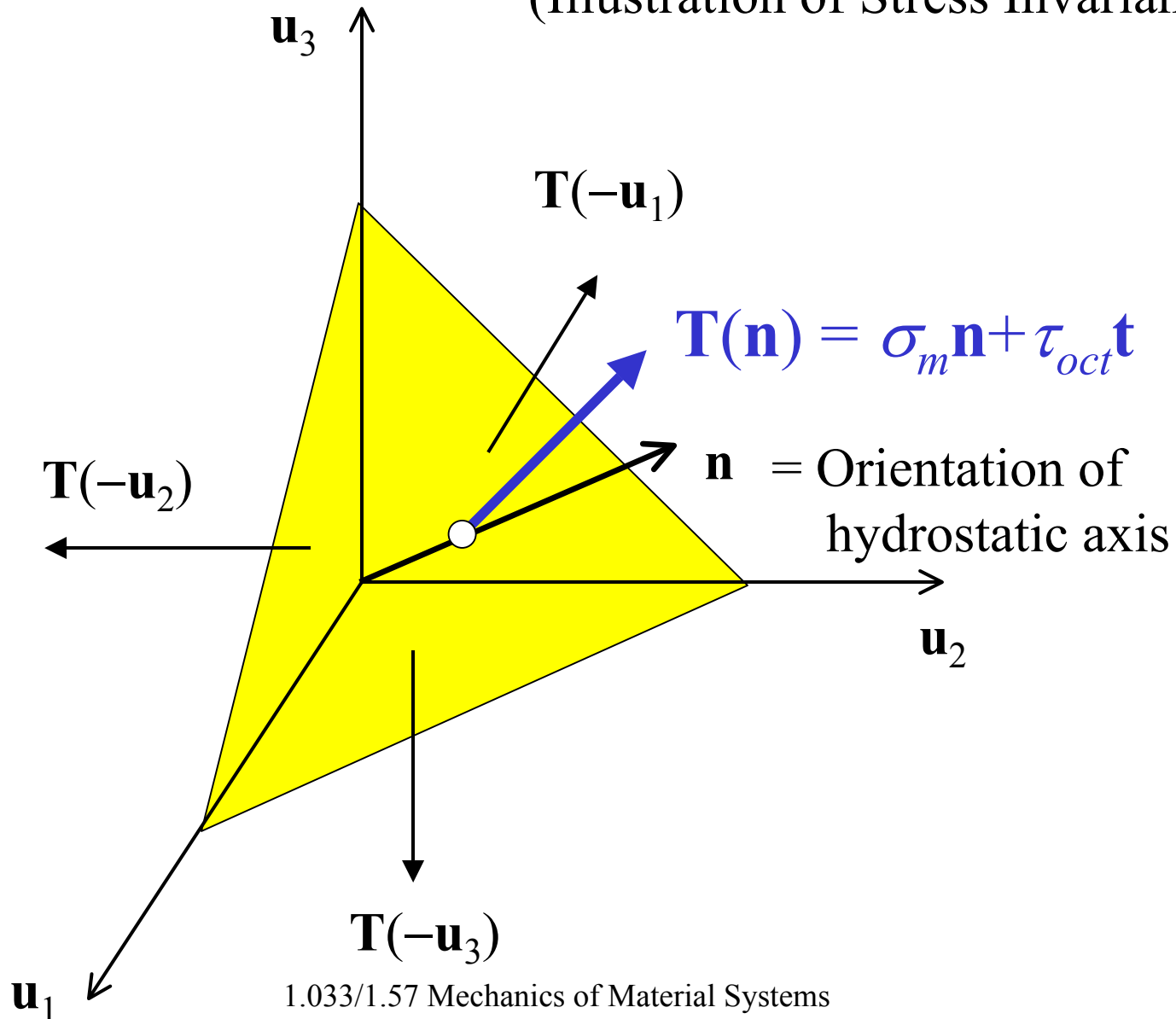
Stress components on a material surface oriented by unit normal  $\mathbf{n}$



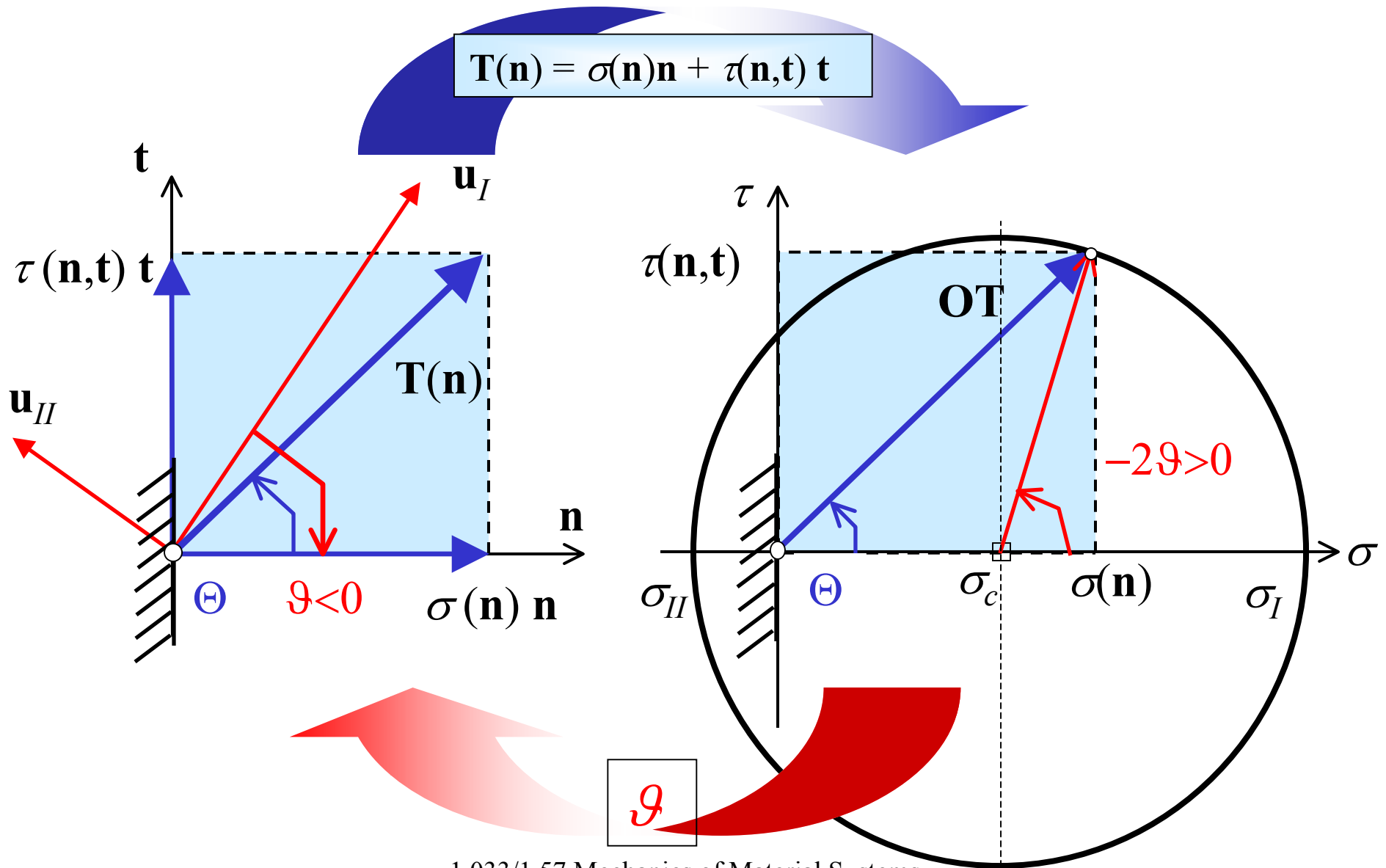
Stress components on a material surface oriented in the principal stress direction  $\mathbf{n} = \mathbf{u}_I$

# Stress Vector in the Principal Stress Space

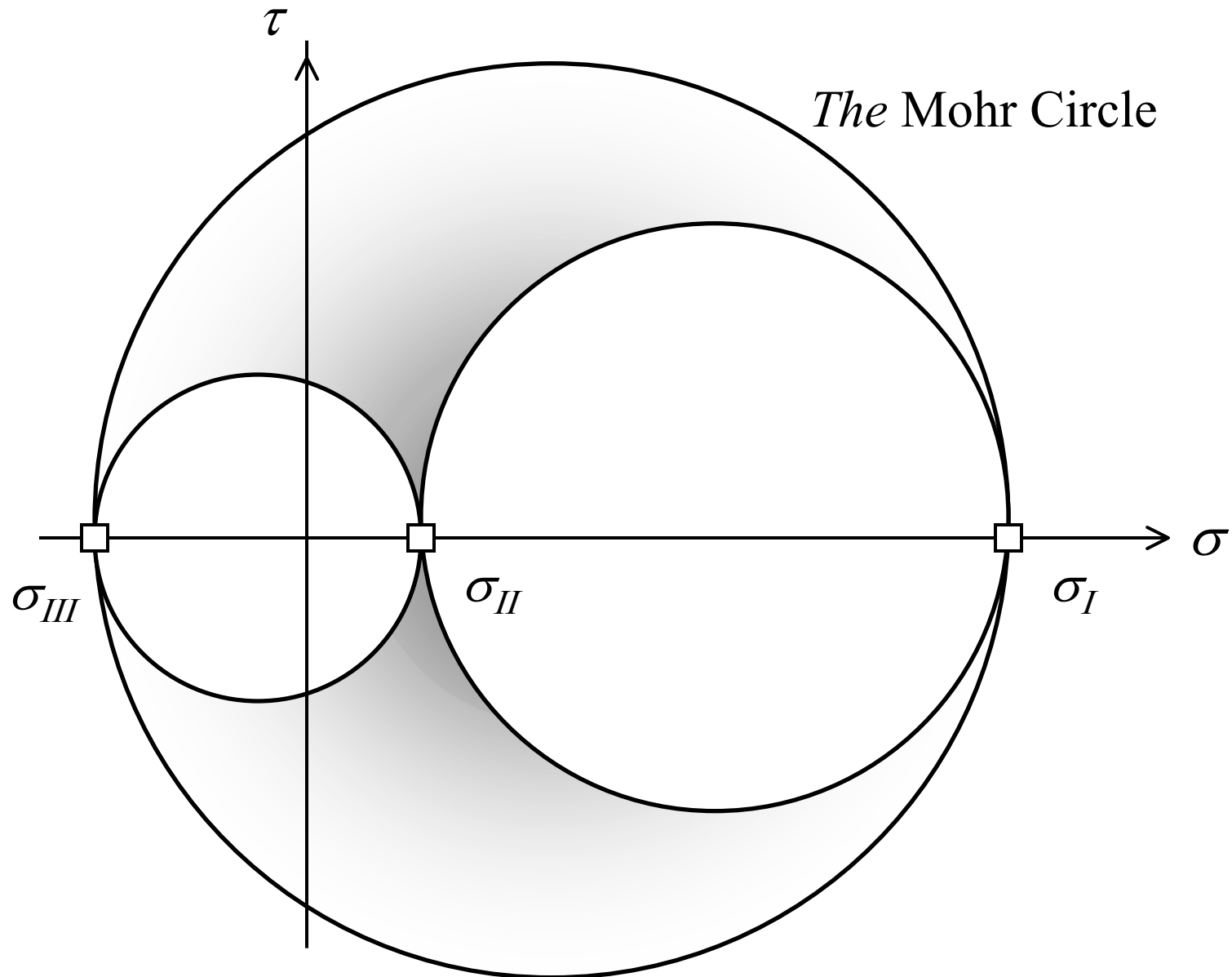
(Illustration of Stress Invariants)



# Stress Vector in the Mohr Plane

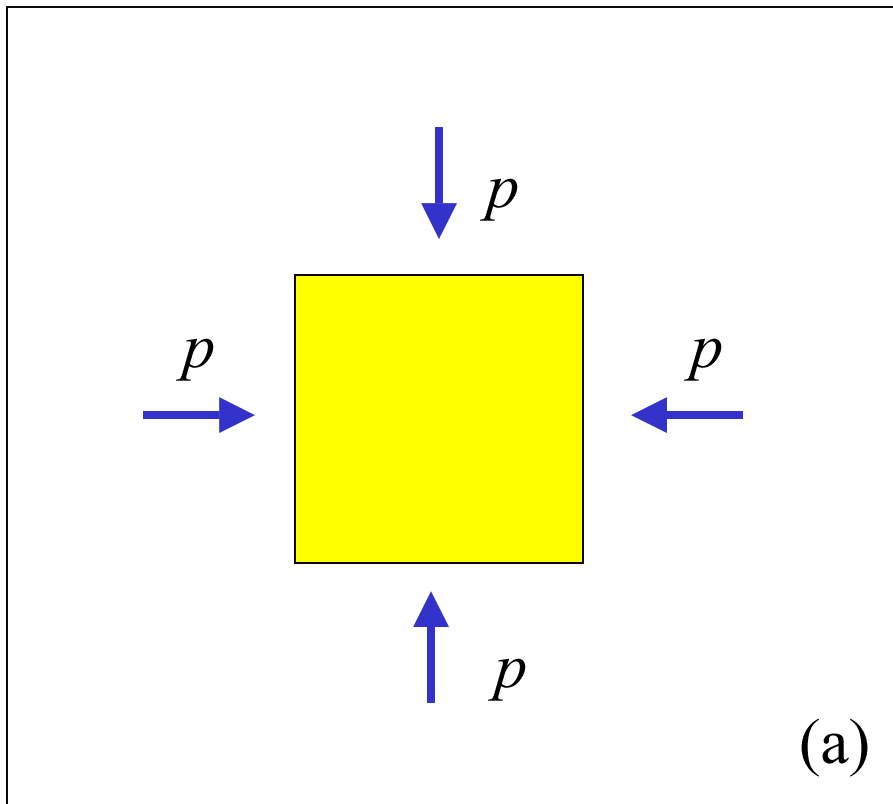


# Mohr Circles and *The* Mohr Circle

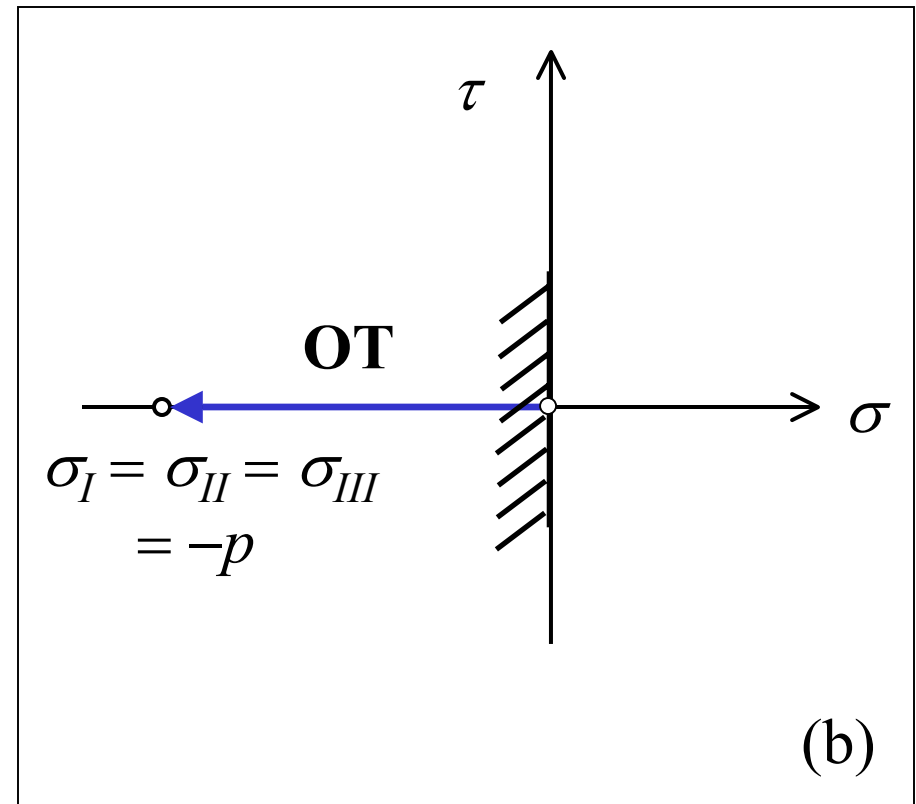


# Selected Stress States: Hydrostatic Pressure

$$\sigma = -p\mathbf{1}$$



*Material Plane*

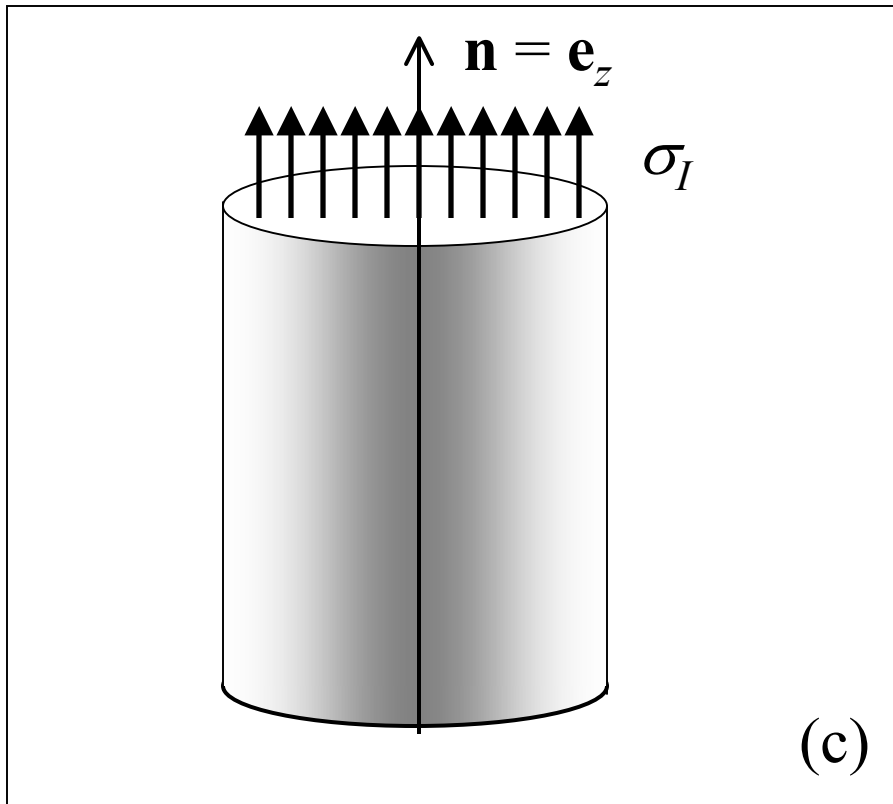


*Mohr Stress Plane*

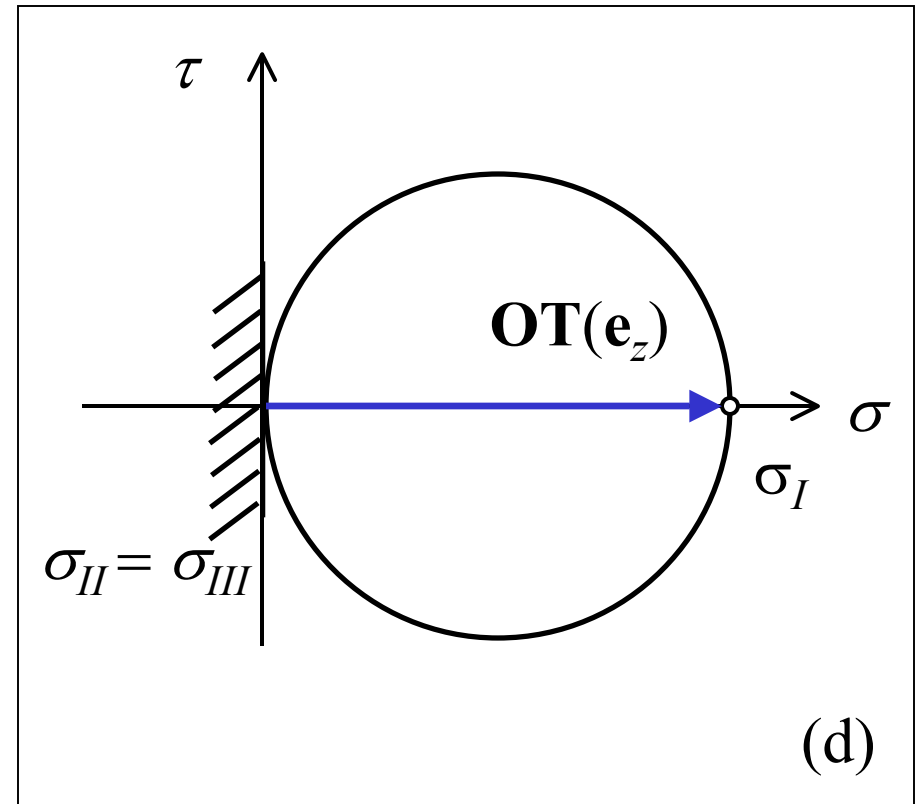


# Selected Stress States: Uniaxial Tension

$$\boldsymbol{\sigma} = \sigma_I \mathbf{e}_z \otimes \mathbf{e}_z$$



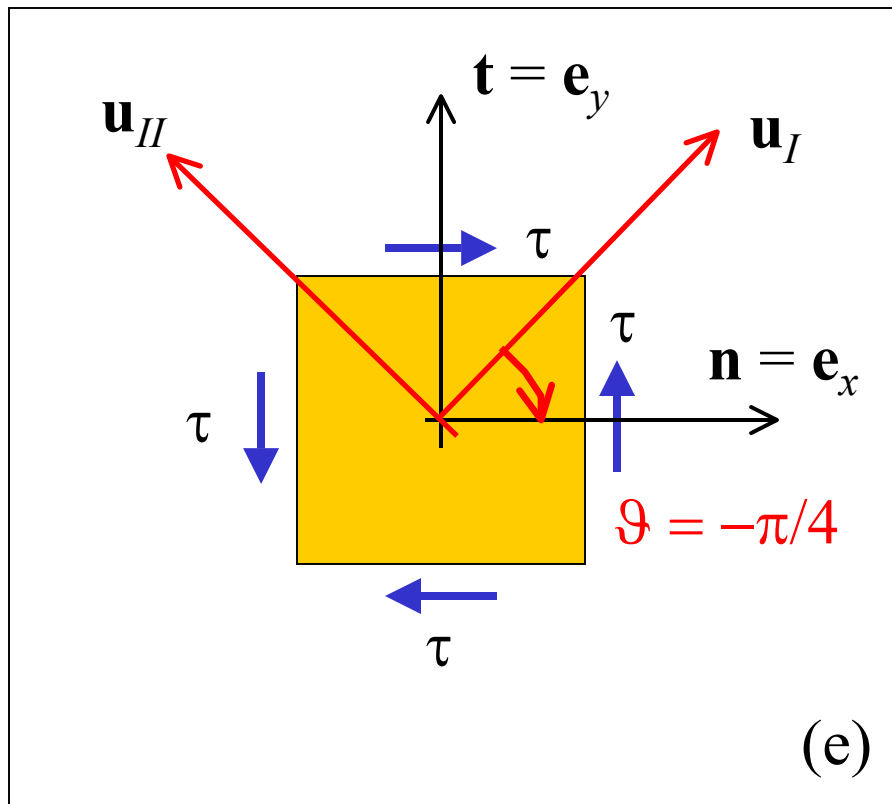
*Material Plane*



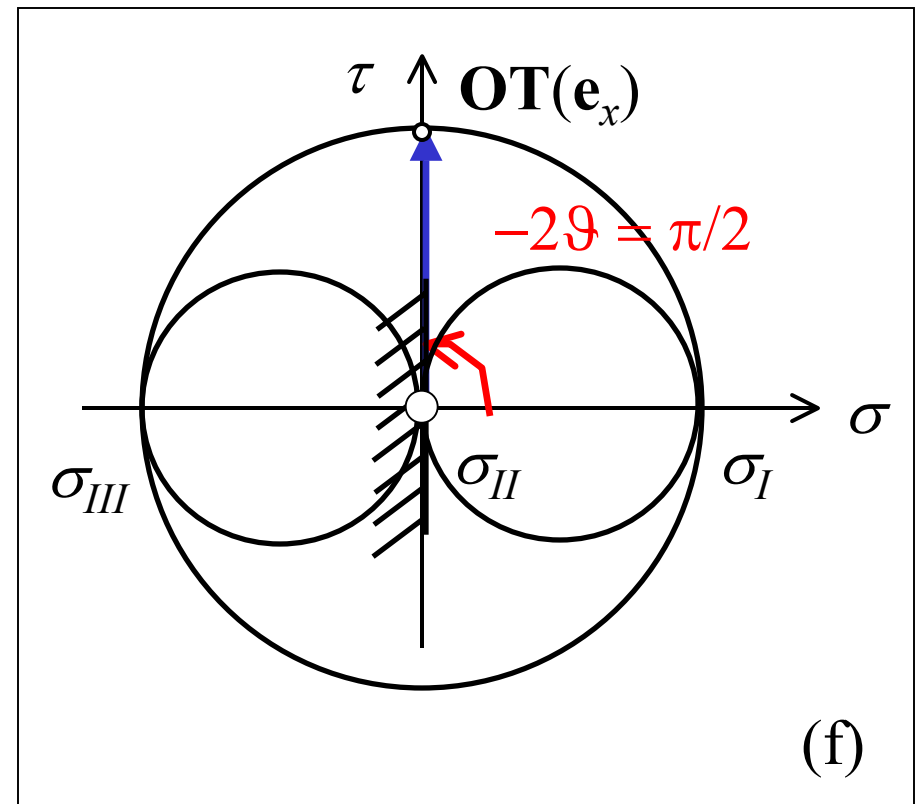
*Mohr Stress Plane*

# Selected Stress States: Pure Shear

$$\boldsymbol{\sigma} = \tau (\mathbf{e}_x \otimes \mathbf{e}_y + \mathbf{e}_y \otimes \mathbf{e}_x)$$



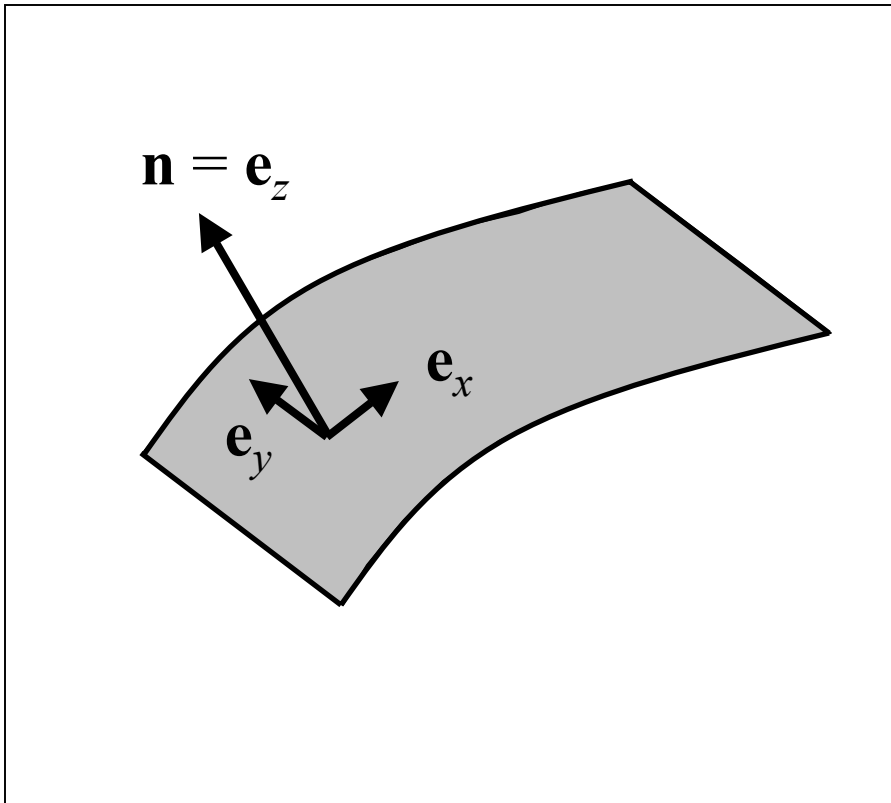
*Material Plane*



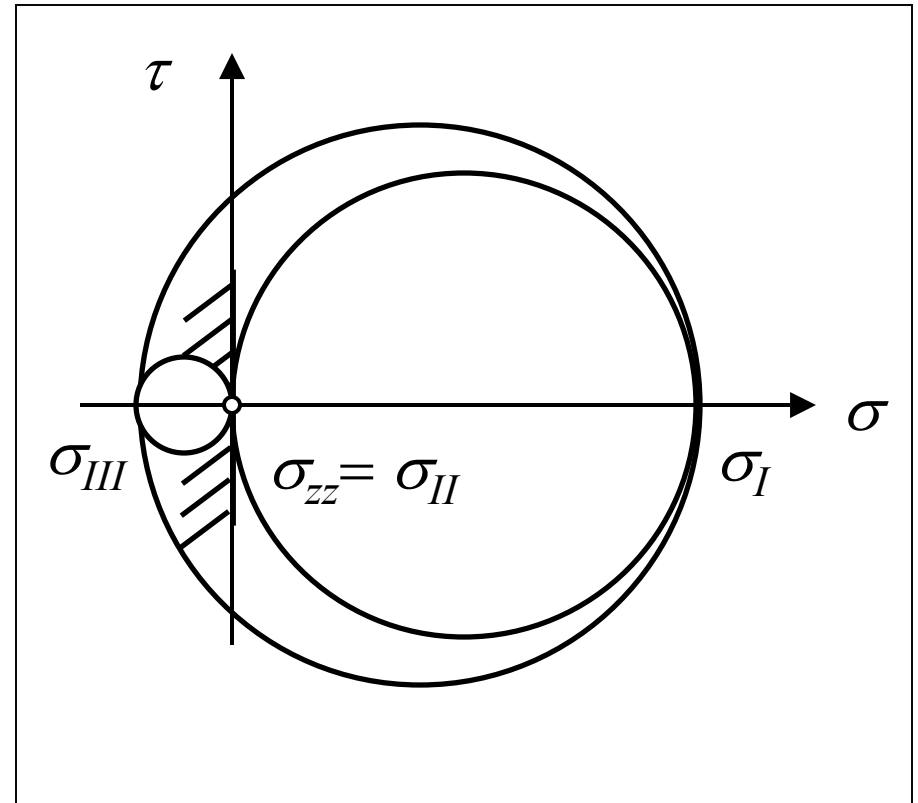
*Mohr Stress Plane*

# Selected Stress States: Plane Stress

$$\mathbf{T}(\mathbf{n}=\mathbf{e}_z) = \boldsymbol{\sigma} \cdot \mathbf{e}_z = \mathbf{0}$$

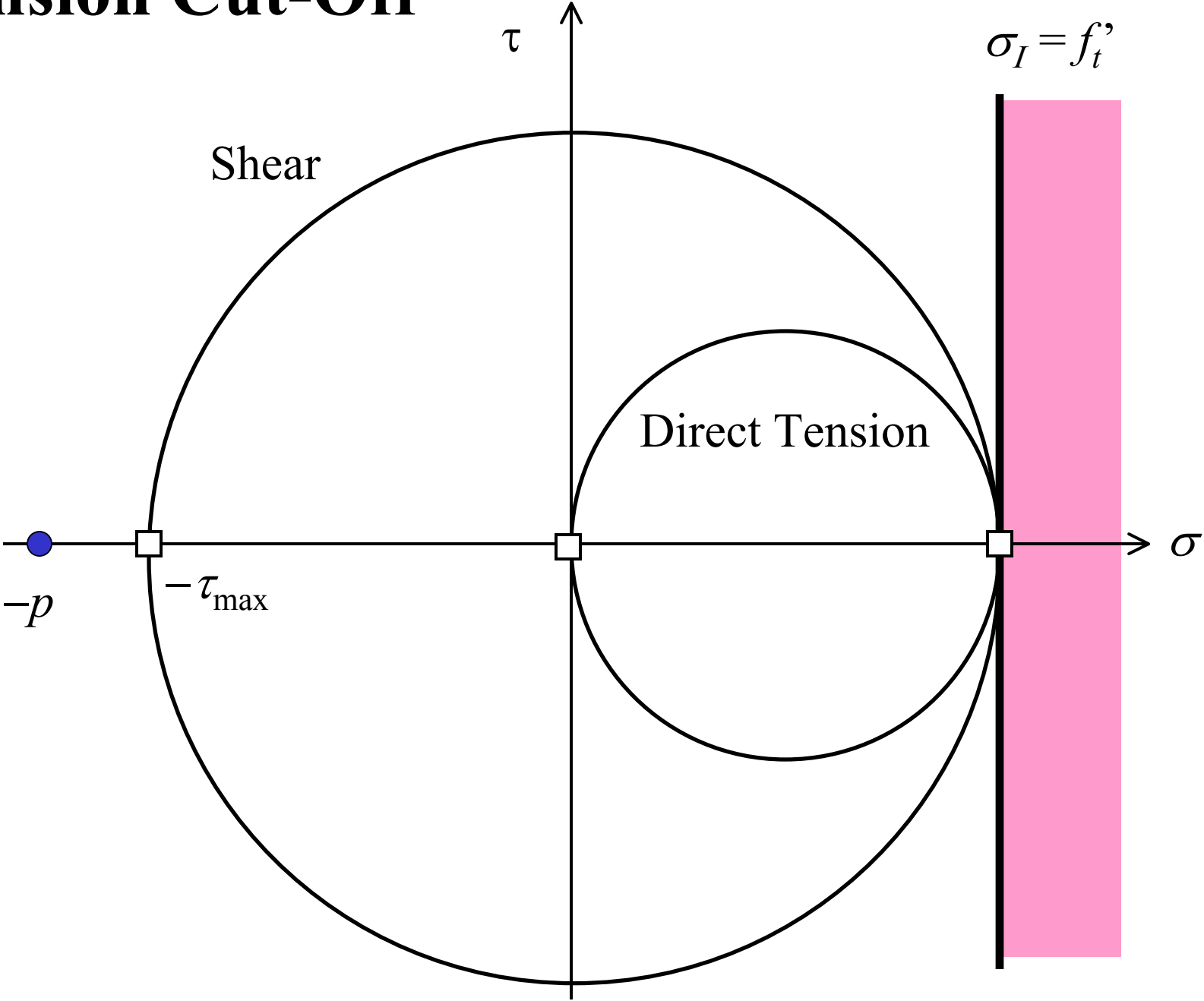


*Material Plane*

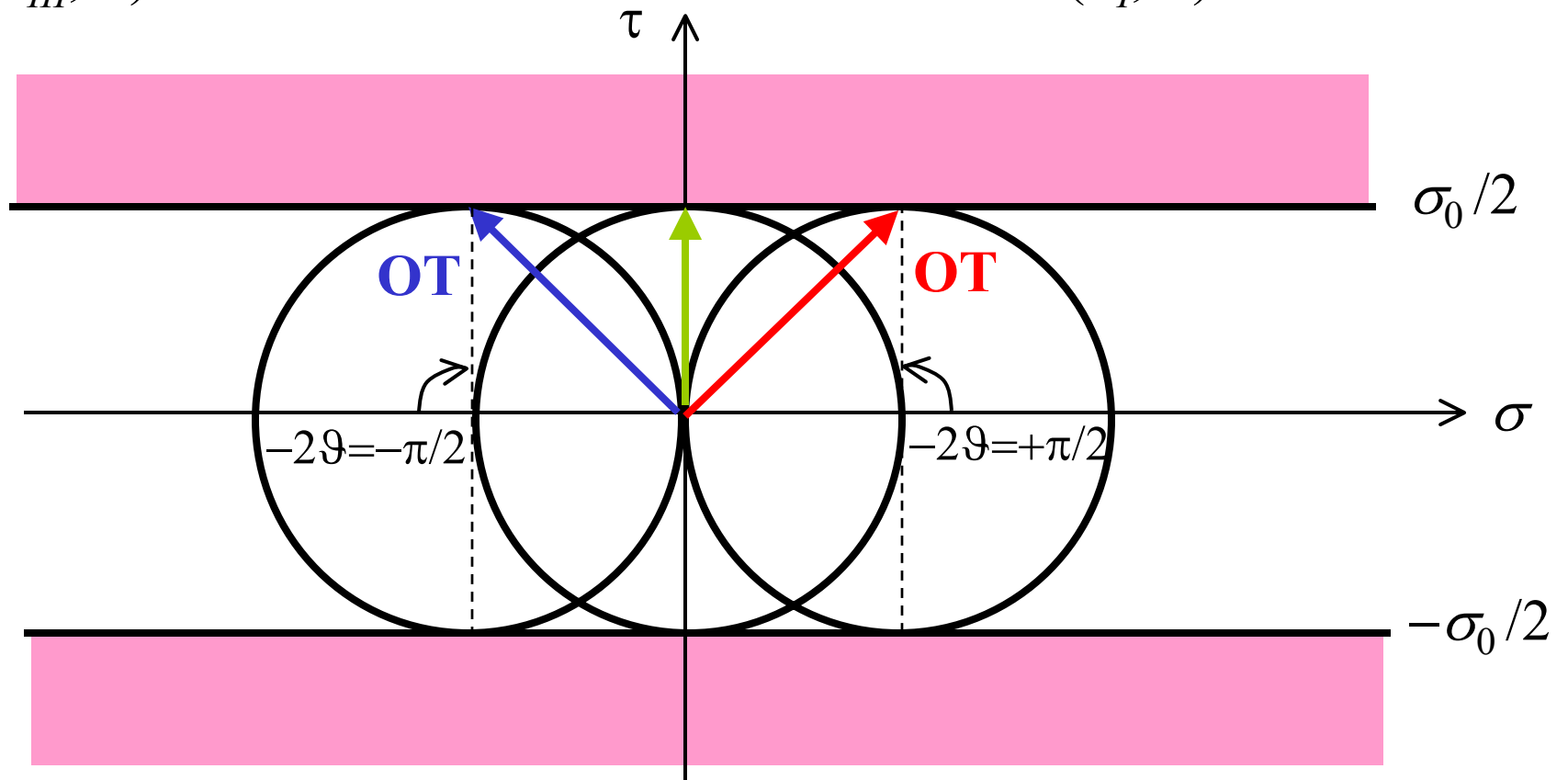
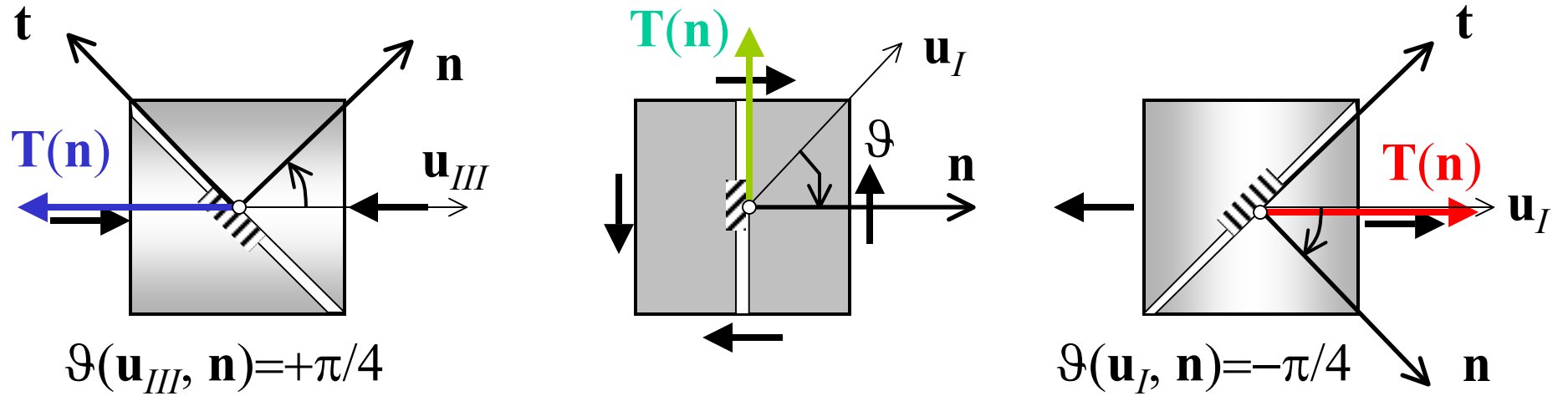


*Mohr Stress Plane*

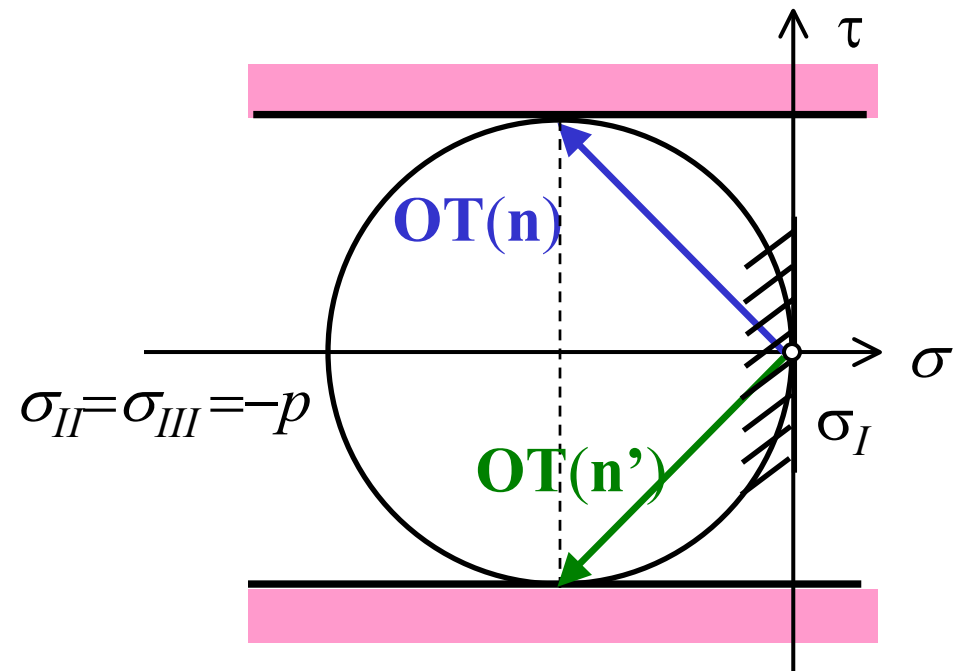
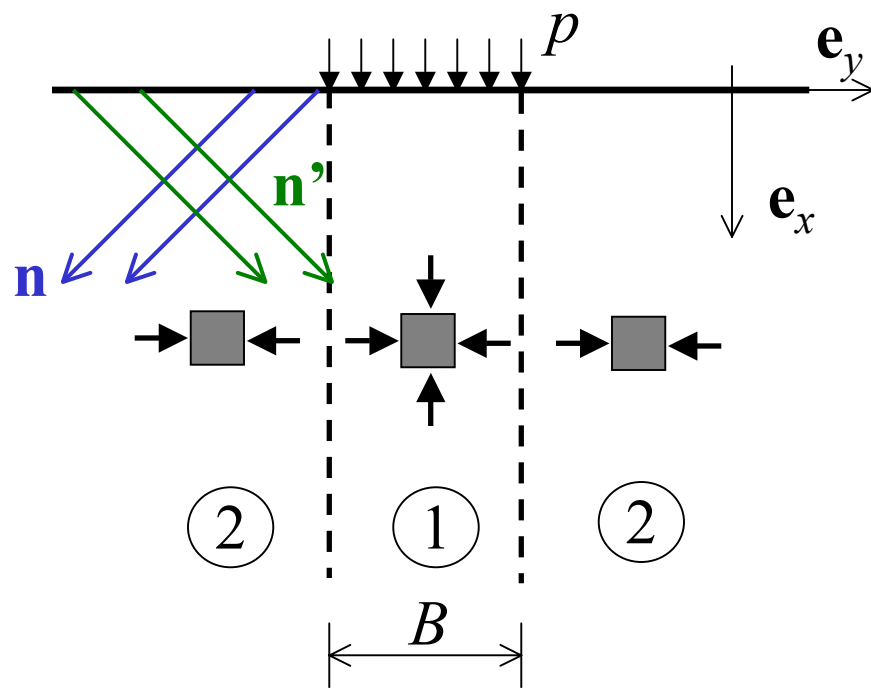
# Tension Cut-Off



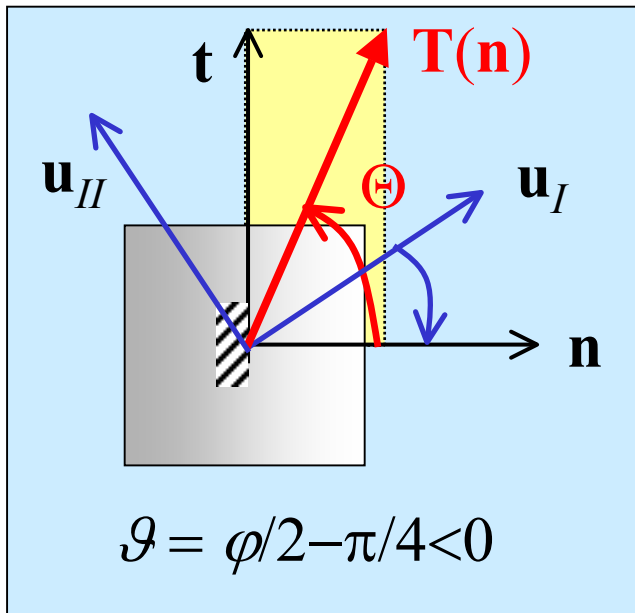
# Tresca Criterion



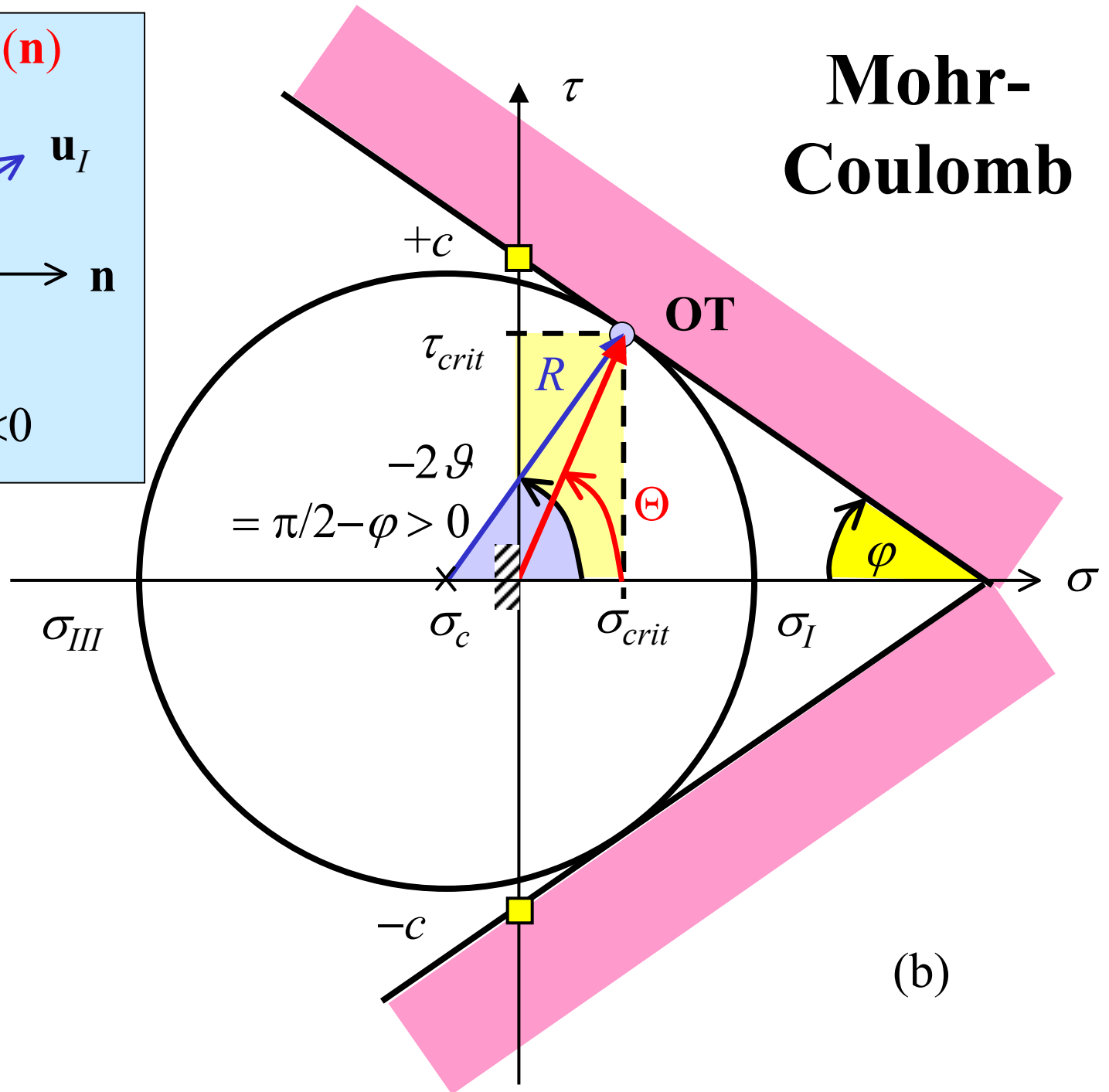
# Tresca Criterion: Application



# Mohr-Coulomb

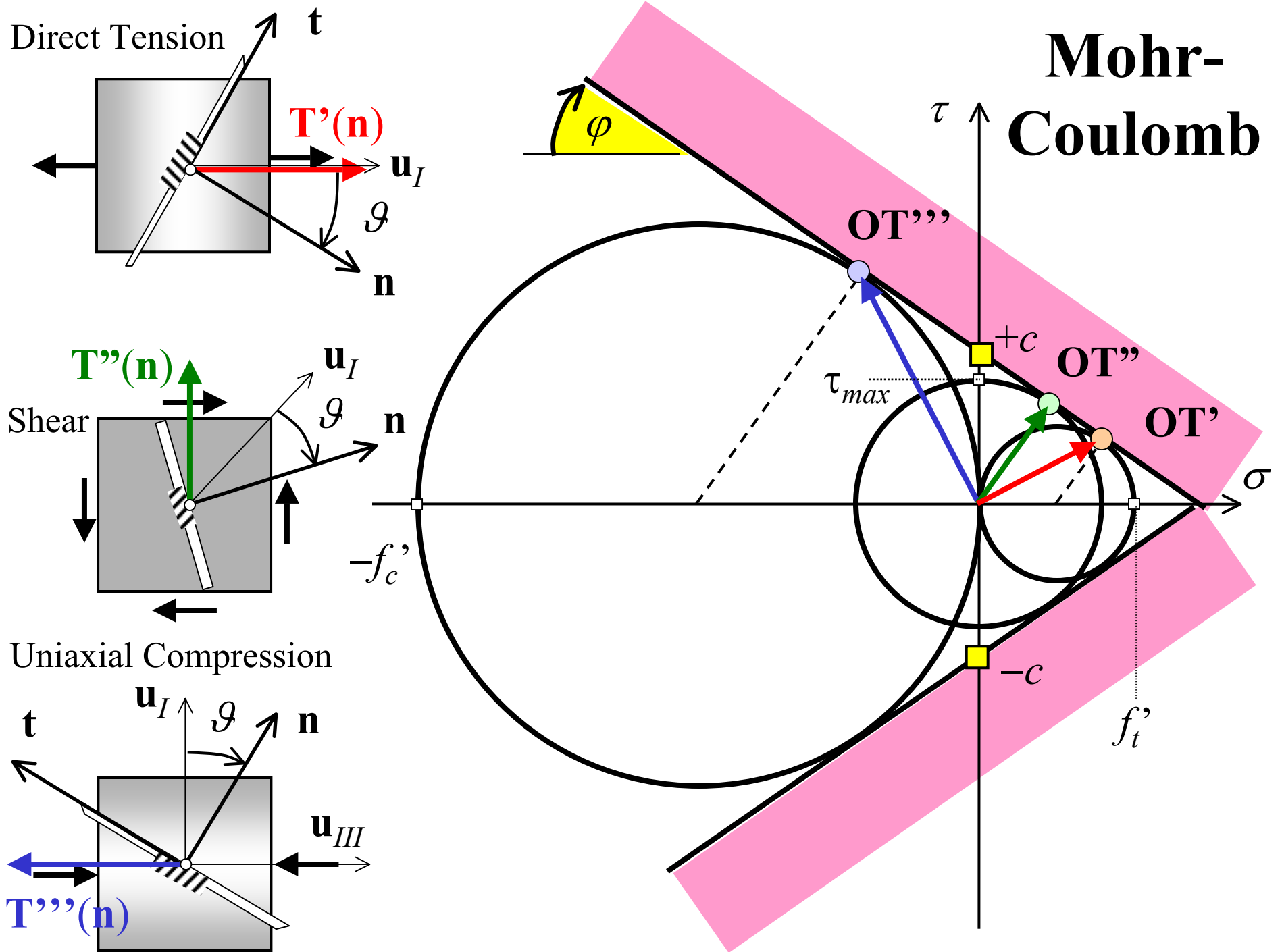


(a)



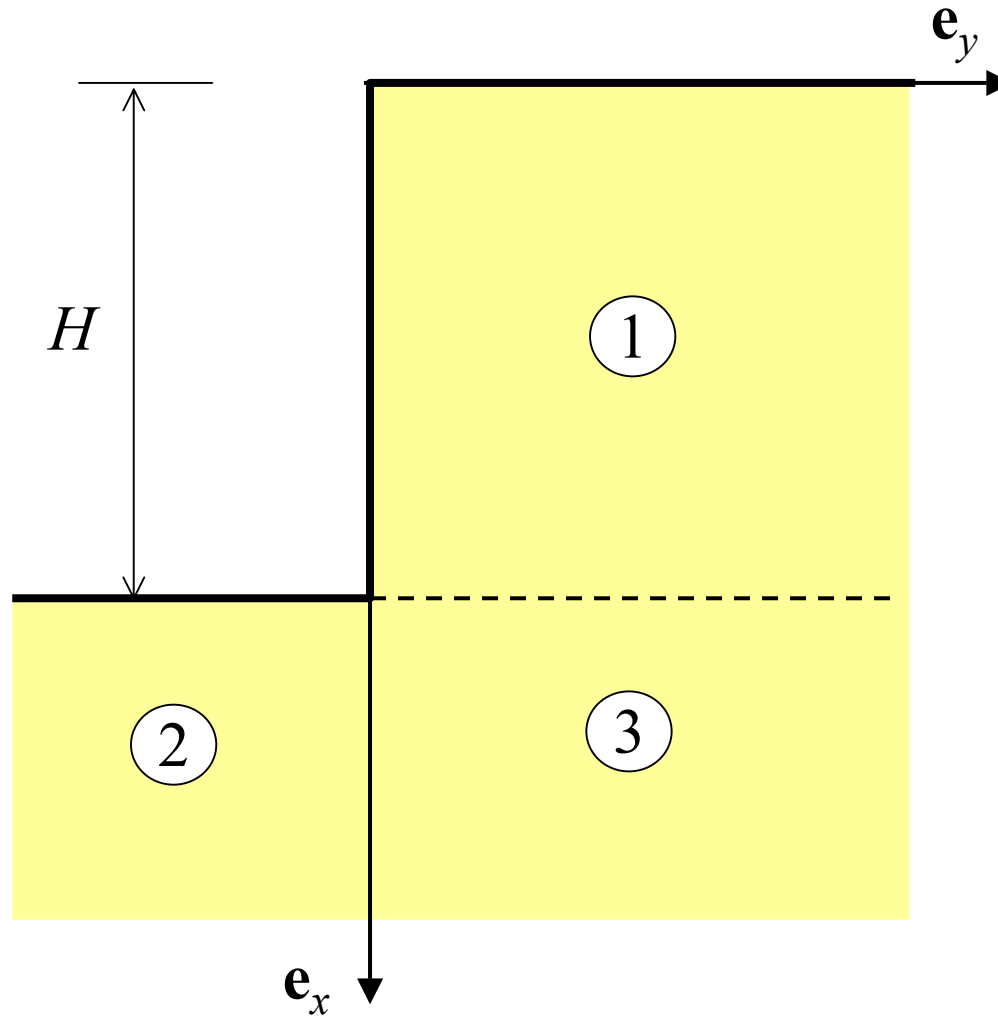
(b)

# Mohr-Coulomb

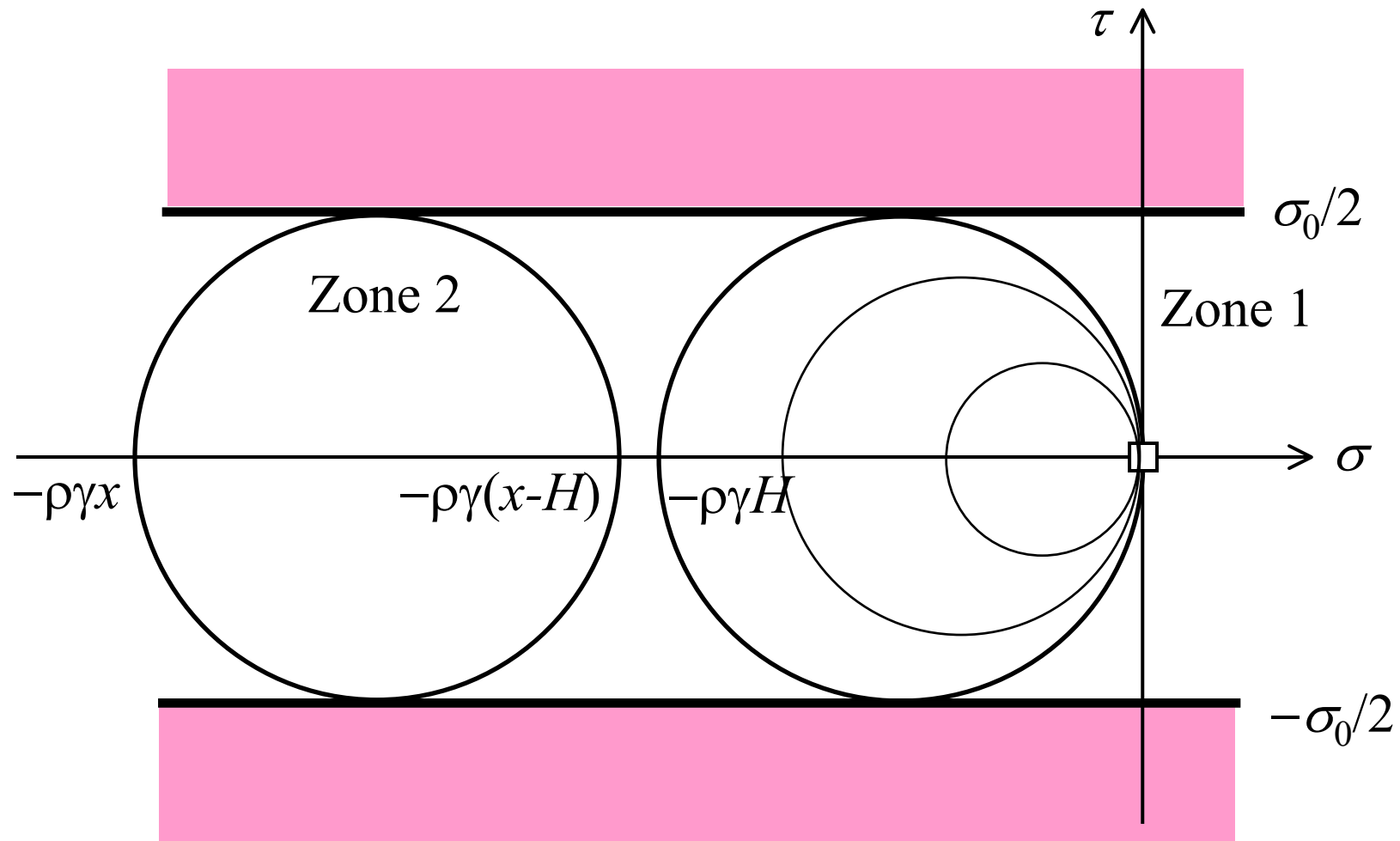




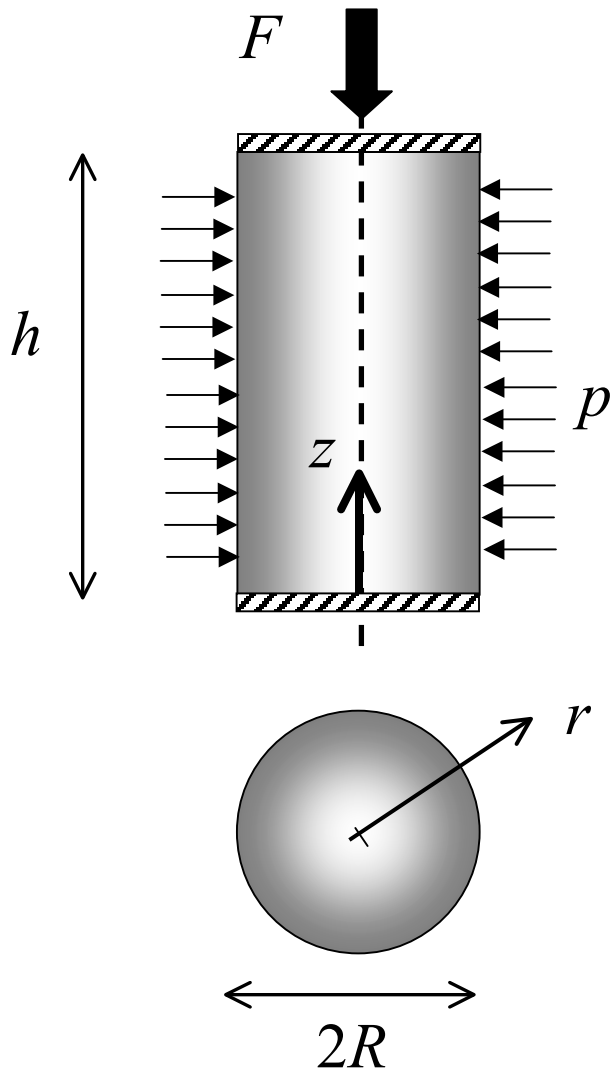
# Training Set: Excavation Set



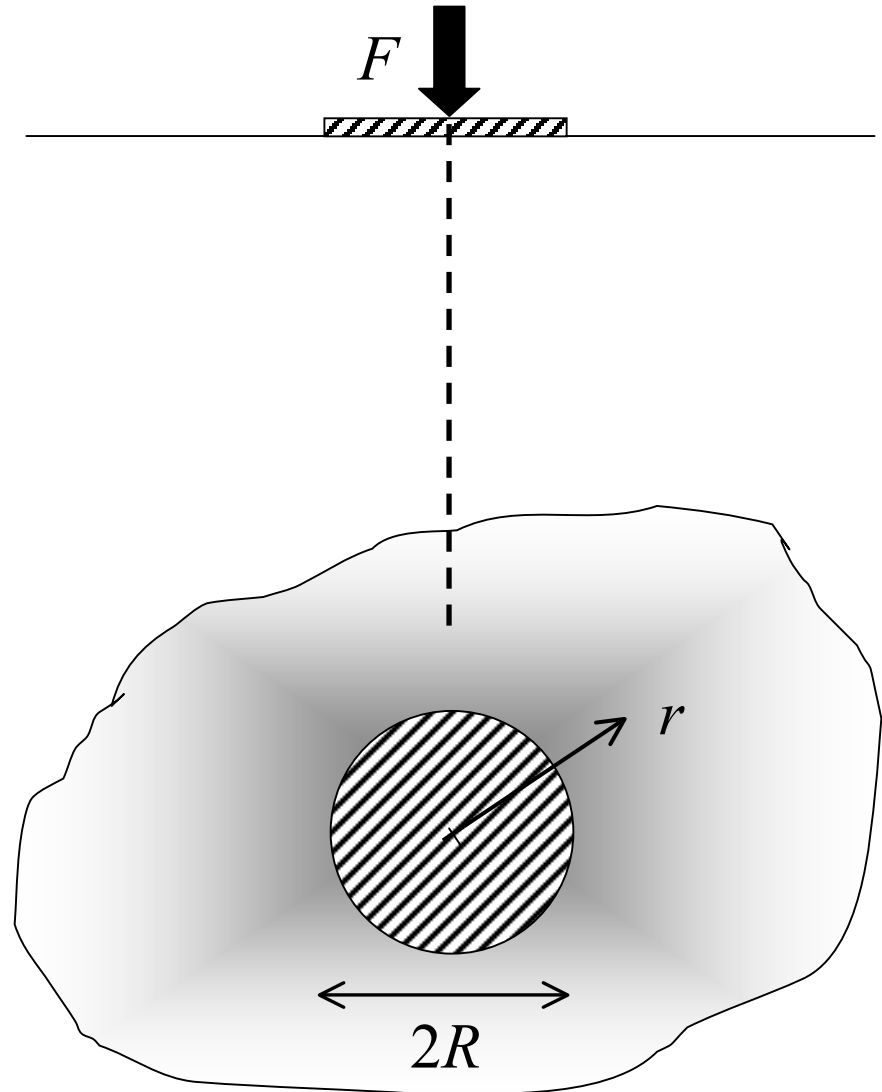
# Max. Excavation Depth of Tresca Material



# Homework Set #2



Part I: Triaxial Test



Part II: Circular Foundation