# Supply Contracts

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# Outline

- ☐ Supply contracts
- ☐ Wholesale contracts
- ☐ Buyback contracts
- ☐ Revenue sharing contracts
- ☐ Option contracts

# The Scenario:

| Supplier | → | Retailer | → | [histogram chart] |
|----------|---|----------|---|-------------------|

- A contract is negotiated
- The retailer places order (a single period)
- The supplier makes and sends the stuff
- The selling season takes place
- Accounting (sales, salvage, etc.)

# Consumer Demand



**Demand Density at Retail Level**

Y-axis: Probability (0, 2, 4, 6, 8, 10, 12, 14, 16)

X-axis: Actual Demand (400, 500, 600, 700, 800, 900, 1,000, 1,100, 1,200, 1,300)
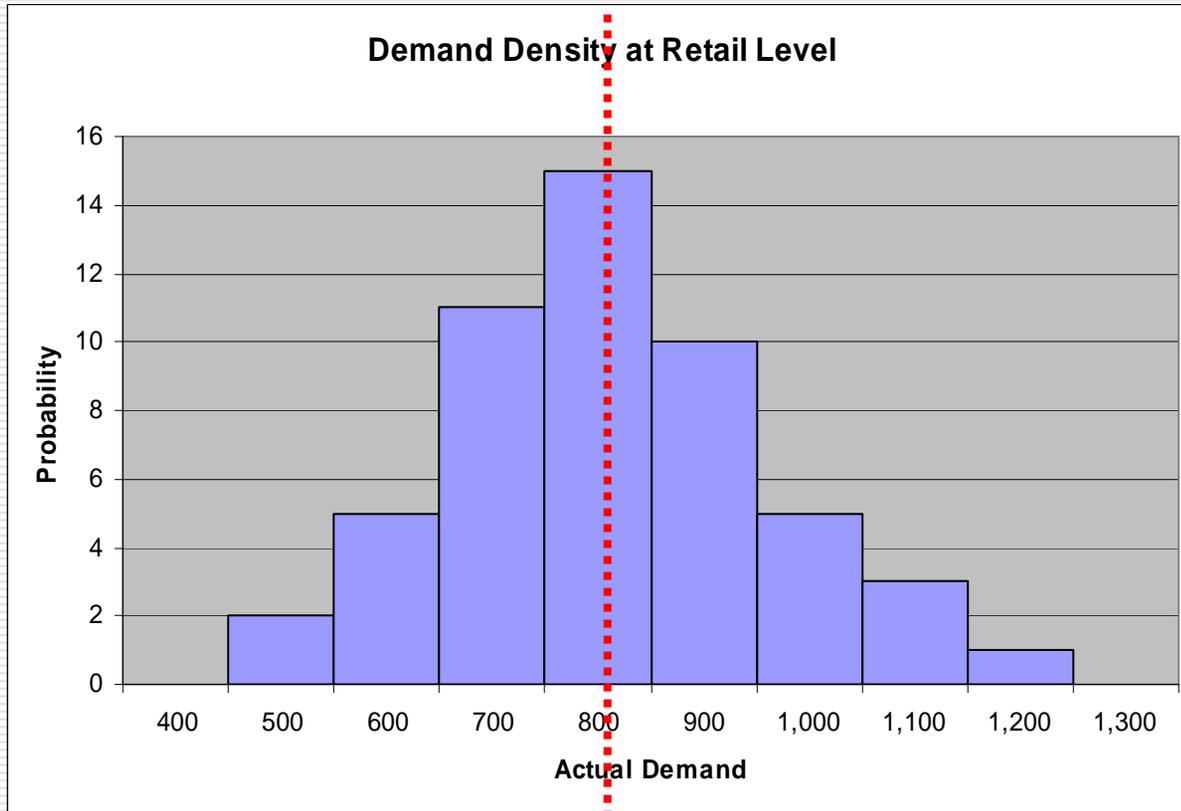
| Bin | Freq. |
|---|---|
| 400 | 0 |
| 500 | 2 |
| 600 | 5 |
| 700 | 11 |
| 800 | 15 |
| 900 | 10 |
| 1,000 | 5 |
| 1,100 | 3 |
| 1,200 | 1 |
| 1,300 | 0 |

Average: 811.54
Std Dev: 154.23

# Demand Distribution

Normal Approximation
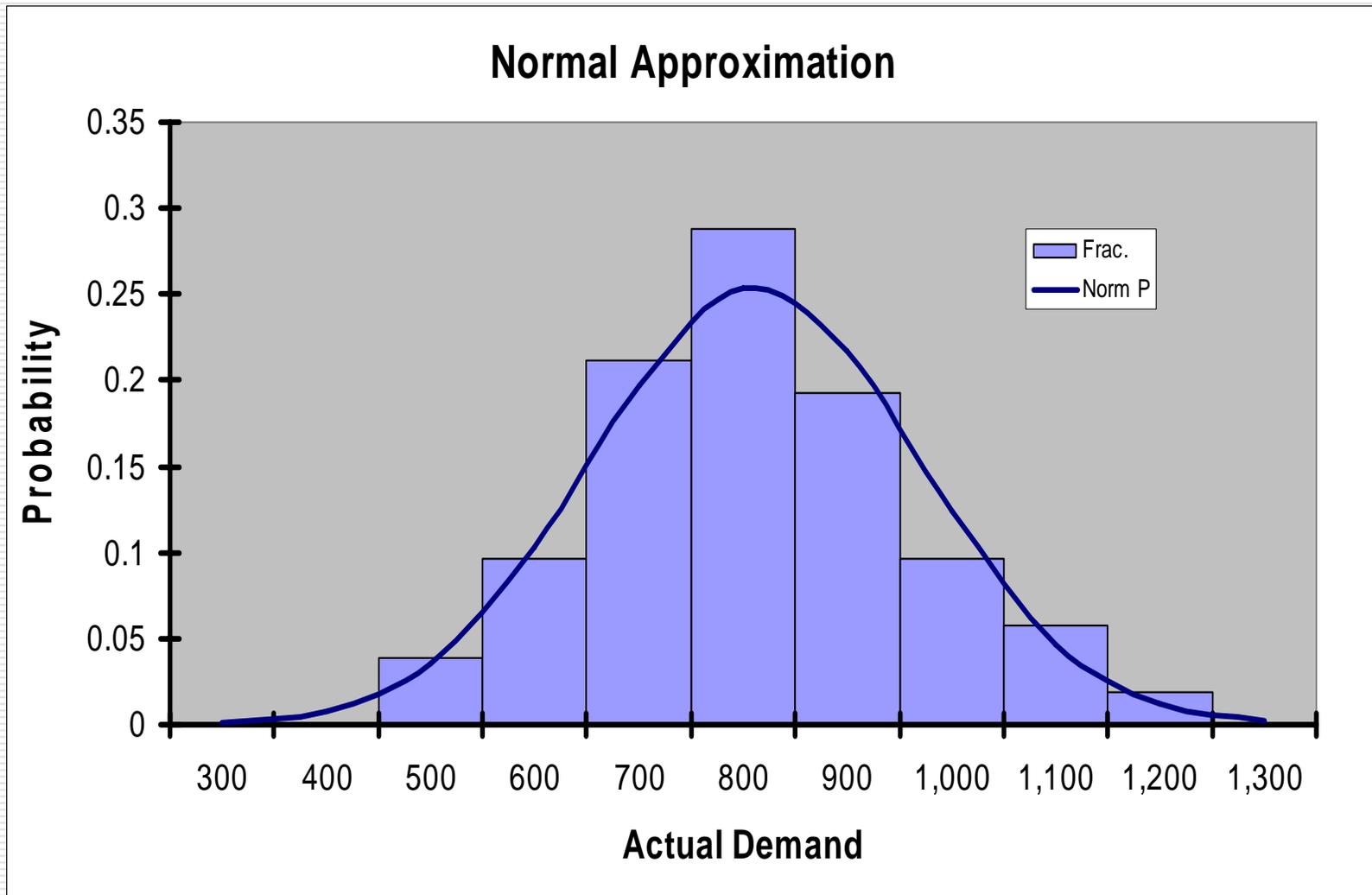
# Notations:

| | |
|---|---|
| Retail Price | $R$ |
| Wholesale Price | $W$ |
| Supplier's Cost | $C$ |
| Salvage Value | $S$ |
| Quantity Ordered | $Q$ |
| Actual Retail Demand | $d$ |
| Retail Demand Density | $f(D)$ |
| Retail Cum. Demand Distribution | $F(D)$ |

# Wholesale Contracts

- Prices and costs:
  - Supplier has a cost to make/purchase (C=$50)
  - Supplier is selling and retail is buying at a wholesale price (W=$135)
  - Retailer is selling for a retail price (R=$200)
  - Retailer can salvage (S=$10)
- How do we optimize the retailer's expected profit?
  - Retailer is facing a Newsboy problem and supplier's profit is trivial
- How do we optimize the supplier's expected profit?

# Wholesale Price Contract

**Retailer:**

| Demand | Prob | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 | 1000 | 1100 | 1200 | 1300 | 1400 | 1500 | 1600 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 400 | 0.00 | $13,000 | $19,500 | $26,000 | $13,500 | $1,000 | -$11,500 | -$24,000 | -$36,500 | -$49,000 | -$61,500 | -$74,000 | -$86,500 | -$99,000 | -$111,500 | -$124,000 |
| 500 | 0.04 | $13,000 | $19,500 | $26,000 | $32,500 | $20,000 | $7,500 | -$5,000 | -$17,500 | -$30,000 | -$42,500 | -$55,000 | -$67,500 | -$80,000 | -$92,500 | -$105,000 |
| 600 | 0.10 | $13,000 | $19,500 | $26,000 | $32,500 | $39,000 | $26,500 | $14,000 | $1,500 | -$11,000 | -$23,500 | -$36,000 | -$48,500 | -$61,000 | -$73,500 | -$86,000 |
| 700 | 0.21 | $13,000 | $19,500 | $26,000 | $32,500 | $39,000 | $45,500 | $33,000 | $20,500 | $8,000 | -$4,500 | -$17,000 | -$29,500 | -$42,000 | -$54,500 | -$67,000 |
| 800 | 0.29 | $13,000 | $19,500 | $26,000 | $32,500 | $39,000 | $45,500 | $52,000 | $39,500 | $27,000 | $14,500 | $2,000 | -$10,500 | -$23,000 | -$35,500 | -$48,000 |
| 900 | 0.19 | $13,000 | $19,500 | $26,000 | $32,500 | $39,000 | $45,500 | $52,000 | $58,500 | $46,000 | $33,500 | $21,000 | $8,500 | -$4,000 | -$16,500 | -$29,000 |
| 1,000 | 0.10 | $13,000 | $19,500 | $26,000 | $32,500 | $39,000 | $45,500 | $52,000 | $58,500 | $65,000 | $52,500 | $40,000 | $27,500 | $15,000 | $2,500 | -$10,000 |
| 1,100 | 0.06 | $13,000 | $19,500 | $26,000 | $32,500 | $39,000 | $45,500 | $52,000 | $58,500 | $65,000 | $71,500 | $59,000 | $46,500 | $34,000 | $21,500 | $9,000 |
| 1,200 | 0.02 | $13,000 | $19,500 | $26,000 | $32,500 | $39,000 | $45,500 | $52,000 | $58,500 | $65,000 | $71,500 | $78,000 | $65,500 | $53,000 | $40,500 | $28,000 |
| 1,300 | 0.00 | $13,000 | $19,500 | $26,000 | $32,500 | $39,000 | $45,500 | $52,000 | $58,500 | $65,000 | $71,500 | $78,000 | $84,500 | $72,000 | $59,500 | $47,000 |
| Expected Profit: | | $13,000 | $19,500 | $26,000 | $32,500 | $38,269 | $42,212 | $42,135 | $36,577 | $27,365 | $16,327 | $4,192 | -$8,308 | -$20,808 | -$33,308 | -$45,808 |
| Maximum Profit: | | | | | | | 1 | | | | | | | | | |

**Supplier:**

| Demand | Prob | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 | 1000 | 1100 | 1200 | 1300 | 1400 | 1500 | 1600 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 400 | 0.00 | $17,000 | $25,500 | $34,000 | $42,500 | $51,000 | $59,500 | $68,000 | $76,500 | $85,000 | $93,500 | $102,000 | $110,500 | $119,000 | $127,500 | $136,000 |
| 500 | 0.04 | $17,000 | $25,500 | $34,000 | $42,500 | $51,000 | $59,500 | $68,000 | $76,500 | $85,000 | $93,500 | $102,000 | $110,500 | $119,000 | $127,500 | $136,000 |
| 600 | 0.10 | $17,000 | $25,500 | $34,000 | $42,500 | $51,000 | $59,500 | $68,000 | $76,500 | $85,000 | $93,500 | $102,000 | $110,500 | $119,000 | $127,500 | $136,000 |
| 700 | 0.21 | $17,000 | $25,500 | $34,000 | $42,500 | $51,000 | $59,500 | $68,000 | $76,500 | $85,000 | $93,500 | $102,000 | $110,500 | $119,000 | $127,500 | $136,000 |
| 800 | 0.29 | $17,000 | $25,500 | $34,000 | $42,500 | $51,000 | $59,500 | $68,000 | $76,500 | $85,000 | $93,500 | $102,000 | $110,500 | $119,000 | $127,500 | $136,000 |
| 900 | 0.19 | $17,000 | $25,500 | $34,000 | $42,500 | $51,000 | $59,500 | $68,000 | $76,500 | $85,000 | $93,500 | $102,000 | $110,500 | $119,000 | $127,500 | $136,000 |
| 1,000 | 0.10 | $17,000 | $25,500 | $34,000 | $42,500 | $51,000 | $59,500 | $68,000 | $76,500 | $85,000 | $93,500 | $102,000 | $110,500 | $119,000 | $127,500 | $136,000 |
| 1,100 | 0.06 | $17,000 | $25,500 | $34,000 | $42,500 | $51,000 | $59,500 | $68,000 | $76,500 | $85,000 | $93,500 | $102,000 | $110,500 | $119,000 | $127,500 | $136,000 |
| 1,200 | 0.02 | $17,000 | $25,500 | $34,000 | $42,500 | $51,000 | $59,500 | $68,000 | $76,500 | $85,000 | $93,500 | $102,000 | $110,500 | $119,000 | $127,500 | $136,000 |
| 1,300 | 0.00 | $17,000 | $25,500 | $34,000 | $42,500 | $51,000 | $59,500 | $68,000 | $76,500 | $85,000 | $93,500 | $102,000 | $110,500 | $119,000 | $127,500 | $136,000 |
| Expected Profit: | | $17,000 | $25,500 | $34,000 | $42,500 | $51,000 | $59,500 | $68,000 | $76,500 | $85,000 | $93,500 | $102,000 | $110,500 | $119,000 | $127,500 | $136,000 |
| Maximum Profit: | | | | | | | | | | | | | | | | !!! |

| | | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 | 1000 | 1100 | 1200 | 1300 | 1400 | 1500 | 1600 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Total Profit:** | | **$30,000** | **$45,000** | **$60,000** | **$75,000** | **$89,269** | **$101,712** | **$110,135** | **$113,077** | **$112,365** | **$109,827** | **$106,192** | **$102,192** | **$98,192** | **$94,192** | **$90,192** |
| Maximum Profit | | | | | | | | | !!! | | | | | | | |

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# Wholesale Price Contract
# Expected Profits
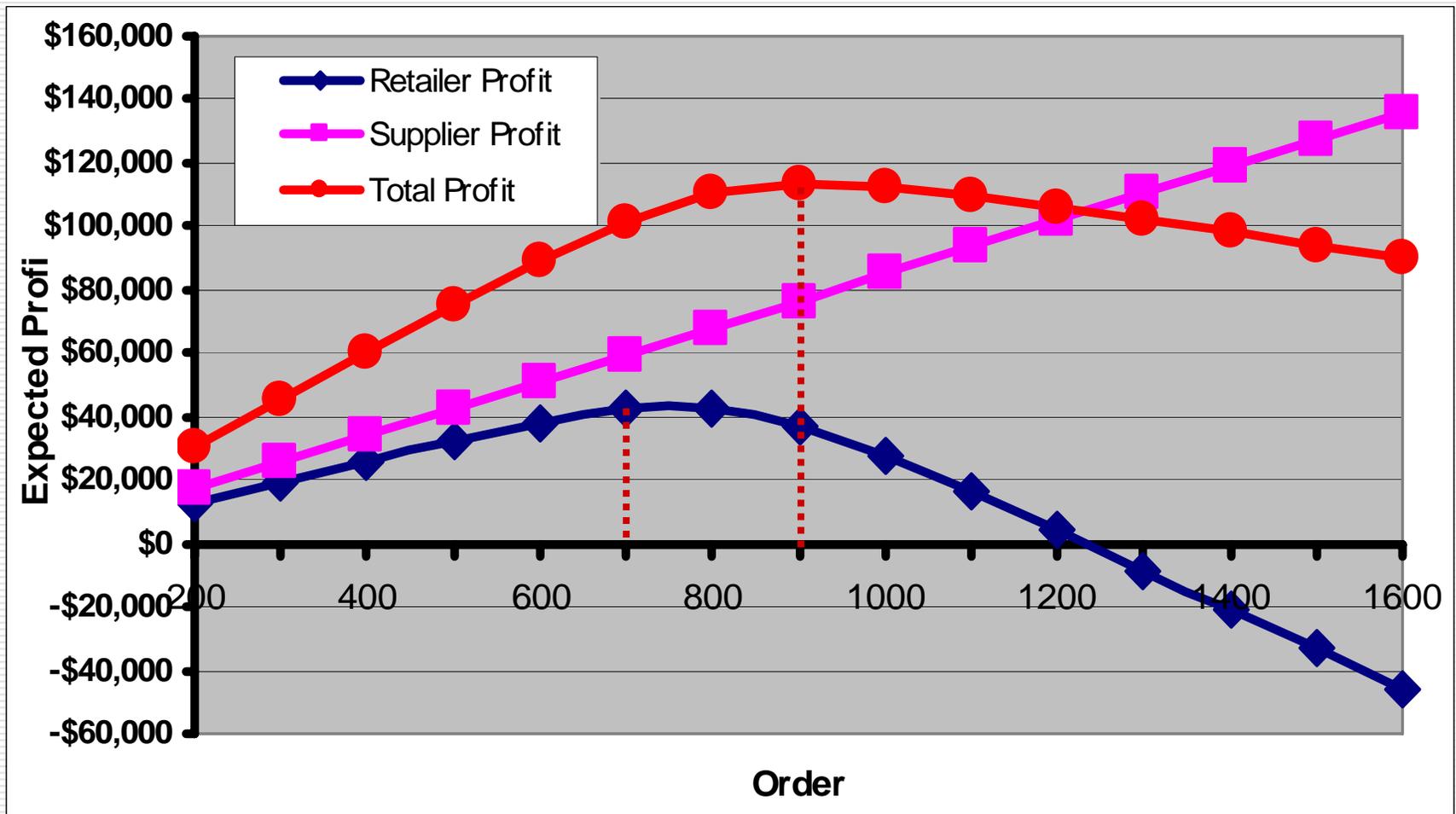
# Wholesale Price Contract
# Exp Profits (Normal Approx)

R=$200
W=$135
C=$50
S=$10



Retail Order: 749
Retailer Profit $41,988
Supplier Profit: $63,649
Total Profit: $105,637

Optimal Order: 936
Retailer Profit $33,745
Supplier Profit: $79,528
Total Profit: $113,273
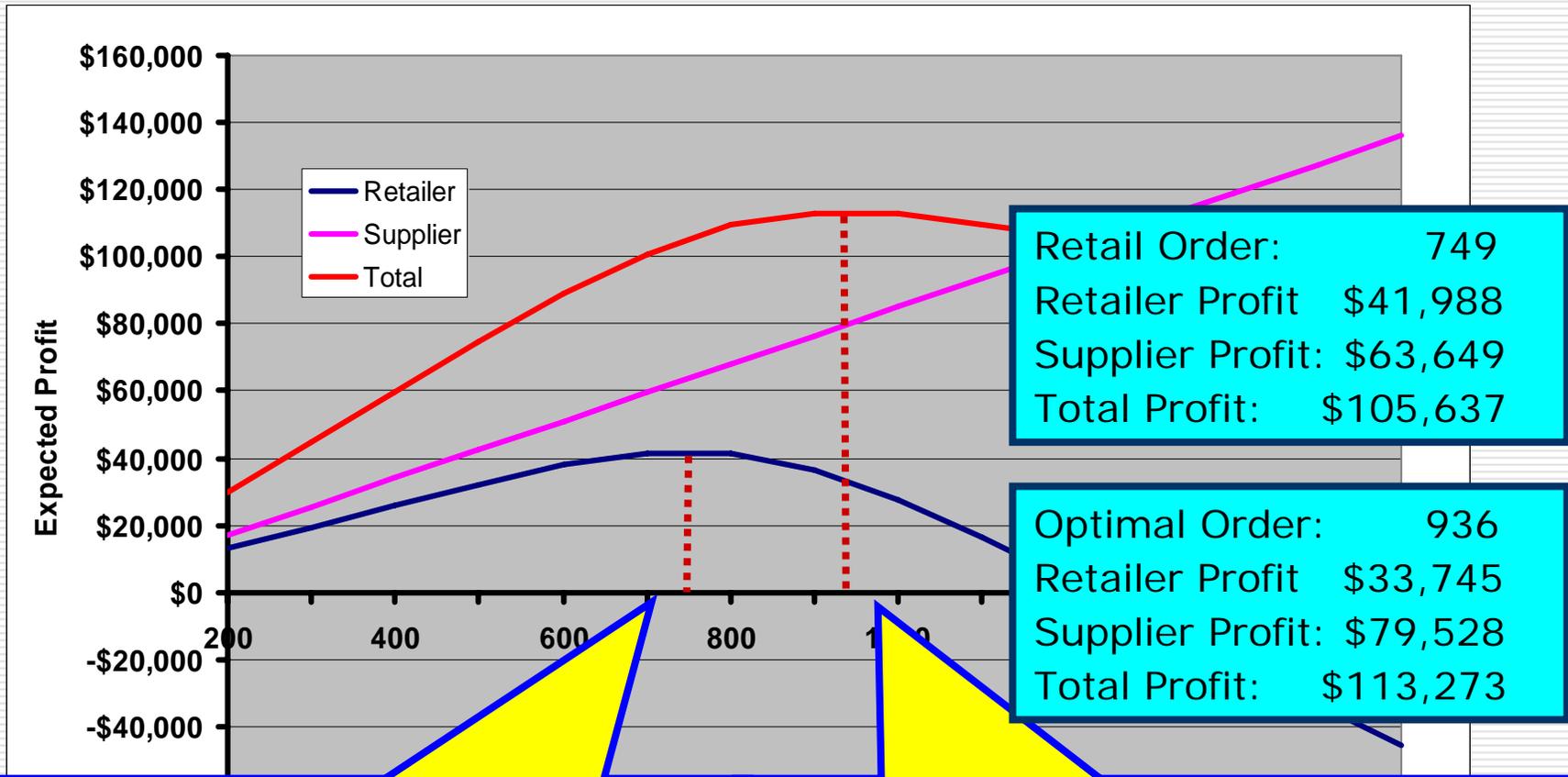
Optimal Retailer order=
$$F^{-1}\left(\frac{R-W}{R-S}\right) = NORMINV\left(\frac{200-135}{200-10}, 811,154\right) = 746$$

Optimal channel order=
$$F^{-1}\left(\frac{R-C}{R-S}\right) = NORMINV\left(\frac{200-50}{200-10}, 811,154\right) = 936$$

# Effects of Wholesale Price on Profits
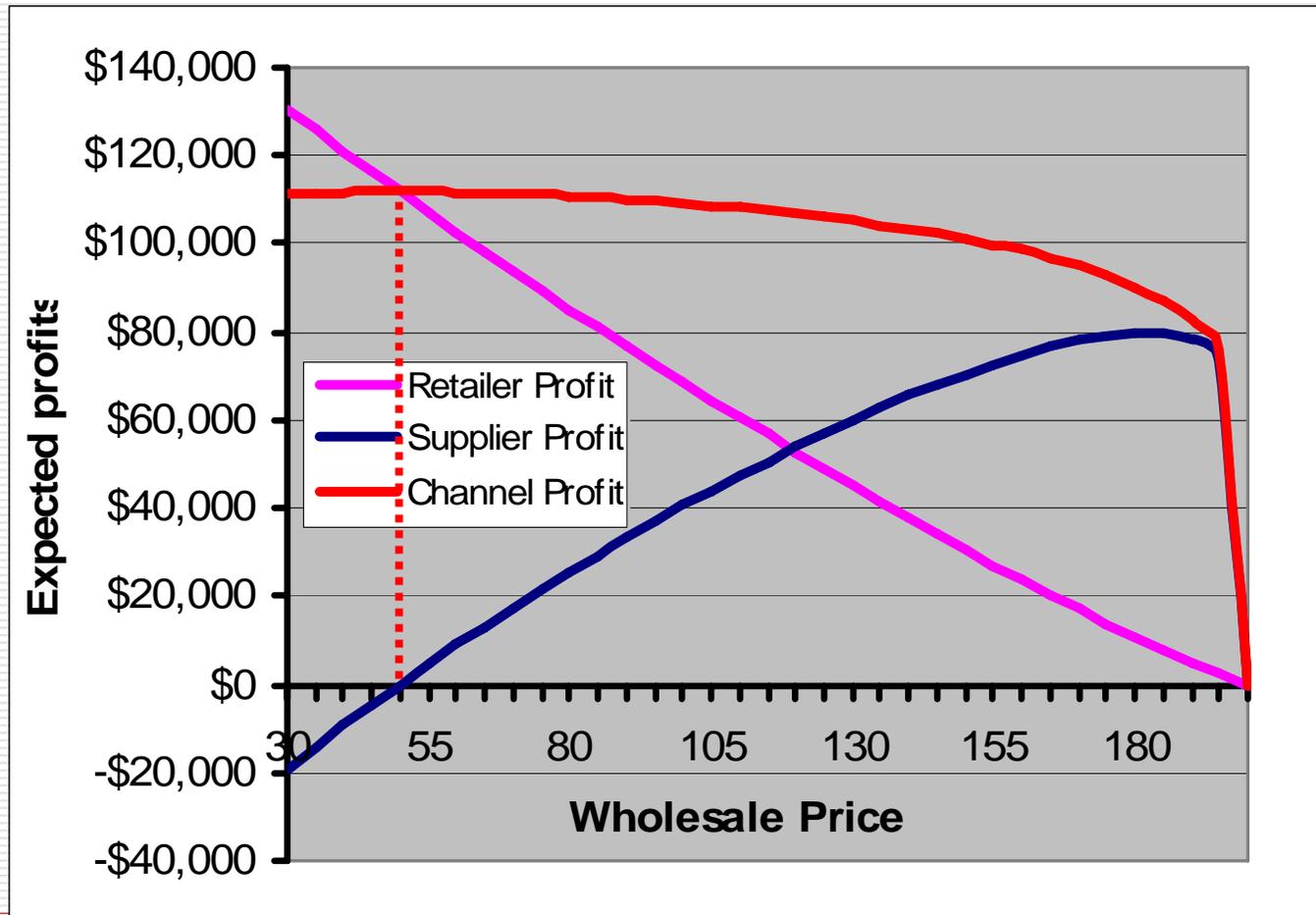
# Optimal Retail Order as a Function of the Wholesale Price
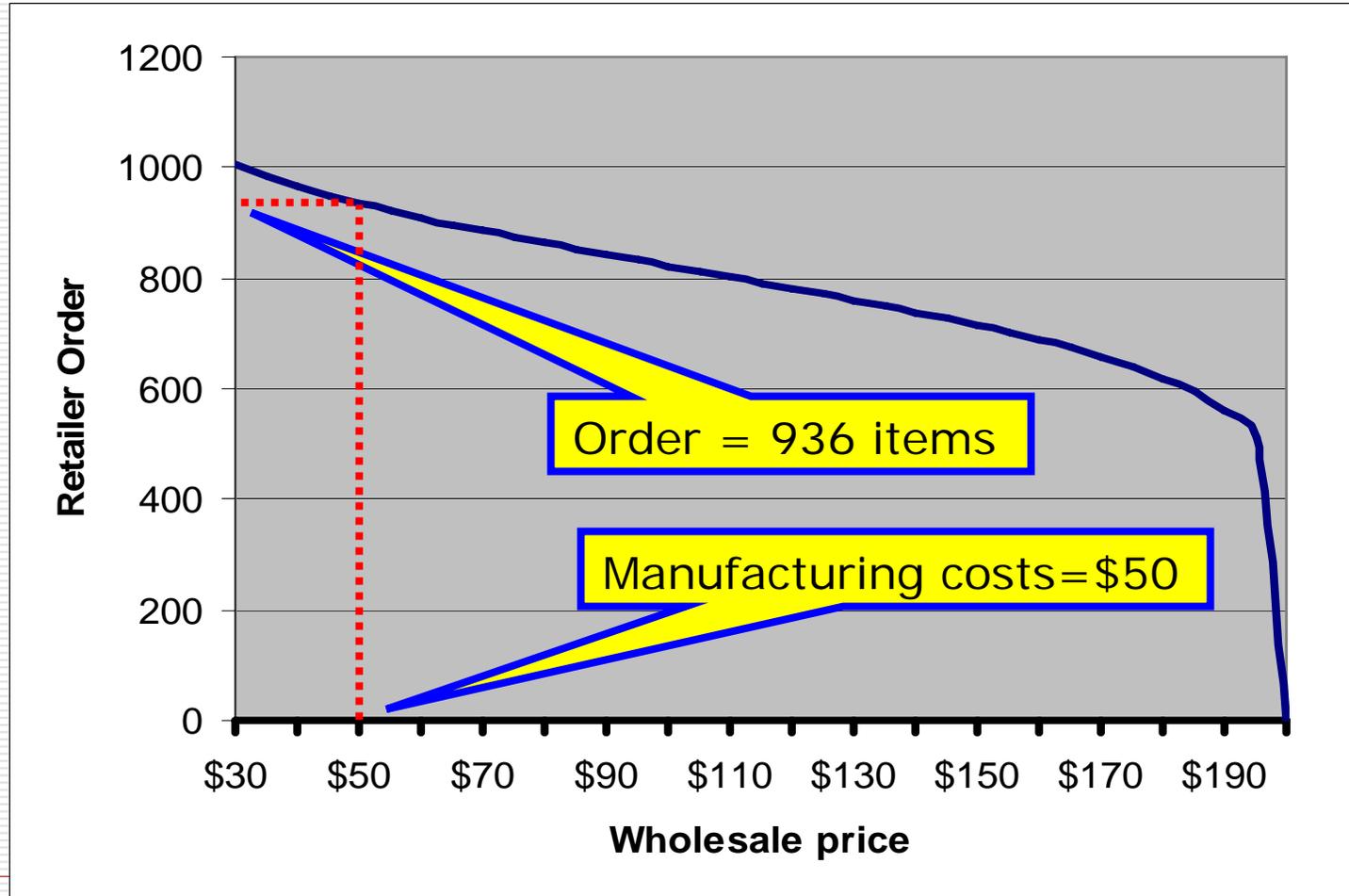
# Wholesale Price Contract
# Coordinating the Channel

$$Q^*_{\text{Re }tailer} = Q^*_{Channel}$$

$$F^{-1}\left(\frac{R-W}{R-S}\right) = F^{-1}\left(\frac{R-C}{R-S}\right)$$

$$\left(\frac{R-W}{R-S}\right) = \left(\frac{R-C}{R-S}\right)$$

$$W = C$$

# Buyback Contract

- The problem: how can the supplier convince the retailer to move towards the optimal order size?
- The supplier offer to the retailer to buy back all unsold items ($B/item).
- For the retailer – this is like a higher salvage value, so he will order more.
- The supplier now shares in the overage risk (he can still salvage, though, at the same price).
- Note: supplier may simply pay ($B-$S) rather than actually buy back (unless he has a better use for it)

# Buyback Contract Calculations

**Retailer:**

| Demand | Prob | Order: 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 | 1000 | 1100 | 1200 | 1300 | 1400 | 1500 | 1600 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 400 | 0.00 | $13,000 | $19,500 | $26,000 | $20,500 | $15,000 | $9,500 | $4,000 | -$1,500 | -$7,000 | -$12,500 | -$18,000 | -$23,500 | -$29,000 | -$34,500 | -$40,000 |
| 500 | 0.04 | $13,000 | $19,500 | $26,000 | $32,500 | $27,000 | $21,500 | $16,000 | $10,500 | $5,000 | -$500 | -$6,000 | -$11,500 | -$17,000 | -$22,500 | -$28,000 |
| 600 | 0.10 | $13,000 | $19,500 | $26,000 | $32,500 | $39,000 | $33,500 | $28,000 | $22,500 | $17,000 | $11,500 | $6,000 | $500 | -$5,000 | -$10,500 | -$16,000 |
| 700 | 0.21 | $13,000 | $19,500 | $26,000 | $32,500 | $39,000 | $45,500 | $40,000 | $34,500 | $29,000 | $23,500 | $18,000 | $12,500 | $7,000 | $1,500 | -$4,000 |
| 800 | 0.29 | $13,000 | $19,500 | $26,000 | $32,500 | $39,000 | $45,500 | $52,000 | $46,500 | $41,000 | $35,500 | $30,000 | $24,500 | $19,000 | $13,500 | $8,000 |
| 900 | 0.19 | $13,000 | $19,500 | $26,000 | $32,500 | $39,000 | $45,500 | $52,000 | $58,500 | $53,000 | $47,500 | $42,000 | $36,500 | $31,000 | $25,500 | $20,000 |
| 1,000 | 0.10 | $13,000 | $19,500 | $26,000 | $32,500 | $39,000 | $45,500 | $52,000 | $58,500 | $65,000 | $59,500 | $54,000 | $48,500 | $43,000 | $37,500 | $32,000 |
| 1,100 | 0.06 | $13,000 | $19,500 | $26,000 | $32,500 | $39,000 | $45,500 | $52,000 | $58,500 | $65,000 | $71,500 | $66,000 | $60,500 | $55,000 | $49,500 | $44,000 |
| 1,200 | 0.02 | $13,000 | $19,500 | $26,000 | $32,500 | $39,000 | $45,500 | $52,000 | $58,500 | $65,000 | $71,500 | $78,000 | $72,500 | $67,000 | $61,500 | $56,000 |
| 1,300 | 0.00 | $13,000 | $19,500 | $26,000 | $32,500 | $39,000 | $45,500 | $52,000 | $58,500 | $65,000 | $71,500 | $78,000 | $84,500 | $79,000 | $73,500 | $68,000 |
| Expected Profit: | | $13,000 | $19,500 | $26,000 | $32,500 | $38,538 | $43,423 | $45,769 | $44,654 | $41,231 | $36,654 | $31,385 | $25,885 | $20,385 | $14,885 | $9,385 |
| **Maximum Profit:** | | | | | | | | 1 | | | | | | | | |

**Supplier:**

| Demand | Prob | Order: 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 | 1000 | 1100 | 1200 | 1300 | 1400 | 1500 | 1600 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 400 | 0.00 | $17,000 | $25,500 | $34,000 | $35,500 | $37,000 | $38,500 | $40,000 | $41,500 | $43,000 | $44,500 | $46,000 | $47,500 | $49,000 | $50,500 | $52,000 |
| 500 | 0.04 | $17,000 | $25,500 | $34,000 | $42,500 | $44,000 | $45,500 | $47,000 | $48,500 | $50,000 | $51,500 | $53,000 | $54,500 | $56,000 | $57,500 | $59,000 |
| 600 | 0.10 | $17,000 | $25,500 | $34,000 | $42,500 | $51,000 | $52,500 | $54,000 | $55,500 | $57,000 | $58,500 | $60,000 | $61,500 | $63,000 | $64,500 | $66,000 |
| 700 | 0.21 | $17,000 | $25,500 | $34,000 | $42,500 | $51,000 | $59,500 | $61,000 | $62,500 | $64,000 | $65,500 | $67,000 | $68,500 | $70,000 | $71,500 | $73,000 |
| 800 | 0.29 | $17,000 | $25,500 | $34,000 | $42,500 | $51,000 | $59,500 | $68,000 | $69,500 | $71,000 | $72,500 | $74,000 | $75,500 | $77,000 | $78,500 | $80,000 |
| 900 | 0.19 | $17,000 | $25,500 | $34,000 | $42,500 | $51,000 | $59,500 | $68,000 | $76,500 | $78,000 | $79,500 | $81,000 | $82,500 | $84,000 | $85,500 | $87,000 |
| 1,000 | 0.10 | $17,000 | $25,500 | $34,000 | $42,500 | $51,000 | $59,500 | $68,000 | $76,500 | $85,000 | $86,500 | $88,000 | $89,500 | $91,000 | $92,500 | $94,000 |
| 1,100 | 0.06 | $17,000 | $25,500 | $34,000 | $42,500 | $51,000 | $59,500 | $68,000 | $76,500 | $85,000 | $93,500 | $95,000 | $96,500 | $98,000 | $99,500 | $101,000 |
| 1,200 | 0.02 | $17,000 | $25,500 | $34,000 | $42,500 | $51,000 | $59,500 | $68,000 | $76,500 | $85,000 | $93,500 | $102,000 | $103,500 | $105,000 | $106,500 | $108,000 |
| 1,300 | 0.00 | $17,000 | $25,500 | $34,000 | $42,500 | $51,000 | $59,500 | $68,000 | $76,500 | $85,000 | $93,500 | $102,000 | $110,500 | $112,000 | $113,500 | $115,000 |
| Expected Profit: | | $17,000 | $25,500 | $34,000 | $42,500 | $50,731 | $58,288 | $64,365 | $68,423 | $71,135 | $73,173 | $74,808 | $76,308 | $77,808 | $79,308 | $80,808 |
| **Maximum Profit:** | | | | | | | | | | | | | | | | !!! |

| | | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 | 1000 | 1100 | 1200 | 1300 | 1400 | 1500 | 1600 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Total Profit:** | | $30,000 | $45,000 | $60,000 | $75,000 | $89,269 | $101,712 | $110,135 | $113,077 | $112,365 | $109,827 | $106,192 | $102,192 | $98,192 | $94,192 | $90,192 |
| **Maximum Profit** | | | | | | | | | !!! | | | | | | | |

# Channel profit with Buyback

R=$200
W=$135
C=$50
S=$10
B=$80
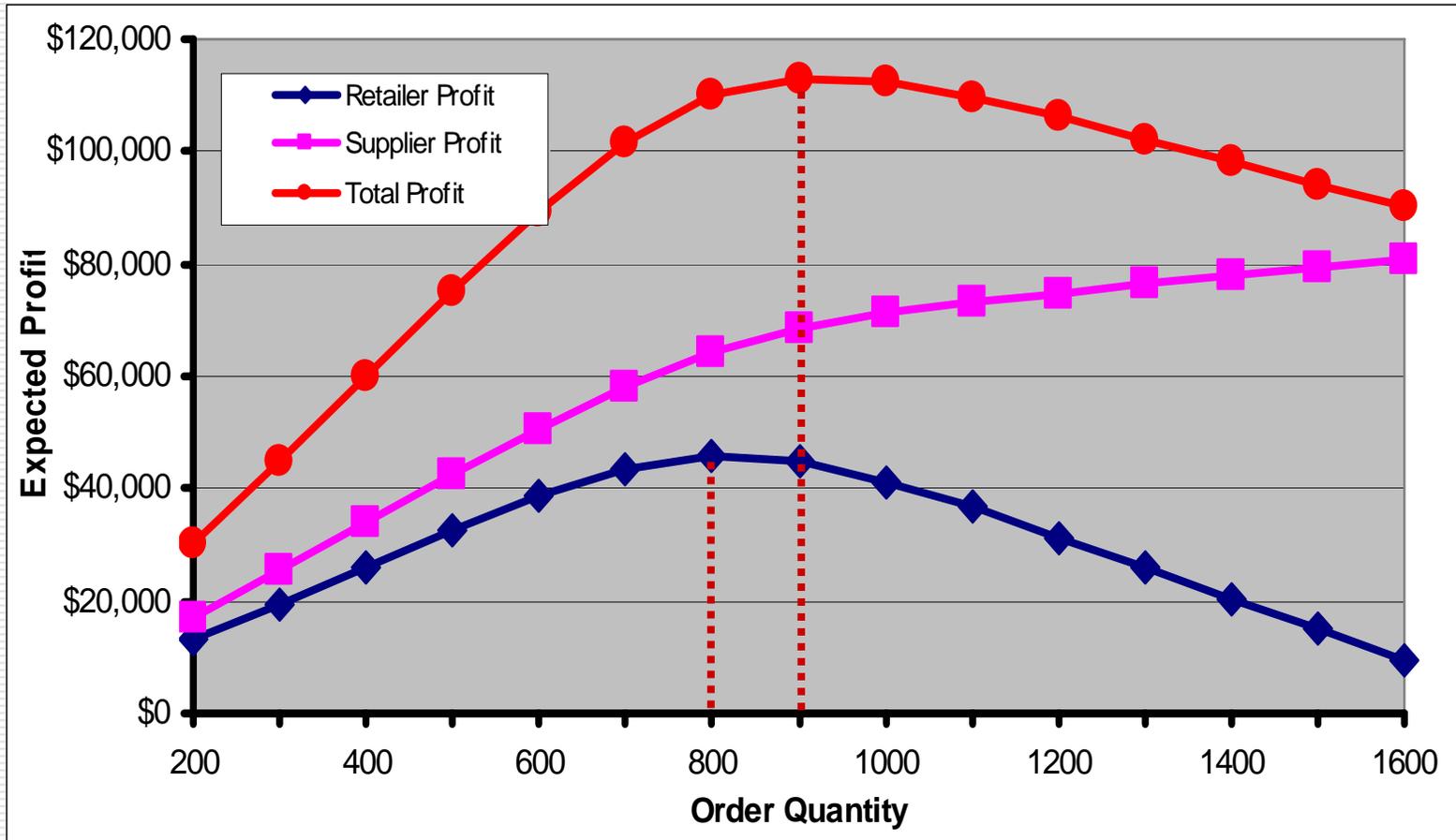
# Optimal Buyback and Wholesale price

- ☐ Higher wholesale price requires a *higher* buyback rate.
- ☐ As the wholesale price (and the buyback rate) grows the supplier's share of the profit *increases.*
  - ■ Wholesale price ranges from $50 (supplier's cost) to $200 (retail price)
  - ■ Buyback rate ranges from $10 (salvage value) to the wholesale price.

# Expected Profits with Buyback Contract

R=$200
W=$135
C=$50
S=$10
B=$80



Retail Order:        749
Retailer Profit    $41,988
Supplier Profit: $63,649
Total Profit:    $105,637

Retail Order:        828
Retailer Profit    $45,407
Supplier Profit: $65,457
Total Profit:    $110,273

Optimal Order:        936
Retailer Profit    $33,745
Supplier Profit: $79,528
Total Profit:    $113,273

Optimal Order:        936
Retailer Profit    $43,718
Supplier Profit: $69,555
Total Profit:    $113,273
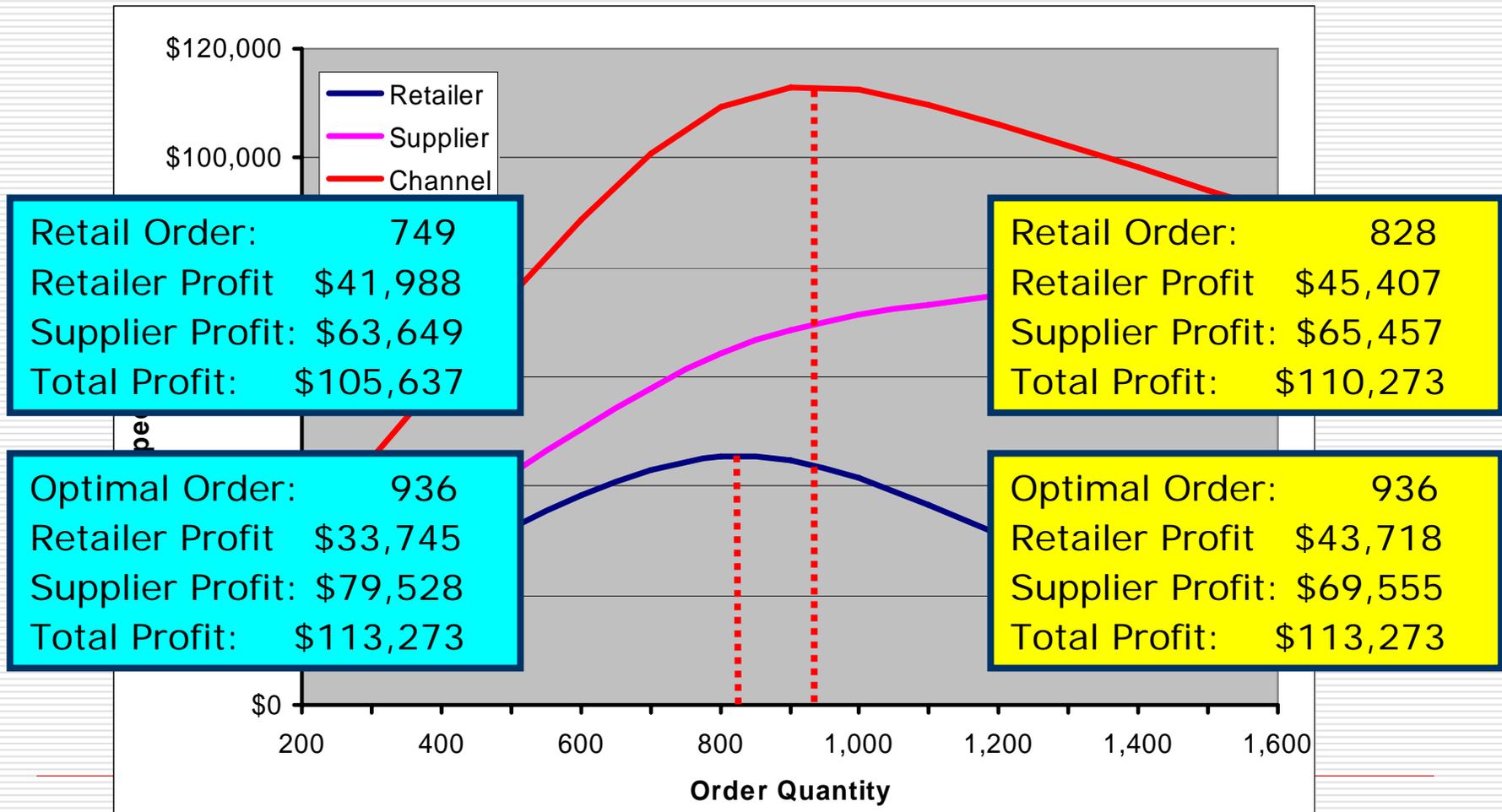
# Optimal Buyback Price

$$Q^*_{\text{Re } tailer} = Q^*_{Channel}$$

The manufacturer's levers are the wholesale price and the buyback rate

$$F^{-1}\left(\frac{R-W}{R-B}\right) = F^{-1}\left(\frac{R-C}{R-S}\right)$$

$$\left(\frac{R-W}{R-B}\right) = \left(\frac{R-C}{R-S}\right)$$

$$B = \left(\frac{R-S}{R-C}\right)\Box W - \left(\frac{R\Box(C-S)}{R-C}\right)$$

# Channel Coordination with Buyback

R=$200
W=
C=$50
S=$10
B=

**Wholesale Price & Buyback Amount that Coordinate the Channel**



B=$118

% Retailer=43

W=$135

- Buyback amount
- Retailer % of Profit
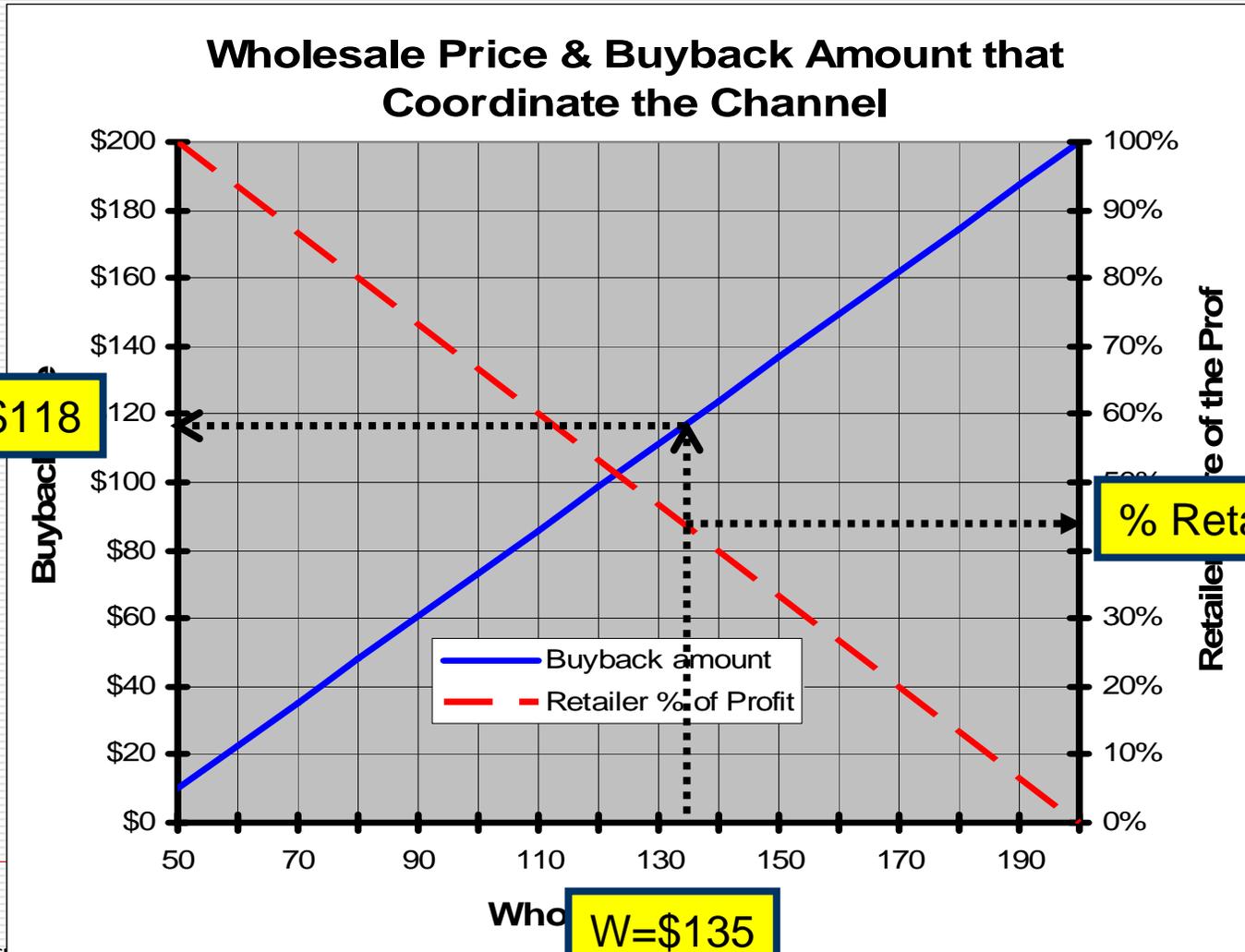
# Expected Profit with Coordinating Buyback Rate

R=$200
W=$135
C=$50
S=$10
**B*=$118**



Retail Order: 749
Retailer Profit $41,988
Supplier Profit: $63,649
Total Profit: $105,637

Optimal Order: 936
Retailer Profit $33,745
Supplier Profit: $79,528
Total Profit: $113,273

Optimal Order: 936
Retailer Profit $49,086
Supplier Profit: $64,188
Total Profit: $113,273

$120,000

$100,000

Retailer

$0

200   400   600   800   1,000   1,200   1,400   1,600

**Order Quantity**
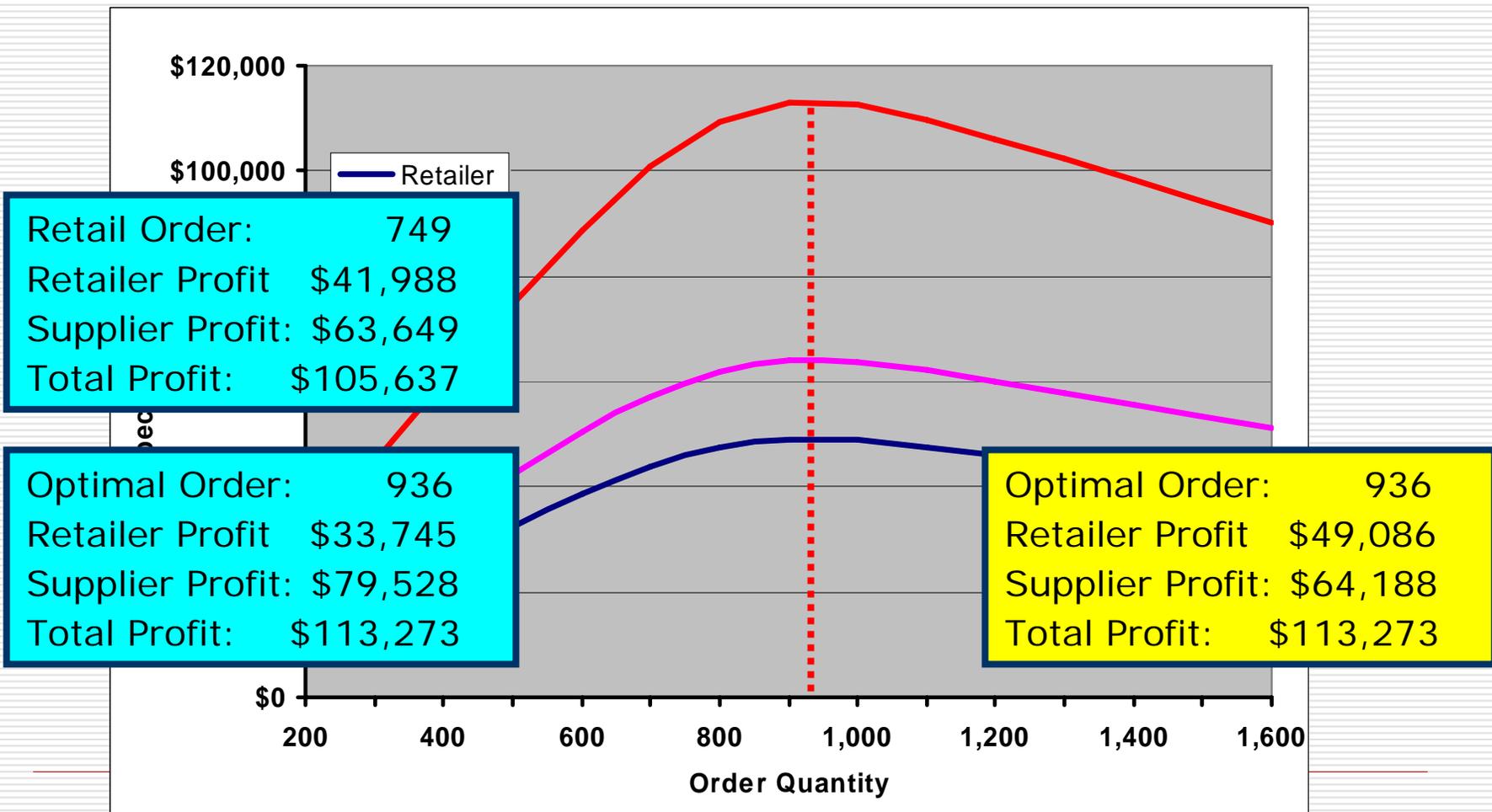
# Buyback Contracts in Practice

☐ Book publishing

☐ Periodicals/newspapers

☐ Price support in consumer electronics

# Revenue Sharing

- Supplier still needs to get the retailer to order more

- Another risk-sharing scheme: supplier lowers the wholesale price but takes a percentage (1-*p*) of the revenue

- Question: how to choose *W* and *p* so the retailer will order the optimal amount

- Note: wholesale price has to be lower than the supplier's cost.

# The Players

Images of the Paramount motion picture company and the Blockbuster video rental chain removed due to copyright restrictions.

# The Economics of Revenue Sharing

In the video industry, revenue sharing may increase profits for the supplier and the retailer alike, as this hypothetical example reveals.

## FOR THE RETAILER

|  | Traditional Pricing | Revenue Sharing |
|---|---|---|
| A.  Number of tapes purchased | 10 | 30 |
| B.  Price per tape | $60 | $9 |
| C.  Purchase cost | $600 | $270 |
| D.  Number of rentals | 300 | 500 |
| E.  Total rental revenue (D x $3/rental) | $900 | $1,500 |
| F.  Retailer's share of rental revenue | $900  (100%) | $750  (50%) |
| G.  Retailer's profit | $300 | $480 |
| H.  Profit per dollar of inventory | $0.50 | $1.78 |

## FOR THE SUPPLIER

|  | Traditional Pricing | Revenue Sharing |
|---|---|---|
| I.  Number of tapes purchased | 10 | 30 |
| J.  Price per tape | $60 | $9 |
| K.  Revenue from selling tapes | $600 | $270 |
| L.  Number of rentals | 300 | 500 |
| M.  Total rental revenue (L x $3/rental) | $900 | $1,500 |
| N.  Supplier's share of rental revenue | $0  (0%) | $750  (50%) |
| O.  Supplier's total revenues | $600 | $1,020 |
| P.  Supplier's production and distribution cost (I x $10/tape) | $100 | $300 |
| Q.  Supplier's profit | $500 | $720 |

Figure by MIT OCW.

# Revenue Sharing

**Retailer:**

| Demand | Prob | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 | 1000 | 1100 | 1200 | 1300 | 1400 | 1500 | 1600 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Order: | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 | 1000 | 1100 | 1200 | 1300 | 1400 | 1500 | 1600 |
| 400 | 0.00 | $8,000 | $12,000 | $16,000 | $13,000 | $10,000 | $7,000 | $4,000 | $1,000 | -$2,000 | -$5,000 | -$8,000 | -$11,000 | -$14,000 | -$17,000 | -$20,000 |
| 500 | 0.04 | $8,000 | $12,000 | $16,000 | $20,000 | $17,000 | $14,000 | $11,000 | $8,000 | $5,000 | $2,000 | -$1,000 | -$4,000 | -$7,000 | -$10,000 | -$13,000 |
| 600 | 0.10 | $8,000 | $12,000 | $16,000 | $20,000 | $24,000 | $21,000 | $18,000 | $15,000 | $12,000 | $9,000 | $6,000 | $3,000 | $0 | -$3,000 | -$6,000 |
| 700 | 0.21 | $8,000 | $12,000 | $16,000 | $20,000 | $24,000 | $28,000 | $25,000 | $22,000 | $19,000 | $16,000 | $13,000 | $10,000 | $7,000 | $4,000 | $1,000 |
| 800 | 0.29 | $8,000 | $12,000 | $16,000 | $20,000 | $24,000 | $28,000 | $32,000 | $29,000 | $26,000 | $23,000 | $20,000 | $17,000 | $14,000 | $11,000 | $8,000 |
| 900 | 0.19 | $8,000 | $12,000 | $16,000 | $20,000 | $24,000 | $28,000 | $32,000 | $36,000 | $33,000 | $30,000 | $27,000 | $24,000 | $21,000 | $18,000 | $15,000 |
| 1,000 | 0.10 | $8,000 | $12,000 | $16,000 | $20,000 | $24,000 | $28,000 | $32,000 | $36,000 | $40,000 | $37,000 | $34,000 | $31,000 | $28,000 | $25,000 | $22,000 |
| 1,100 | 0.06 | $8,000 | $12,000 | $16,000 | $20,000 | $24,000 | $28,000 | $32,000 | $36,000 | $40,000 | $44,000 | $41,000 | $38,000 | $35,000 | $32,000 | $29,000 |
| 1,200 | 0.02 | $8,000 | $12,000 | $16,000 | $20,000 | $24,000 | $28,000 | $32,000 | $36,000 | $40,000 | $44,000 | $48,000 | $45,000 | $42,000 | $39,000 | $36,000 |
| 1,300 | 0.00 | $8,000 | $12,000 | $16,000 | $20,000 | $24,000 | $28,000 | $32,000 | $36,000 | $40,000 | $44,000 | $48,000 | $52,000 | $49,000 | $46,000 | $43,000 |
| Expected Profit: | | $8,000 | $12,000 | $16,000 | $20,000 | $23,731 | $26,788 | $28,365 | $27,923 | $26,135 | $23,673 | $20,808 | $17,808 | $14,808 | $11,808 | $8,808 |
| Maximum Profit: | | | | | | | | **1** | | | | | | | | |

**Supplier:**

| Demand | Prob | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 | 1000 | 1100 | 1200 | 1300 | 1400 | 1500 | 1600 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Order: | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 | 1000 | 1100 | 1200 | 1300 | 1400 | 1500 | 1600 |
| 400 | 0.00 | $22,000 | $33,000 | $44,000 | $43,000 | $42,000 | $41,000 | $40,000 | $39,000 | $38,000 | $37,000 | $36,000 | $35,000 | $34,000 | $33,000 | $32,000 |
| 500 | 0.04 | $22,000 | $33,000 | $44,000 | $55,000 | $54,000 | $53,000 | $52,000 | $51,000 | $50,000 | $49,000 | $48,000 | $47,000 | $46,000 | $45,000 | $44,000 |
| 600 | 0.10 | $22,000 | $33,000 | $44,000 | $55,000 | $66,000 | $65,000 | $64,000 | $63,000 | $62,000 | $61,000 | $60,000 | $59,000 | $58,000 | $57,000 | $56,000 |
| 700 | 0.21 | $22,000 | $33,000 | $44,000 | $55,000 | $66,000 | $77,000 | $76,000 | $75,000 | $74,000 | $73,000 | $72,000 | $71,000 | $70,000 | $69,000 | $68,000 |
| 800 | 0.29 | $22,000 | $33,000 | $44,000 | $55,000 | $66,000 | $77,000 | $88,000 | $87,000 | $86,000 | $85,000 | $84,000 | $83,000 | $82,000 | $81,000 | $80,000 |
| 900 | 0.19 | $22,000 | $33,000 | $44,000 | $55,000 | $66,000 | $77,000 | $88,000 | $99,000 | $98,000 | $97,000 | $96,000 | $95,000 | $94,000 | $93,000 | $92,000 |
| 1,000 | 0.10 | $22,000 | $33,000 | $44,000 | $55,000 | $66,000 | $77,000 | $88,000 | $99,000 | $110,000 | $109,000 | $108,000 | $107,000 | $106,000 | $105,000 | $104,000 |
| 1,100 | 0.06 | $22,000 | $33,000 | $44,000 | $55,000 | $66,000 | $77,000 | $88,000 | $99,000 | $110,000 | $121,000 | $120,000 | $119,000 | $118,000 | $117,000 | $116,000 |
| 1,200 | 0.02 | $22,000 | $33,000 | $44,000 | $55,000 | $66,000 | $77,000 | $88,000 | $99,000 | $110,000 | $121,000 | $132,000 | $131,000 | $130,000 | $129,000 | $128,000 |
| 1,300 | 0.00 | $22,000 | $33,000 | $44,000 | $55,000 | $66,000 | $77,000 | $88,000 | $99,000 | $110,000 | $121,000 | $132,000 | $143,000 | $142,000 | $141,000 | $140,000 |
| Expected Profit: | | $22,000 | $33,000 | $44,000 | $55,000 | $65,538 | $74,923 | $81,769 | $85,154 | $86,231 | $86,154 | $85,385 | $84,385 | $83,385 | $82,385 | $81,385 |
| Maximum Profit: | | | | | | | | | | **!!!** | | | | | | |
| Total Profit: | | $30,000 | $45,000 | $60,000 | $75,000 | $89,269 | $101,712 | $110,135 | $113,077 | $112,365 | $109,827 | $106,192 | $102,192 | $98,192 | $94,192 | $90,192 |
| **Maximum Profit** | | | | | | | | | **!!!** | | | | | | | |

# Expected Profit with Revenue Sharing

R= $200
W= $40
C= $50
S= $10
p= 0.40
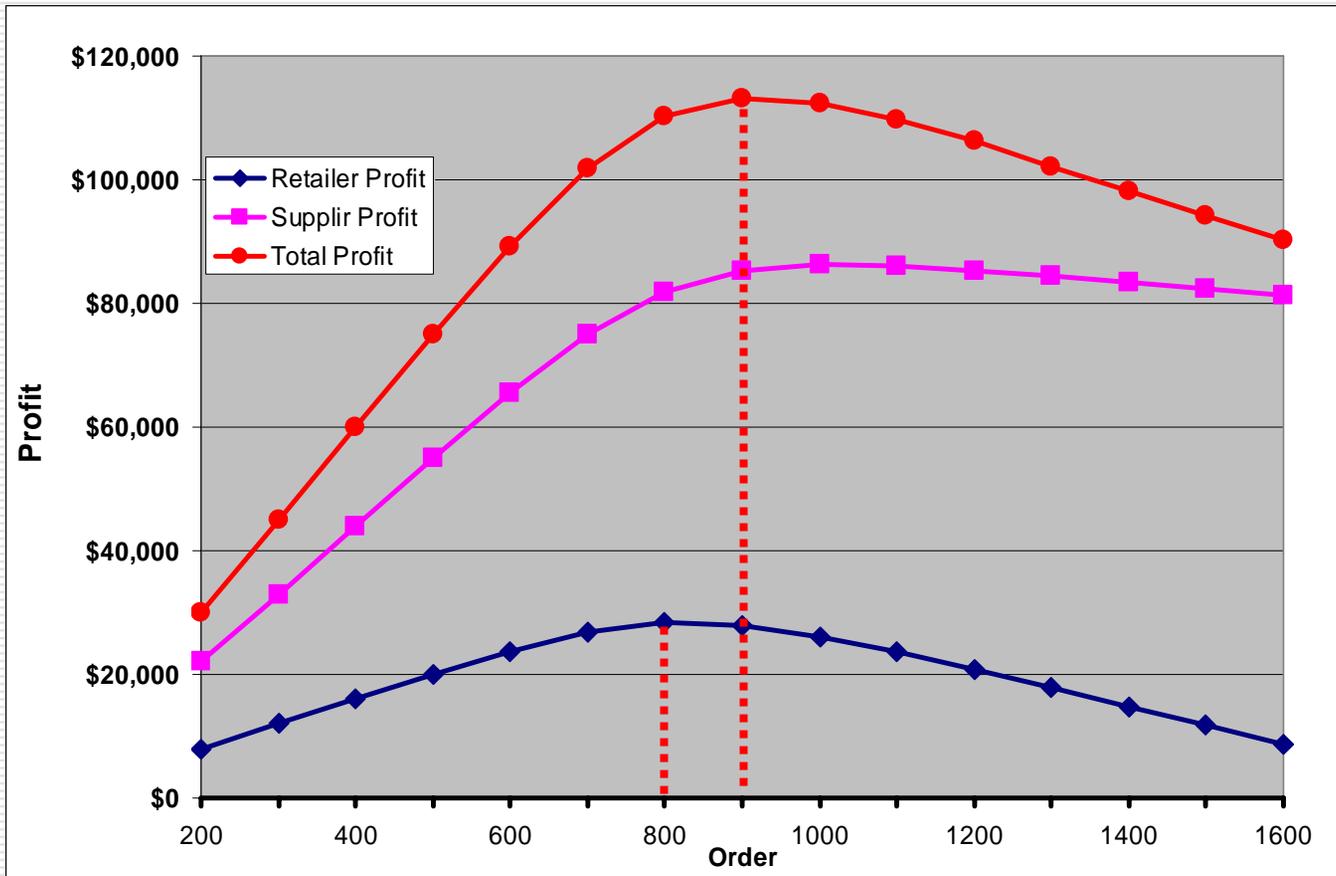
# Optimal Revenue Share

Retailer's optimal order:

$$Q_R^* = F^{-1}\left(\frac{p\square R - W}{p\square R - S}\right)$$

Condition for coordination:

$$Q_{\text{Re }tailer}^* = Q_{Channel}^*$$

$$\left(\frac{p\square R - W}{p\square R - S}\right) = \left(\frac{R - C}{R - S}\right)$$

$$p = W \bullet \frac{(R - S)}{R\square(C - S)} - \frac{S\square(R - C)}{R\square(C - S)}$$

# Coordination with Rev. Sharing



Revenue Sharing

(y-axis) % Retailer: 0% to 100%
(x-axis) Wholesale Price: 10 to 50
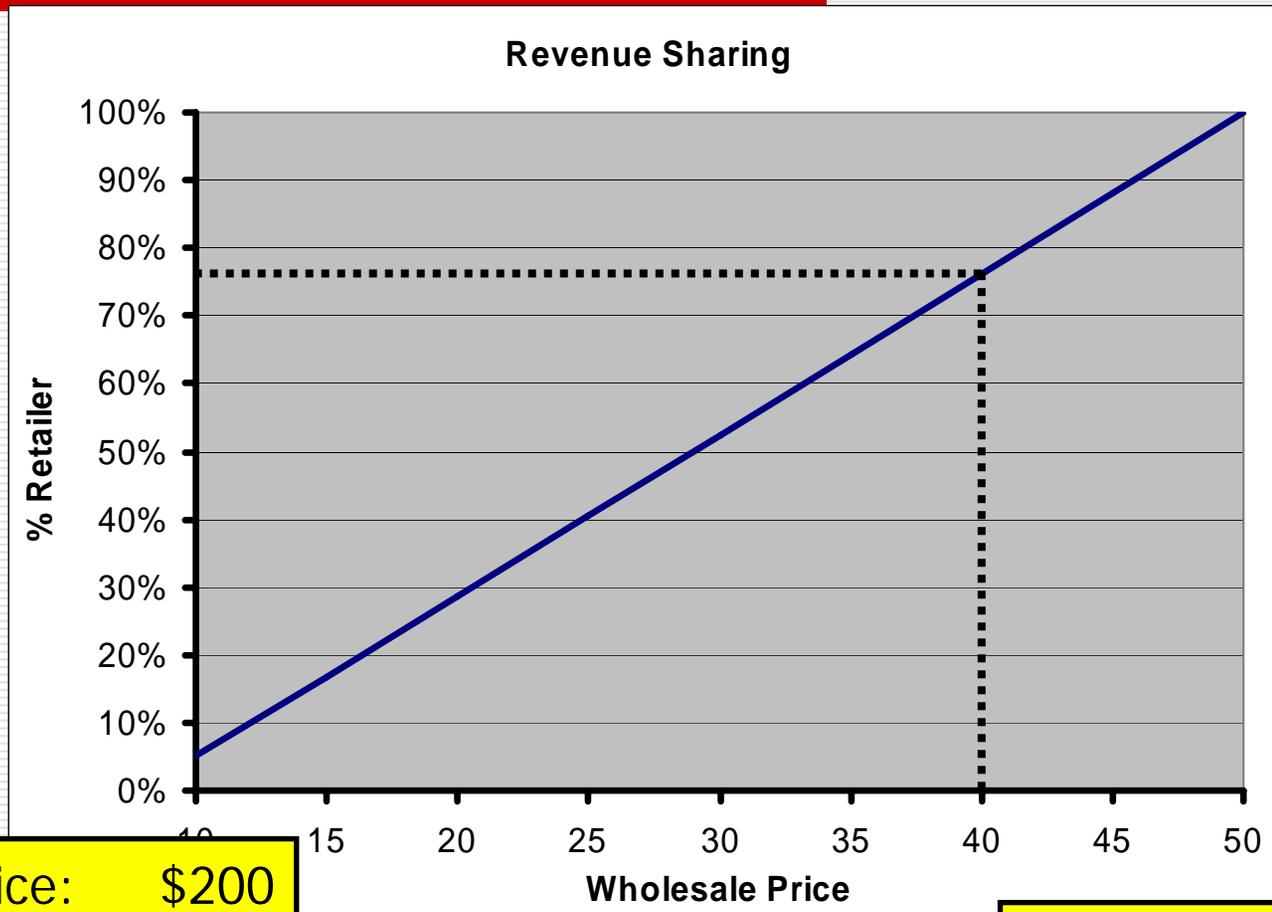
Retail Price:     $200
Supplier Costs:   $50
Salvage value:    $10

Wholesale price: $40
Retail Rev Share:  76%

# Revenue Sharing (Normal Approx)

R= $200
W= $40
C= $50
S= $10
p= 0.76
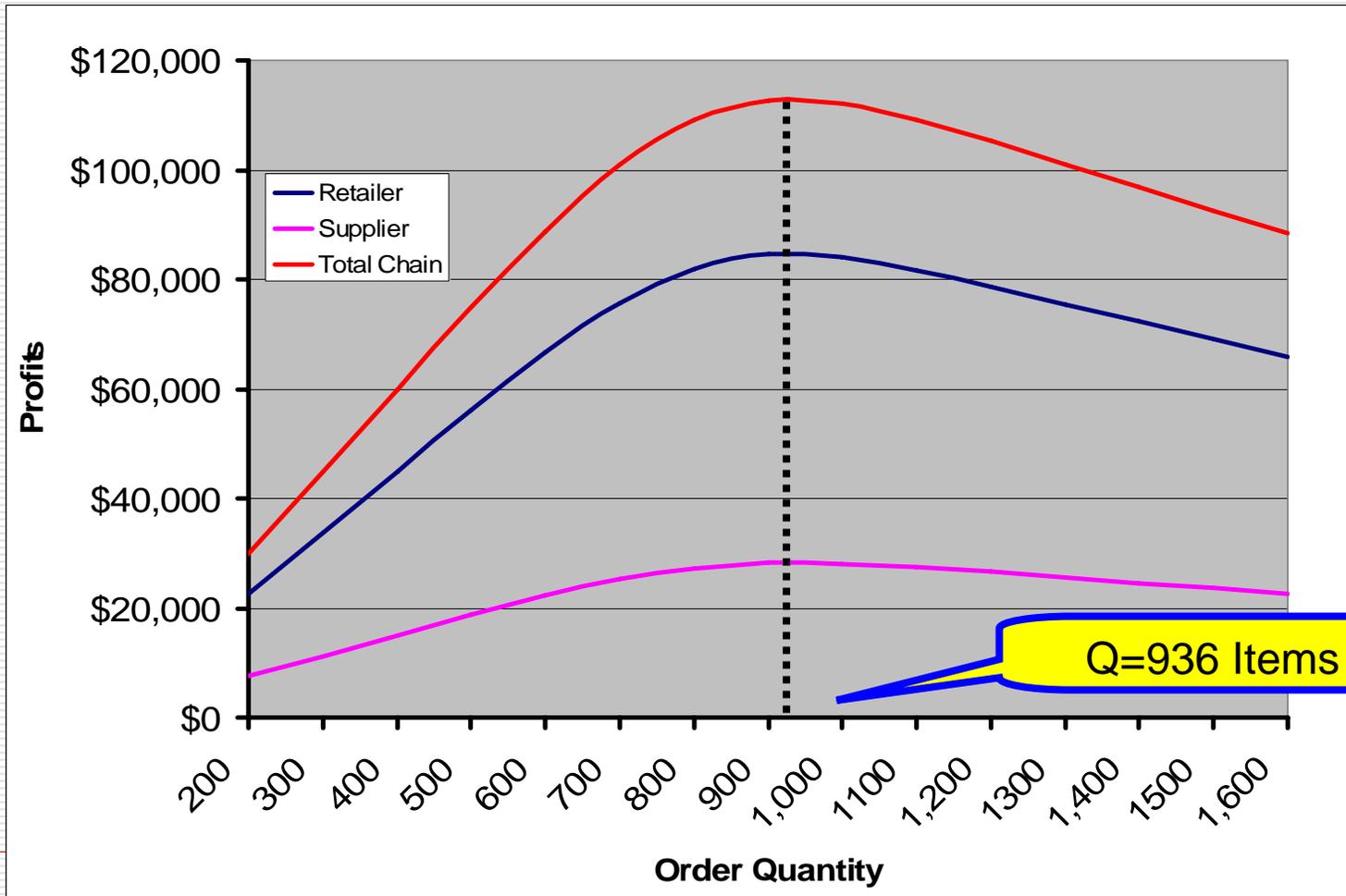


Q=936 Items

# Equivalence of Contracts

- In buyback ($W_B$, B), The retailer:
  - Pays $W_B$, for each unit ordered; gets B for each unit unsold
  - Same as: pays $W_B$-B for each unit ordered and <u>additional</u> B for each unit sold
- In revenue sharing ($W_R$, $p$), the retailer:
  - Pays $W_R$ for each unit ordered and an <u>additional</u> (1-$p$)R for each unit sold
- The contracts are identical when:
  - Retailer payment for order: $W_R$= $W_B$-B
  - Retailer additional payment: (1-$p$)R = B
    $$\text{or } p = (R\text{-}B)/R$$

# Real Options

- The retailer buys $Q$ call options at a price $w$.
- The supplier makes $Q$ items.
- Each option can be exercised at a unit price $E$.
  - As demand materializes the retailer can take deliveries for an additional payment of $E$.
- No more than $Q$ items can be bought from the supplier

# Real Options

R= $200
W= $20
C= $50
S= $10
E= $60

**Retailer:**

| Order: | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 | 1000 | 1100 | 1200 | 1300 | 1400 | 1500 | 1600 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Demand / Prob | | | | | | | | | | | | | | | |
| 400 / 0.00 | $24,000 | $36,000 | $48,000 | $46,000 | $44,000 | $42,000 | $40,000 | $38,000 | $36,000 | $34,000 | $32,000 | $30,000 | $28,000 | $26,000 | $24,000 |
| 500 / 0.04 | $24,000 | $36,000 | $48,000 | $60,000 | $58,000 | $56,000 | $54,000 | $52,000 | $50,000 | $48,00 | $46,000 | $44,000 | $42,000 | $40,000 | $38,000 |
| 600 / 0.10 | $24,000 | $36,000 | $48,000 | $60,000 | $72,000 | $70,000 | $68,000 | $66,000 | $64,000 | $62,000 | $60,000 | $58,000 | $56,000 | $54,000 | $52,000 |
| 700 / 0.21 | $24,000 | $36,000 | $48,000 | $60,000 | $72,000 | $84,000 | $82,000 | $80,000 | $78,000 | $76,000 | $74,000 | $72,000 | $70,000 | $68,000 | $66,000 |
| 800 / 0.29 | $24,000 | $36,000 | $48,000 | $60,000 | $72,000 | $84,000 | $96,000 | $94,000 | $92,000 | $90,000 | $88,000 | $86,000 | $84,000 | $82,000 | $80,000 |
| 900 / 0.19 | $24,000 | $36,000 | $48,000 | $60,000 | $72,000 | $84,000 | $96,000 | $108,000 | $106,000 | $104,000 | $102,000 | $100,000 | $98,000 | $96,000 | $94,000 |
| 1,000 / 0.10 | $24,000 | $36,000 | $48,000 | $60,000 | $72,000 | $84,000 | $96,000 | $108,000 | $120,000 | $118,000 | $116,000 | $114,000 | $112,000 | $110,000 | $108,000 |
| 1,100 / 0.06 | $24,000 | $36,000 | $48,000 | $60,000 | $72,000 | $84,000 | $96,000 | $108,000 | $120,000 | $132,000 | $130,000 | $128,000 | $126,000 | $124,000 | $122,000 |
| 1,200 / 0.02 | $24,000 | $36,000 | $48,000 | $60,000 | $72,000 | $84,000 | $96,000 | $108,000 | $120,000 | $132,000 | $144,000 | $142,000 | $140,000 | $138,000 | $136,000 |
| 1,300 / 0.00 | $24,000 | $36,000 | $48,000 | $60,000 | $72,000 | $84,000 | $96,000 | $108,000 | $120,000 | $132,000 | $144,000 | $156,000 | $154,000 | $152,000 | $150,000 |
| Expected Profit: | $24,000 | $36,000 | $48,000 | $60,000 | $71,462 | $81,577 | $88,731 | $91,846 | $92,269 | $91,346 | $89,615 | $87,615 | $85,615 | $83,615 | $81,615 |
| Maximum Profit: | | | | | | | | | 1 | | | | | | |

**Supplier:**

| Order: | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 | 1000 | 1100 | 1200 | 1300 | 1400 | 1500 | 1600 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Demand / Prob | | | | | | | | | | | | | | | |
| 400 / 0.00 | $6,000 | $9,000 | $12,000 | $10,000 | $8,000 | $6,000 | $4,000 | $2,000 | $0 | -$2,000 | -$4,000 | -$6,000 | -$8,000 | -$10,000 | -$12,000 |
| 500 / 0.04 | $6,000 | $9,000 | $12,000 | $15,000 | $13,000 | $11,000 | $9,000 | $7,000 | $5,000 | $3,000 | $1,000 | -$1,000 | -$3,000 | -$5,000 | -$7,000 |
| 600 / 0.10 | $6,000 | $9,000 | $12,000 | $15,000 | $18,000 | $16,000 | $14,000 | $12,000 | $10,000 | $8,000 | $6,000 | $4,000 | $2,000 | $0 | -$2,000 |
| 700 / 0.21 | $6,000 | $9,000 | $12,000 | $15,000 | $18,000 | $21,000 | $19,000 | $17,000 | $15,000 | $13,000 | $11,000 | $9,000 | $7,000 | $5,000 | $3,000 |
| 800 / 0.29 | $6,000 | $9,000 | $12,000 | $15,000 | $18,000 | $21,000 | $24,000 | $22,000 | $20,000 | $18,000 | $16,000 | $14,000 | $12,000 | $10,000 | $8,000 |
| 900 / 0.19 | $6,000 | $9,000 | $12,000 | $15,000 | $18,000 | $21,000 | $24,000 | $27,000 | $25,000 | $23,000 | $21,000 | $19,000 | $17,000 | $15,000 | $13,000 |
| 1,000 / 0.10 | $6,000 | $9,000 | $12,000 | $15,000 | $18,000 | $21,000 | $24,000 | $27,000 | $30,000 | $28,000 | $26,000 | $24,000 | $22,000 | $20,000 | $18,000 |
| 1,100 / 0.06 | $6,000 | $9,000 | $12,000 | $15,000 | $18,000 | $21,000 | $24,000 | $27,000 | $30,000 | $33,000 | $31,000 | $29,000 | $27,000 | $25,000 | $23,000 |
| 1,200 / 0.02 | $6,000 | $9,000 | $12,000 | $15,000 | $18,000 | $21,000 | $24,000 | $27,000 | $30,000 | $33,000 | $36,000 | $34,000 | $32,000 | $30,000 | $28,000 |
| 1,300 / 0.00 | $6,000 | $9,000 | $12,000 | $15,000 | $18,000 | $21,000 | $24,000 | $27,000 | $30,000 | $33,000 | $36,000 | $39,000 | $37,000 | $35,000 | $33,000 |
| Expected Profit: | $6,000 | $9,000 | $12,000 | $15,000 | $17,808 | $20,135 | $21,404 | $21,231 | $20,096 | $18,481 | $16,577 | $14,577 | $12,577 | $10,577 | $8,577 |
| Maximum Profit: | | | | | | | !!! | | | | | | | | |

| Total Profit: | $30,000 | $45,000 | $60,000 | $75,000 | $89,269 | $101,712 | $110,135 | $113,077 | $112,365 | $109,827 | $106,192 | $102,192 | $98,192 | $94,192 | $90,192 |
| **Maximum Profit** | | | | | | | | !!! | | | | | | | |

# Real Options

**Real Options**

Profit vs. Number of calls

Legend:
- Retailer
- Supplier
- Total
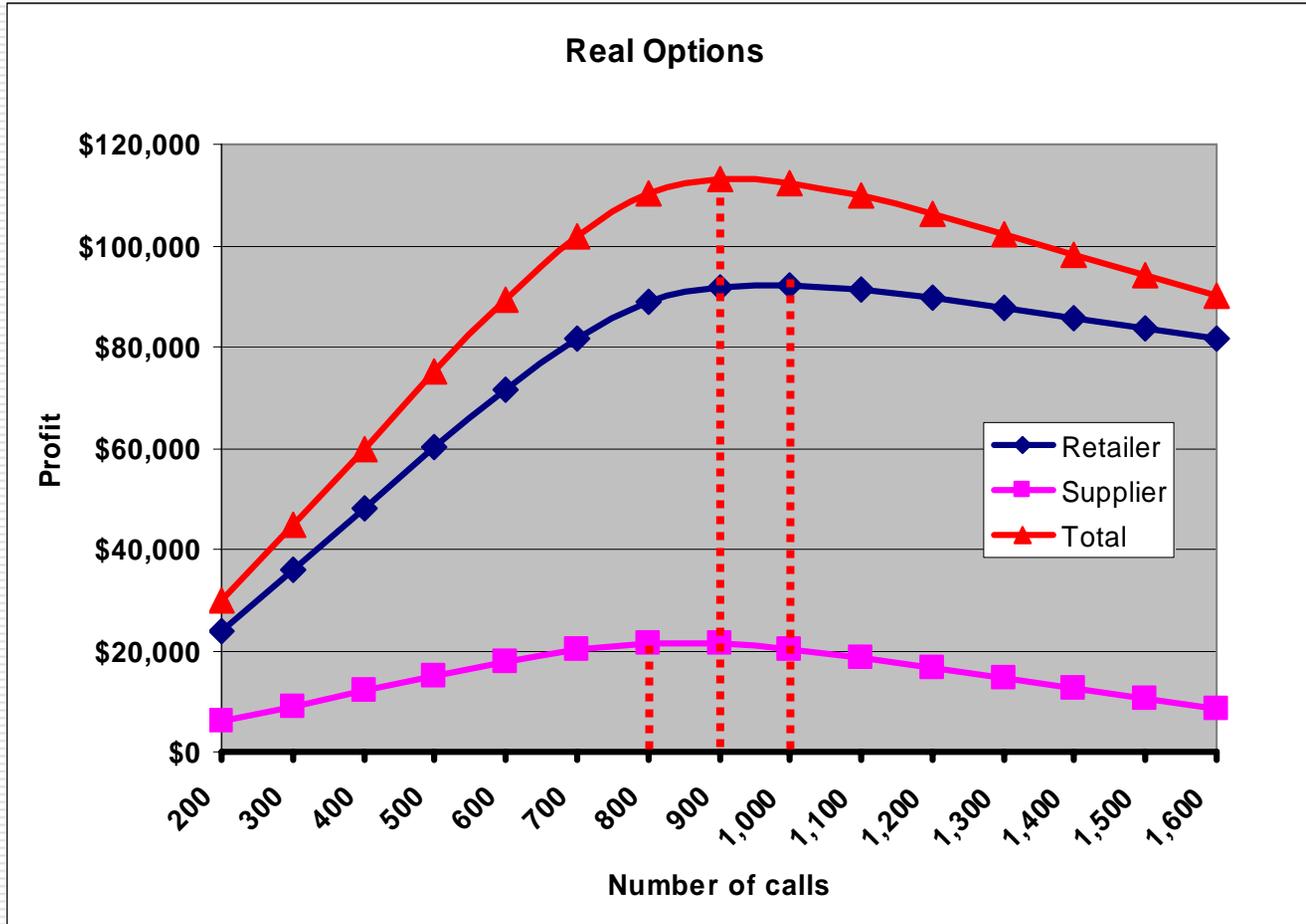
# Coordination with Real Options

Retailer's optimal order:

$$\frac{d\Pi_R(Q)}{dQ} = 0 \qquad \Longrightarrow \qquad Q^* = F^{-1}\left(\frac{R - E - W}{R - E}\right)$$

Comparing to a single channel:

$$Q^* = F^{-1}\left(\frac{R - C}{R - S}\right) \qquad \Longrightarrow \qquad E = -\left(\frac{R - S}{C - S}\right) \cdot W + R$$

$$\text{Where:} \quad W \in \left(0, R \cdot \frac{C - S}{R - S}\right)$$

# Coordination with Real Options



**Option Price and Excercise Price**

Y-axis: **Excercise Price (E)** — $0, $20, $40, $60, $80, $100, $120, $140, $160, $180, $200

X-axis: **Option Price (W)** — 0, 5, 10, 15, 20, 25, 30, 35, 40, 45
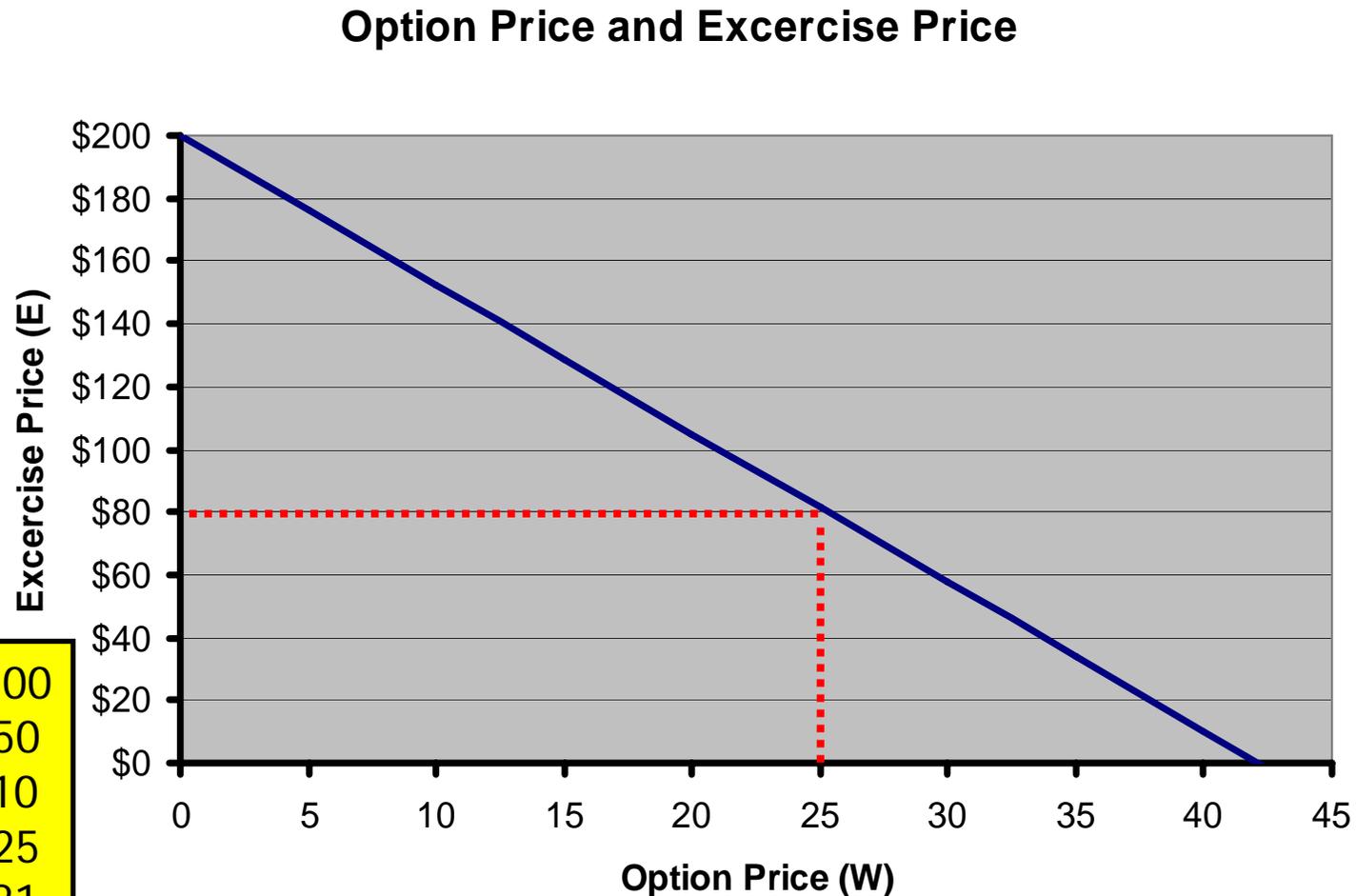
R = $200
C = $50
S = $10
W = $25
E = $81

# Expected Profits with Options



R = $200
C = $50
S = $10
W = $25
E = $81
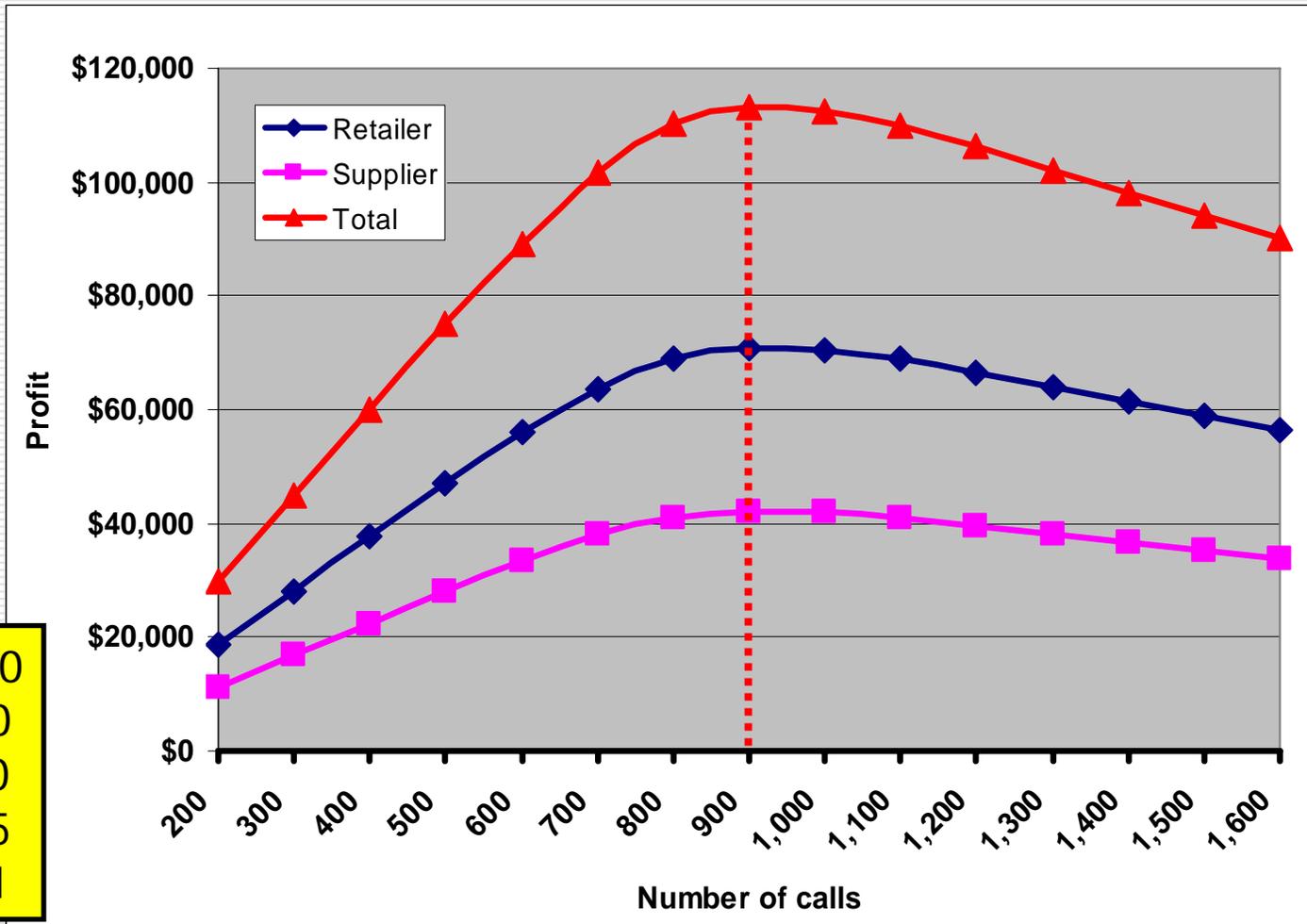
# Summary

- ❑ Wholesale contracts give too much risk and not enough expected reward to the retailer
- ❑ To make the retailer order more (and increase the channel's profit) the supplier has to take on part of the risk
- ❑ Risk sharing mechanisms covered include:
  - ■ Buybacks; revenue sharing, option contract
- ❑ Other mechanisms
  - ■ Quantity-flexibility (refund on a portion of the unsold units)
  - ■ Sales-rebate (rebate on unsold units above a threshold)
  - ■ There are many other mechanisms
- ❑ Each mechanism can coordinate the channel with various allocations of the profit between the retailer and the supplier.

# Any Questions?

? ? ? ? ? ?

Yossi Sheffi