Spatially Distributed Queues

M/G/1

2 Servers

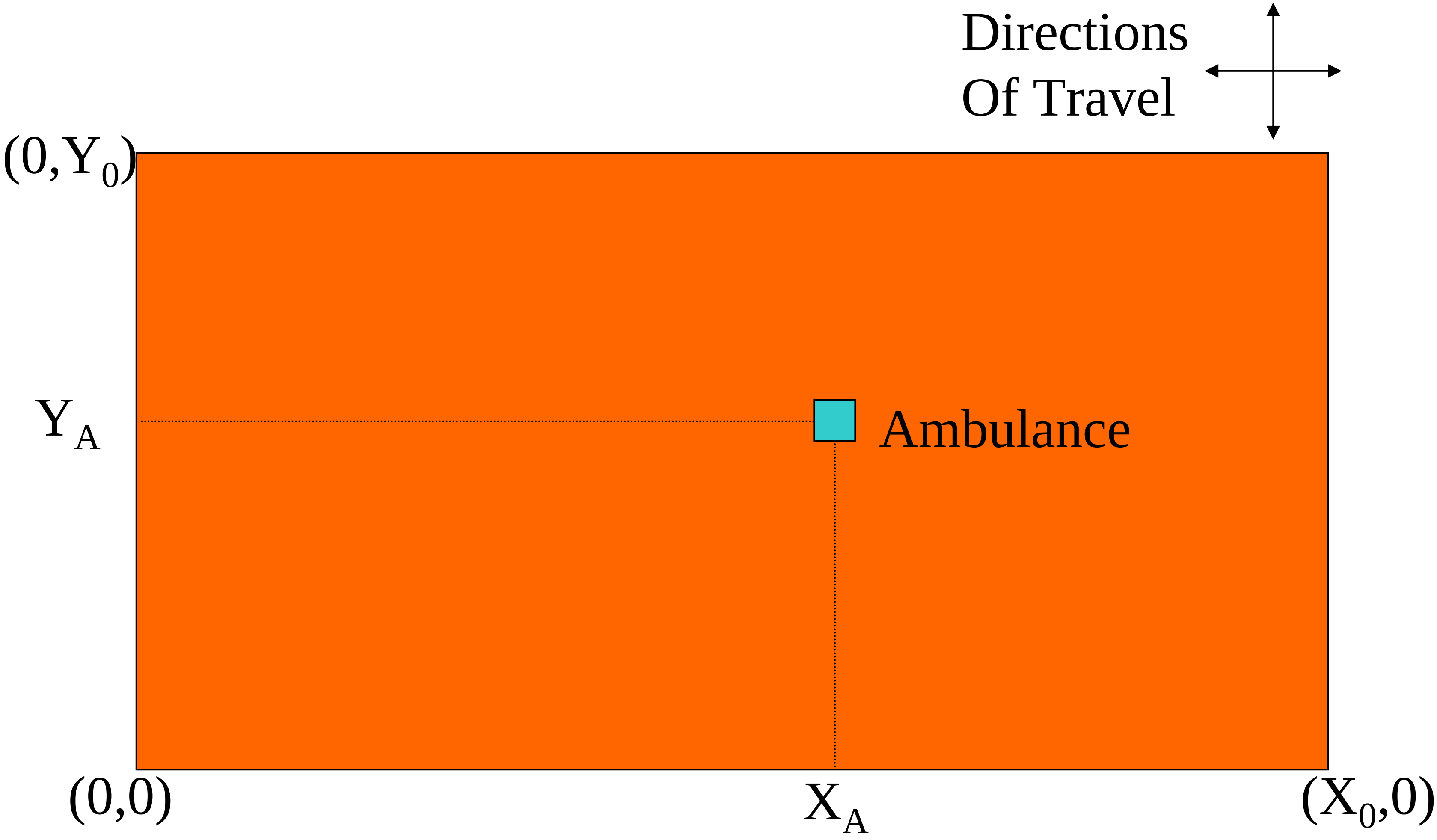
N servers

Approximations

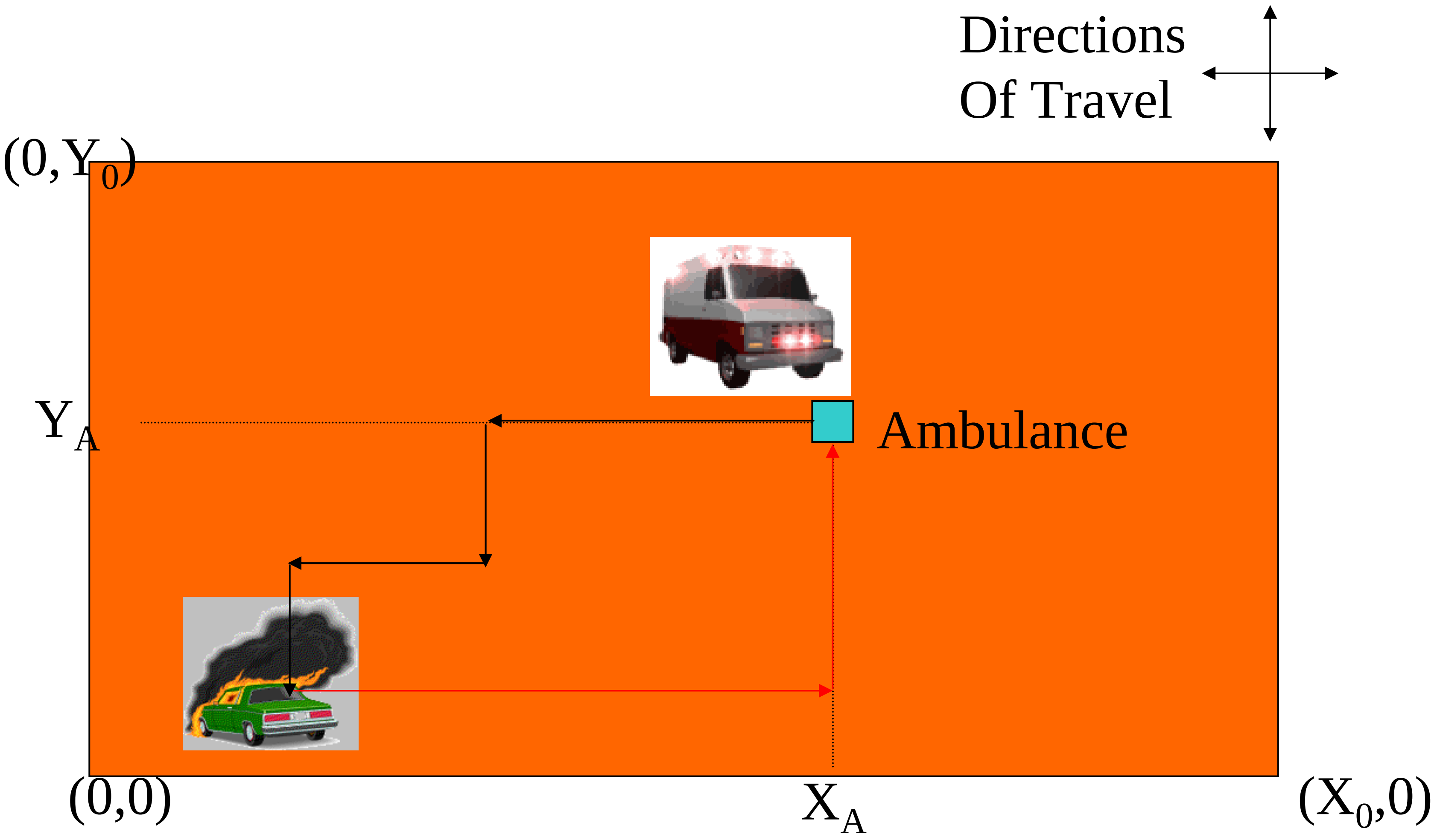
Why Spatial Queues?

- Demand responsive transportation systems
- Organ donation queues
- Warehouses
- Supply chains
- Cell phone systems
- People waiting to be evacuated in a hurricane

M/G/1



M/G/1



M/G/1

- Ambulance always returns home with each service; standard M/G/1 applies
- But suppose we have an emergency repair vehicle that travels directly from one customer to the next?……

M/G/1 with different 1st service time

$\overline{S}_1, \sigma^2_{S_1}$ = expected value and variance,respectively,

of the 1st service time in a busy period

$\overline{S}_2, \sigma^2_{S_2}$ = expected value and variance,respectively,

of the 2nd & all succeeding service times in a busy period

$\lambda \overline{S}_2 < 1$

$\rho = 1 - P_0$ = fraction of time server is busy

M/G/1 with different 1st service time

$$\rho = \frac{\lambda \overline{S_1}}{1 - \lambda(\overline{S_2} - \overline{S_1})}$$

$$L = \rho + \frac{\lambda^2}{1 - \lambda(\overline{S_2} - \overline{S_1})}\left[\frac{\sigma_{S_1}^2 + \overline{S}_1^2 + \lambda\{\overline{S}_1(\sigma_{S_2}^2 + \overline{S}_2^2) - \overline{S}_2(\sigma_{S_1}^2 + \overline{S}_1^2)}{2(1 - \lambda\overline{S}_2)}\right]$$

M/G/1 with different 1st service time

Little's Law: Buy one, get three others for free!

$$L = \lambda W$$

$$L_q = \lambda W_q$$

See the book, Eqs. (5.0) - (5.5)

M/G/1 with different 1st service time

- Does this new more general M/G/1 model apply exactly to the ambulance problem?

- Why or why not?

Two-Server "Hypercube" Queueing Model

- Distinguishable servers
- Different workloads (due to geography)
- Can appear with or without queueing
  - With -- usually FCFS
  - Without -- usually means a backup contract service is in place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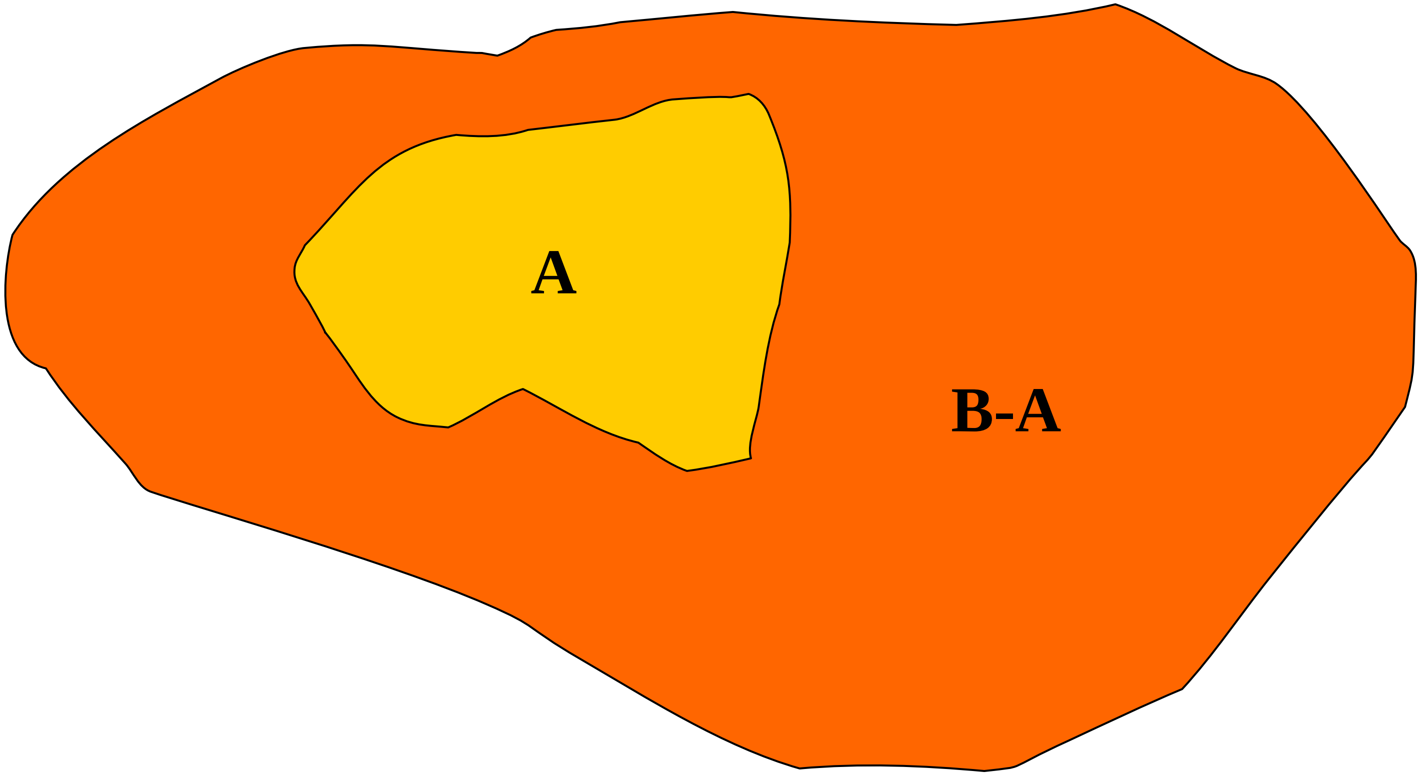
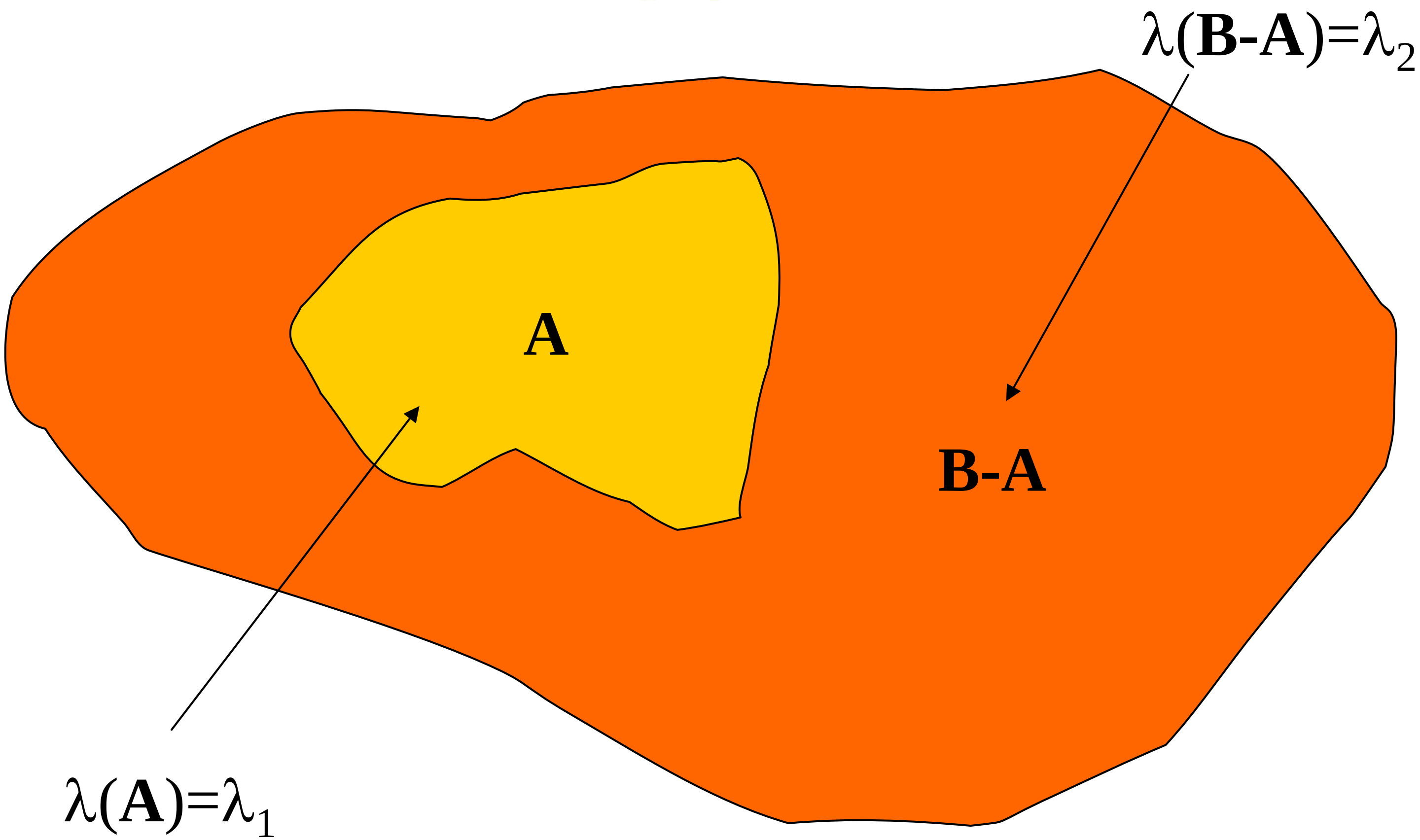


B = "Service Region"

Poisson Arrivals from any sub-region **A**

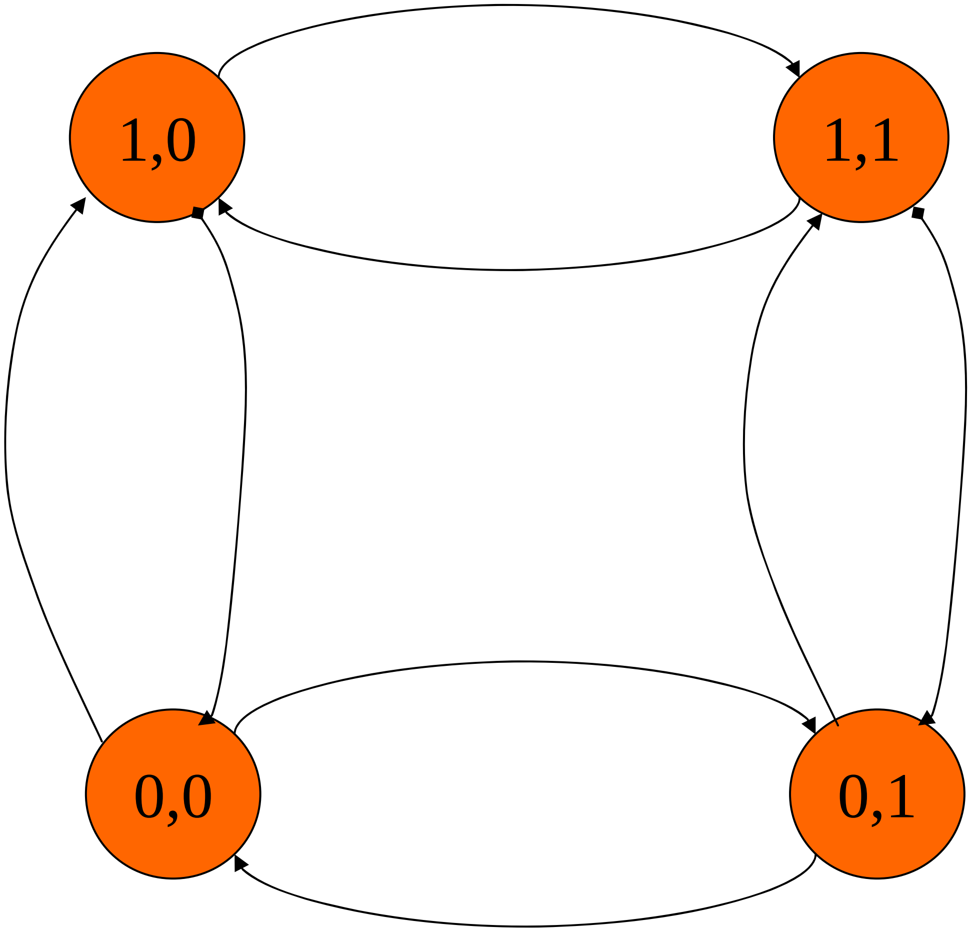


B = "Service Region"

$\lambda = \lambda_1 + \lambda_2$

Service Discipline

- 1st Dispatch Preference to ‘primary server’
- Otherwise, assign customer to other server, if available
- Otherwise, job is ‘lost” (What happens in practice?)


1,0
1,1
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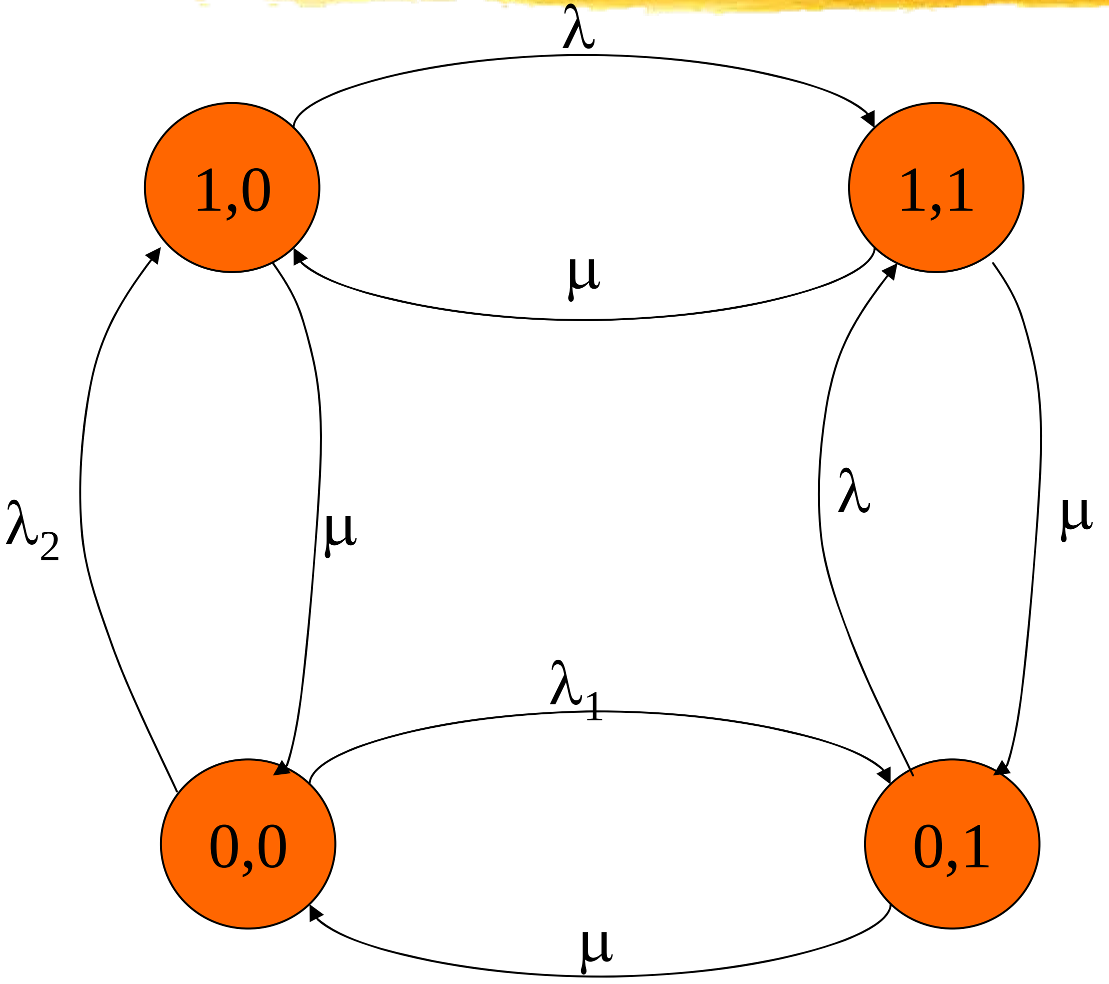
λ
1,0
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μ
λ2
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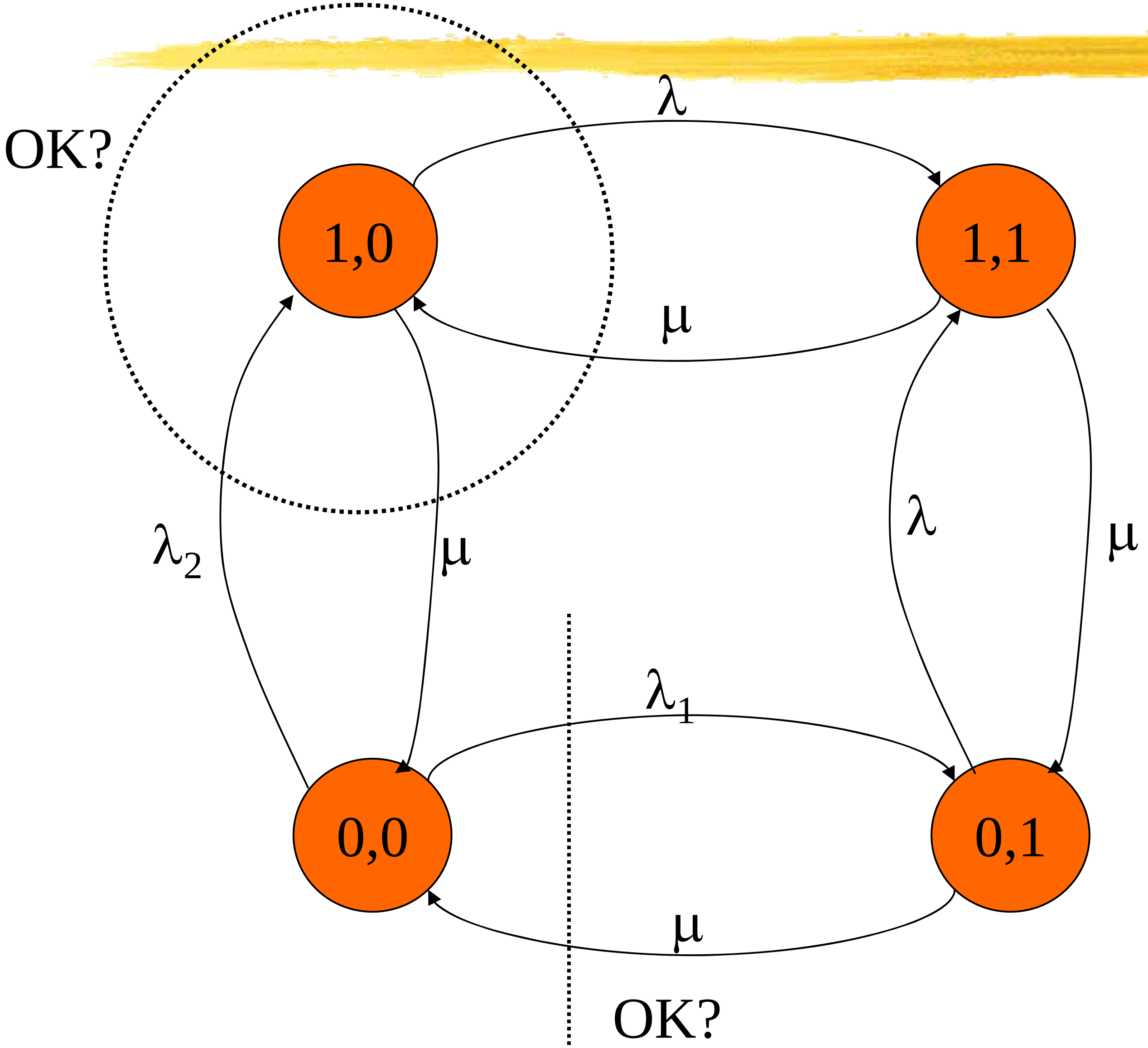
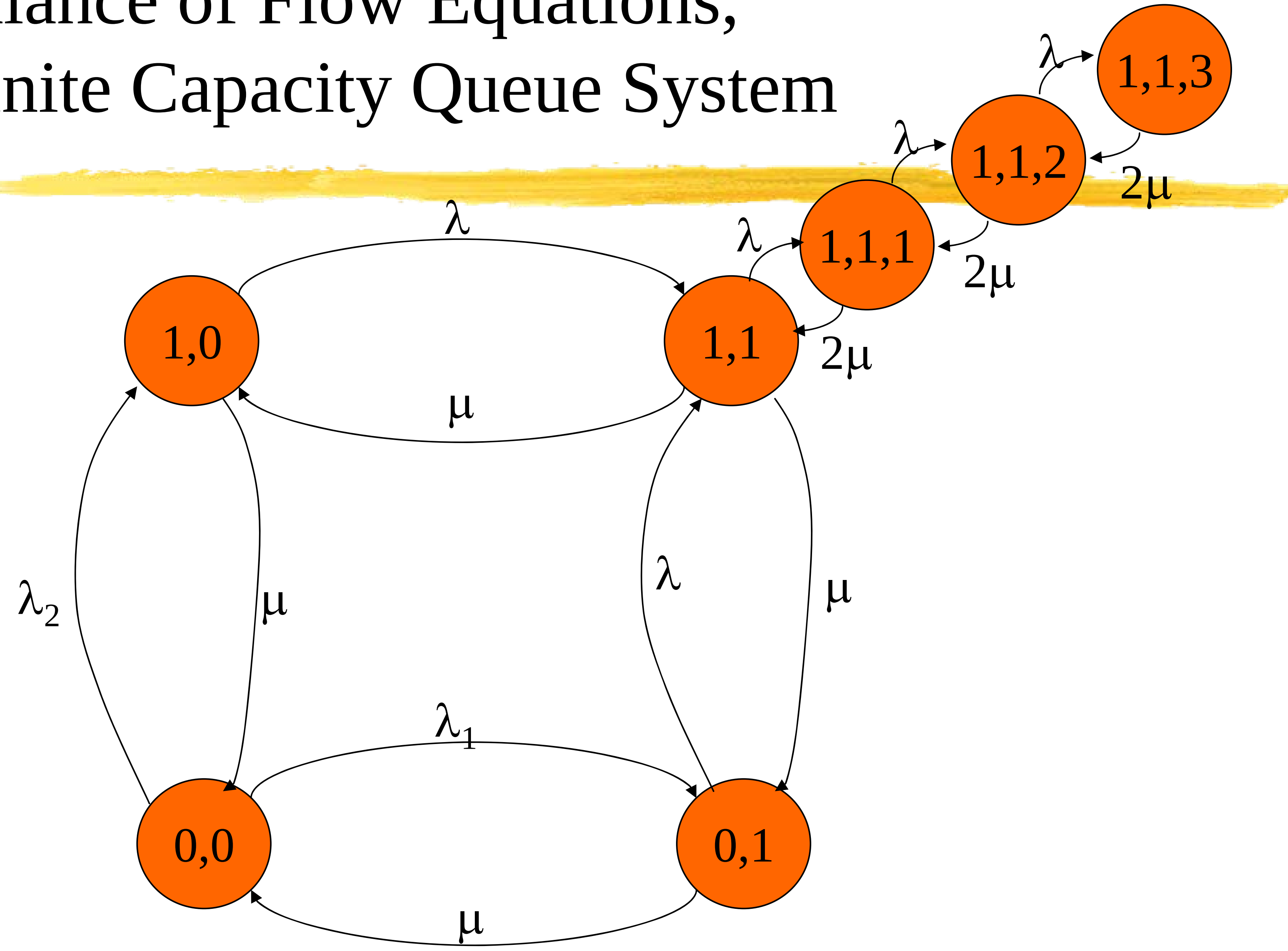
μ

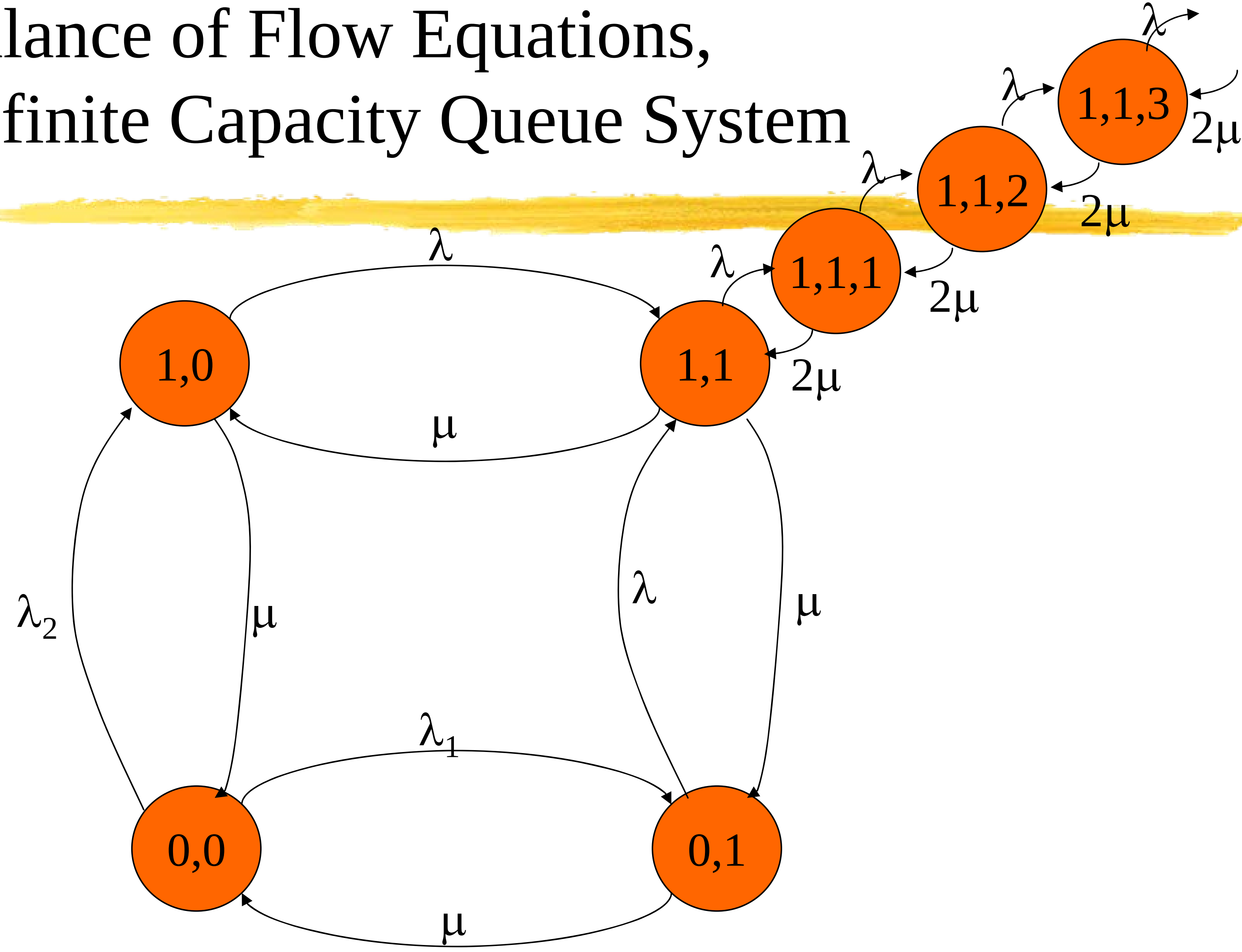
Balance of Flow Equations, Loss System



Balance of Flow Equations, Finite Capacity Queue System



Balance of Flow Equations, Infinite Capacity Queue System



Balance of Flow Equations, Loss System

$$P_{00}\,(\lambda_1 + \lambda_2) = P_{01}\mu + P_{10}\mu$$

$$P_{01}\,(\lambda + \mu) = P_{11}\mu + P_{00}\,\lambda_1$$

Etc.

$$P_{00} + P_{10} + P_{01} + P_{11} = 1$$

Workload and Imbalances

- $\rho_1 = W_1 = P_{01} + P_{11}$
- $\rho_2 = W_2 = P_{10} + P_{11}$
- *Workload Imbalance* $= \Delta W = |W_1 - W_2|$

To Obtain Travel Times, We Must Have Server Response Patterns

- ⌘ f_{nj} = fraction of dispatches that are server n to response area j
- ⌘ $T_n(\mathbf{C})$ = average time for server n to travel to a customer in region **C**
- ⌘ $T(\mathbf{A})$ = average system-wide travel time, assuming that server 1's primary response area is region **A**.

Average System-Wide Travel Time

$$T(\mathbf{A}) = f_{11} T_1(\mathbf{A}) + f_{22} T_2(\mathbf{B\text{-}A}) + f_{12} T_1(\mathbf{B\text{-}A}) + f_{21} T_2(\mathbf{A})$$

Average System-Wide Travel Time

$$T(\mathbf{A}) = f_{11} T_1(\mathbf{A}) + f_{22} T_2(\mathbf{B\text{-}A}) + f_{12} T_1(\mathbf{B\text{-}A}) + f_{21} T_2(\mathbf{A})$$

Queueing

Average System-Wide Travel Time

$$T(\mathbf{A}) = f_{11} T_1(\mathbf{A}) + f_{22} T_2(\mathbf{B\text{-}A}) + f_{12} T_1(\mathbf{B\text{-}A}) + f_{21} T_2(\mathbf{A})$$

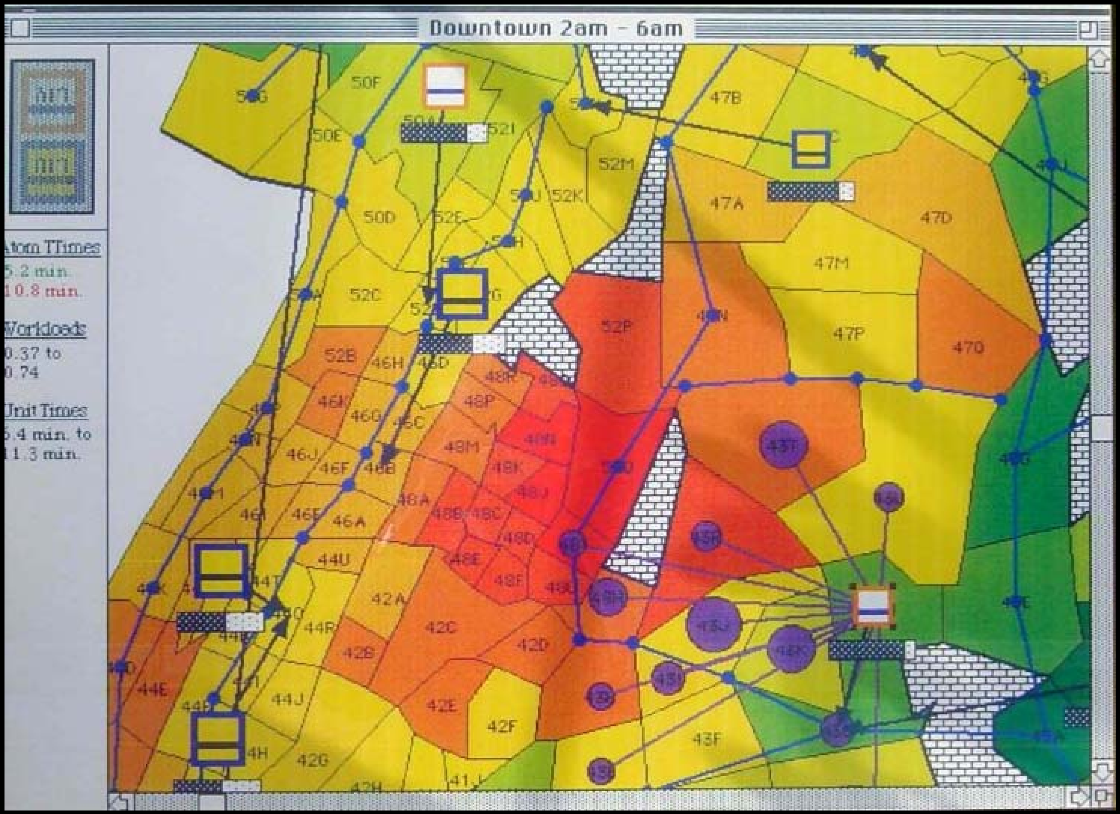
Geometry

How do we obtain the f_{nj}'s?

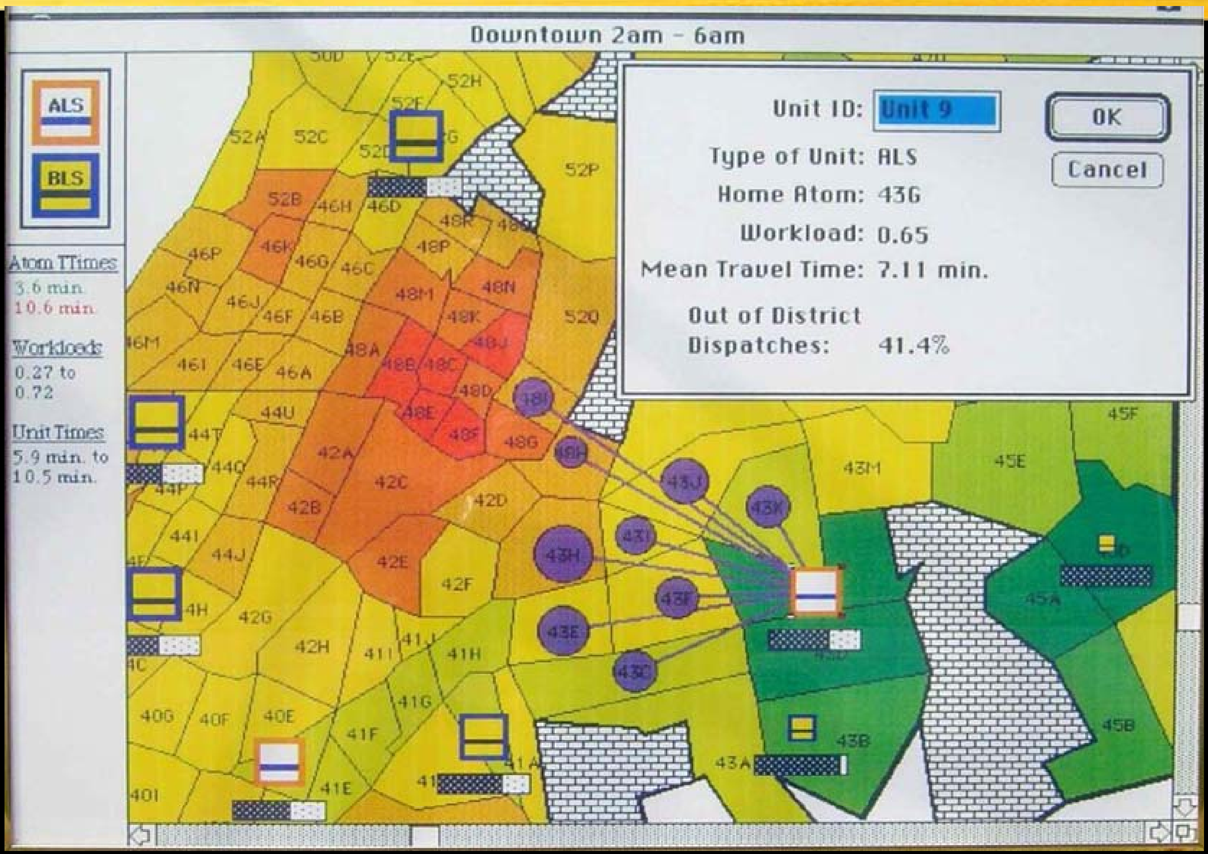
- Consider a long time interval T
- f_{12}=(# requests that assign unit 1 to area 2)/ (total # requests answered)
- Total # requests answered = $(1-P_{11})\lambda T$
- Average # requests that are "server 1 to area 2" is $\lambda_2 T P_{10}$. Why?
- Therefore $f_{12} = (\lambda_2 T P_{10} / [1-P_{11}]\lambda T) = \{\lambda_2 / ((1-P_{11})\lambda)\} P_{10}$

How do we generalize this to *N* servers?

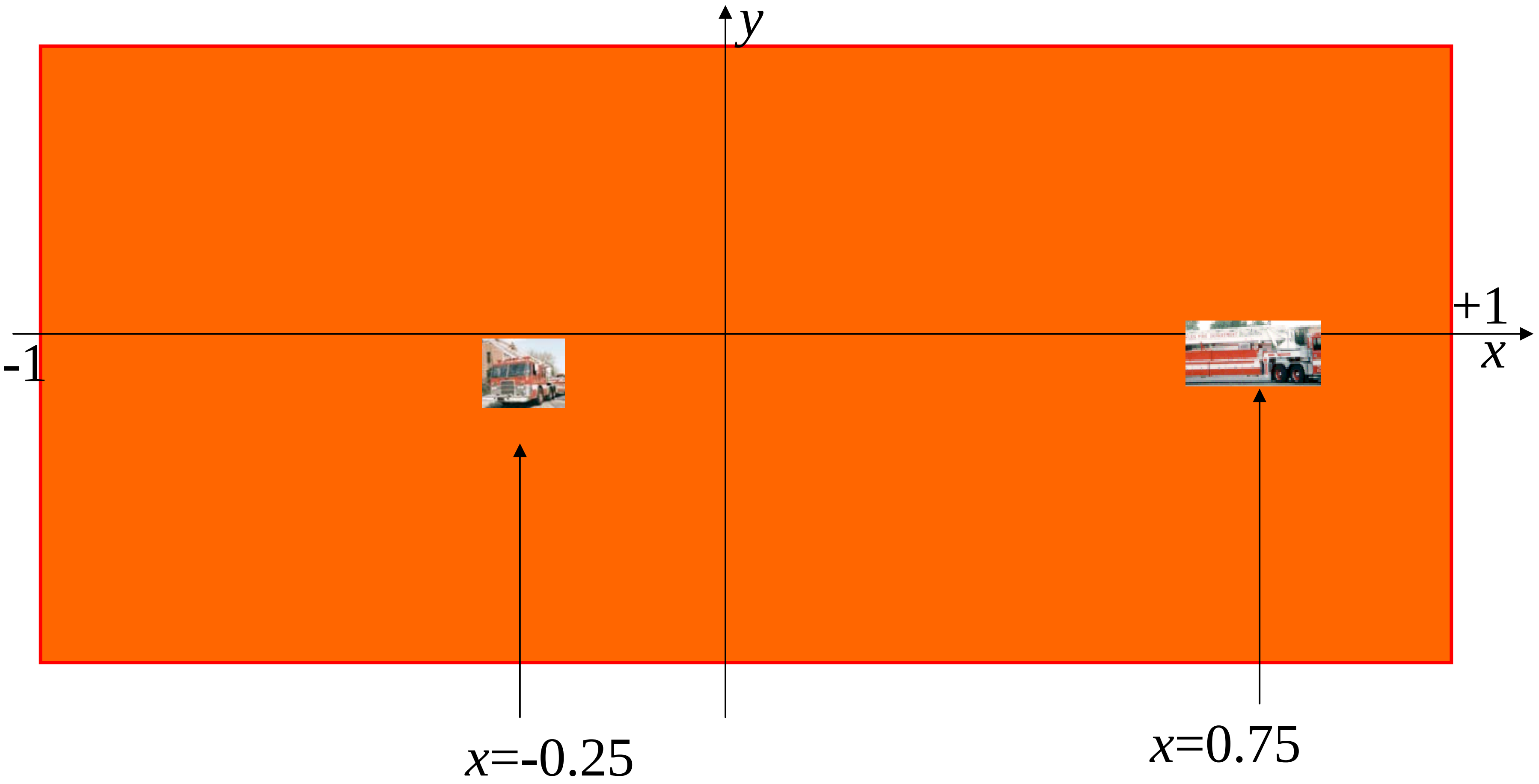
New York City
EMS Hypercube



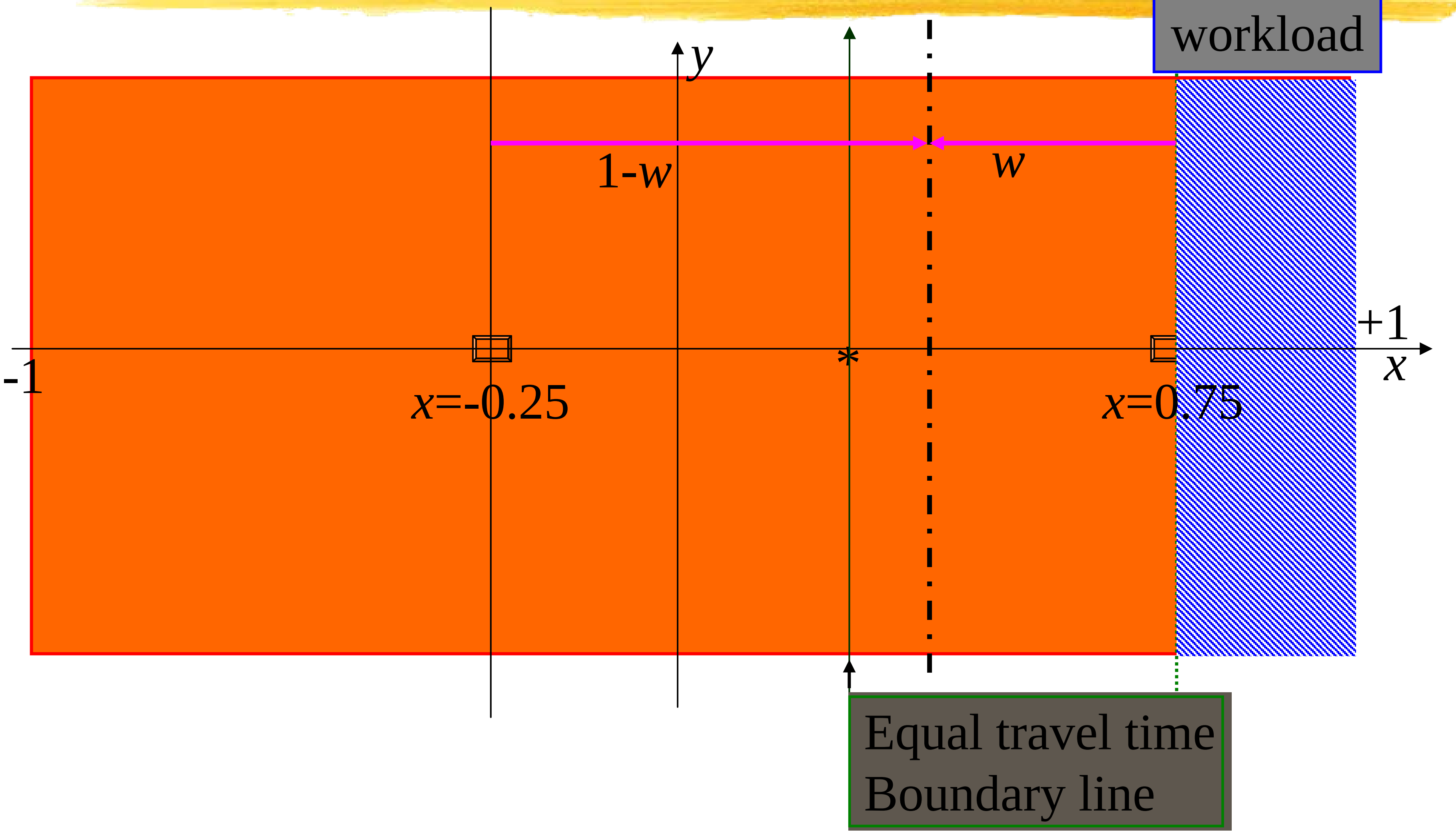
New York City EMS Hypercube



Rectangular City Example



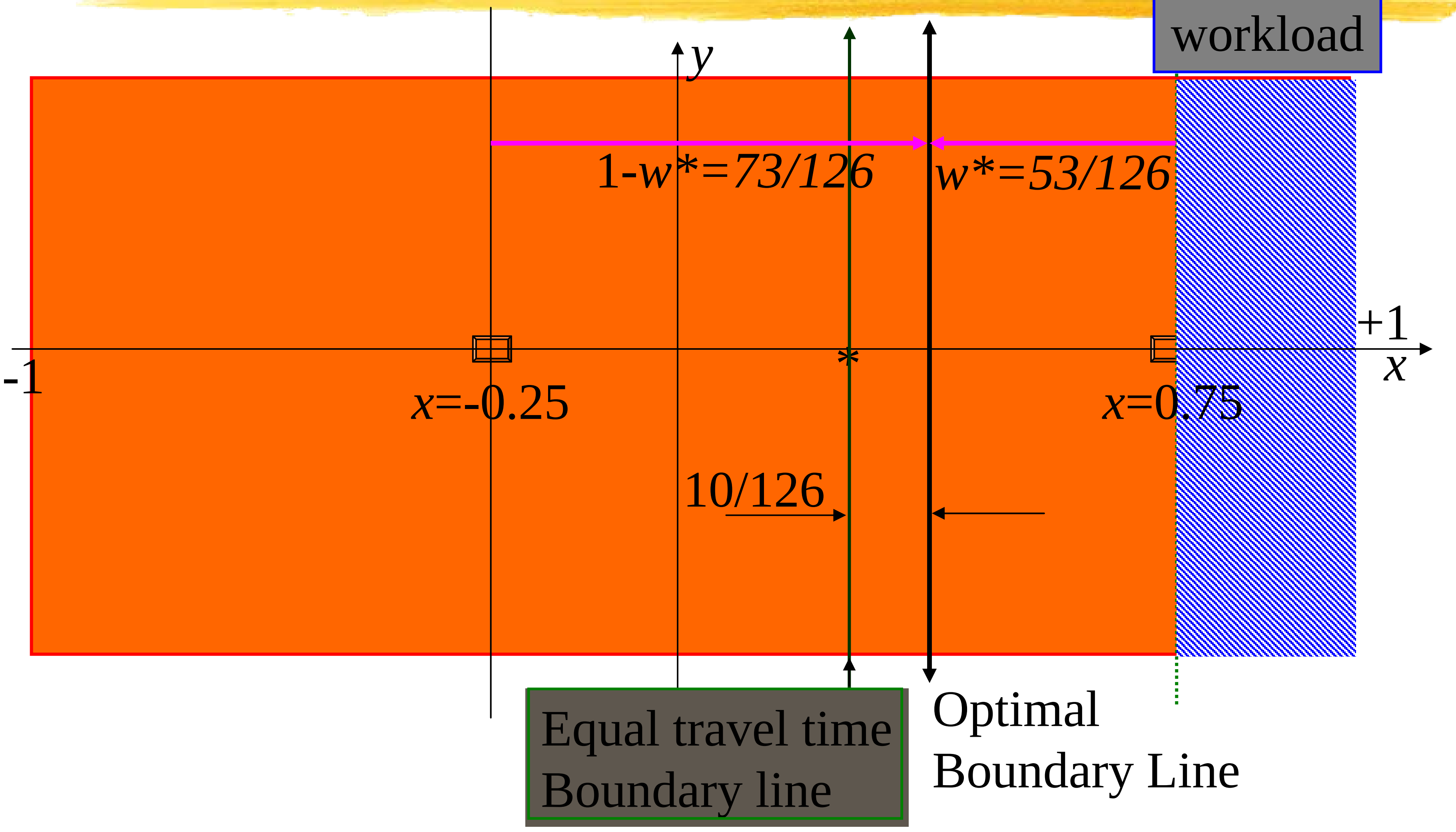
Rectangular City Example



Optimal Districting

- “Dispatch the closest available server’ is often not optimal, where ‘optimal’ implies minimizing mean travel time
- May not be good for reducing workload imbalance either
- With numerical example in book, the optimal boundary line is shifted to the right by 10/126 miles.

Rectangular City Example



Boundary Line Comparison

- Equal travel time boundary line
  - $T(A_{w=1/2})=0.46246$
  - $\Delta W = 0.05236$
- Optimal boundary line
  - $T(A_{w^*})=0.46166$
  - $\Delta W = 0.04405$

Two server Loss Model: Boundary Line Result

- **To minimize mean city-wide mean travel time:**
- ***The optimal partitioning consists of a set of points within the region that is a constant travel time s_0 closer to facility 1 than to facility 2.*** (Carter, Chaiken, Ignall, 1972)
- Does our rectangular city example work for this?

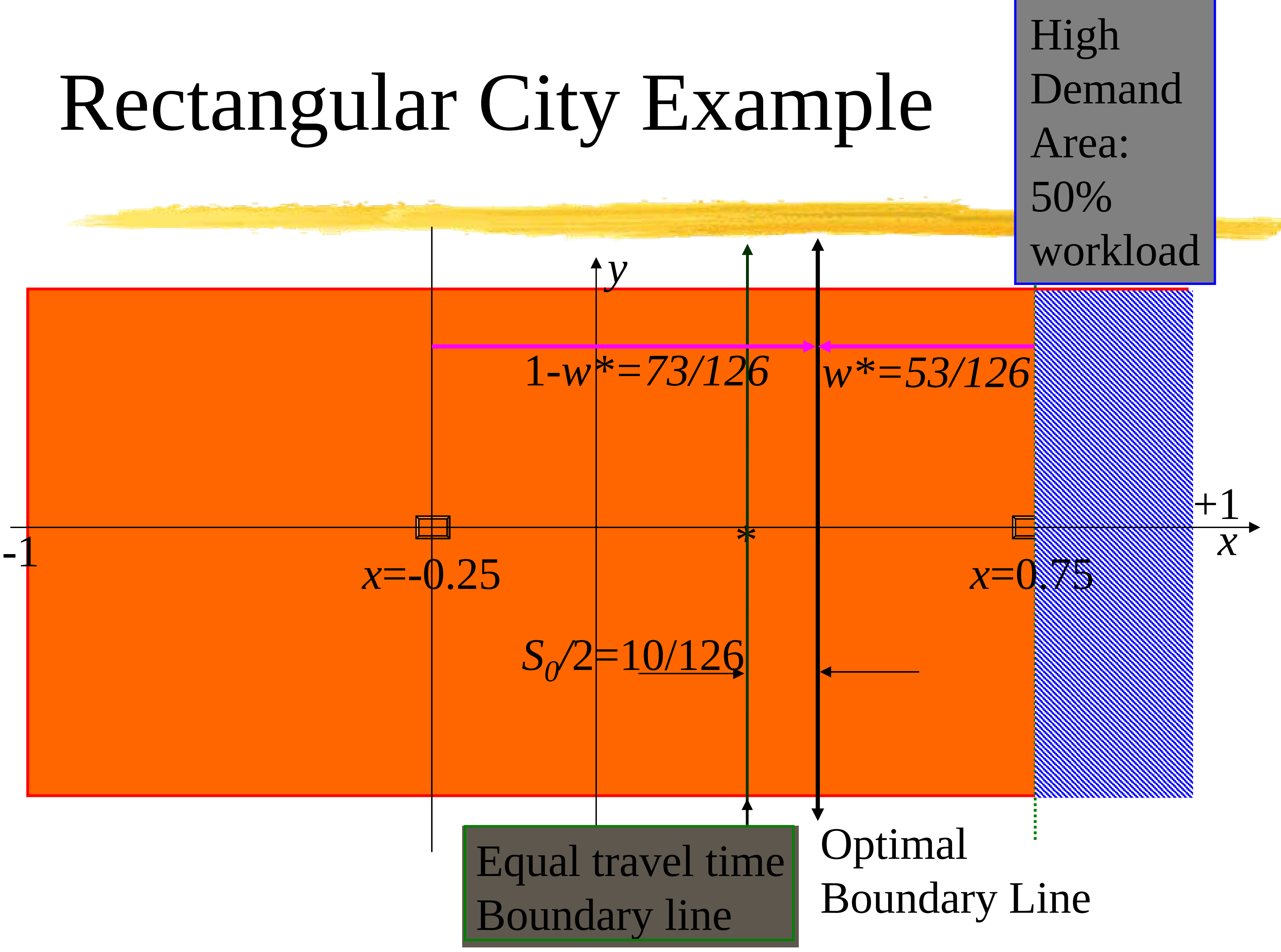
S_0: Optimal Partitioning

$$\alpha \equiv \lambda / 2\mu$$

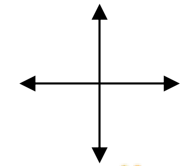
$$\mu_1 = \mu_2$$

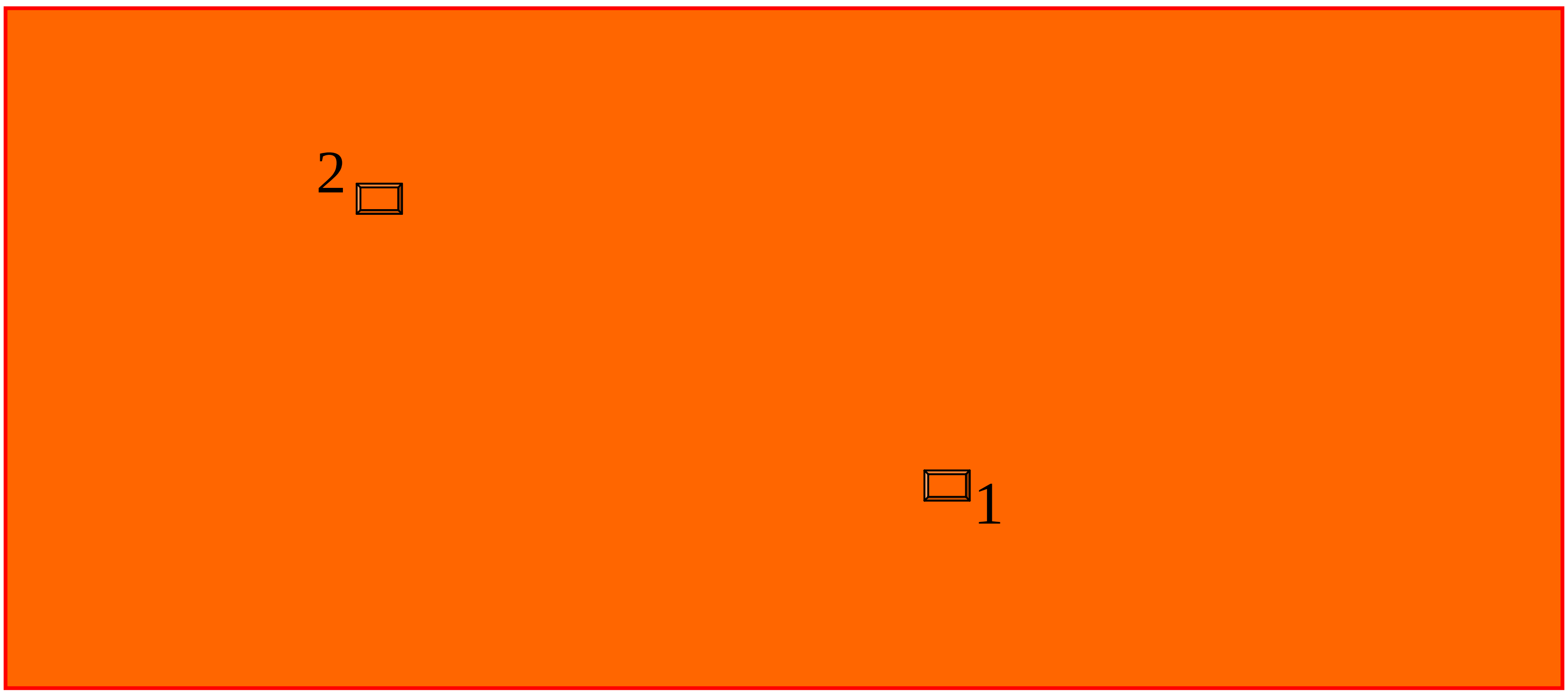
$$S_0 = [2\alpha /(2\alpha + 1)]\{T_2(B) - T_1(B)\}$$

Rectangular City Example

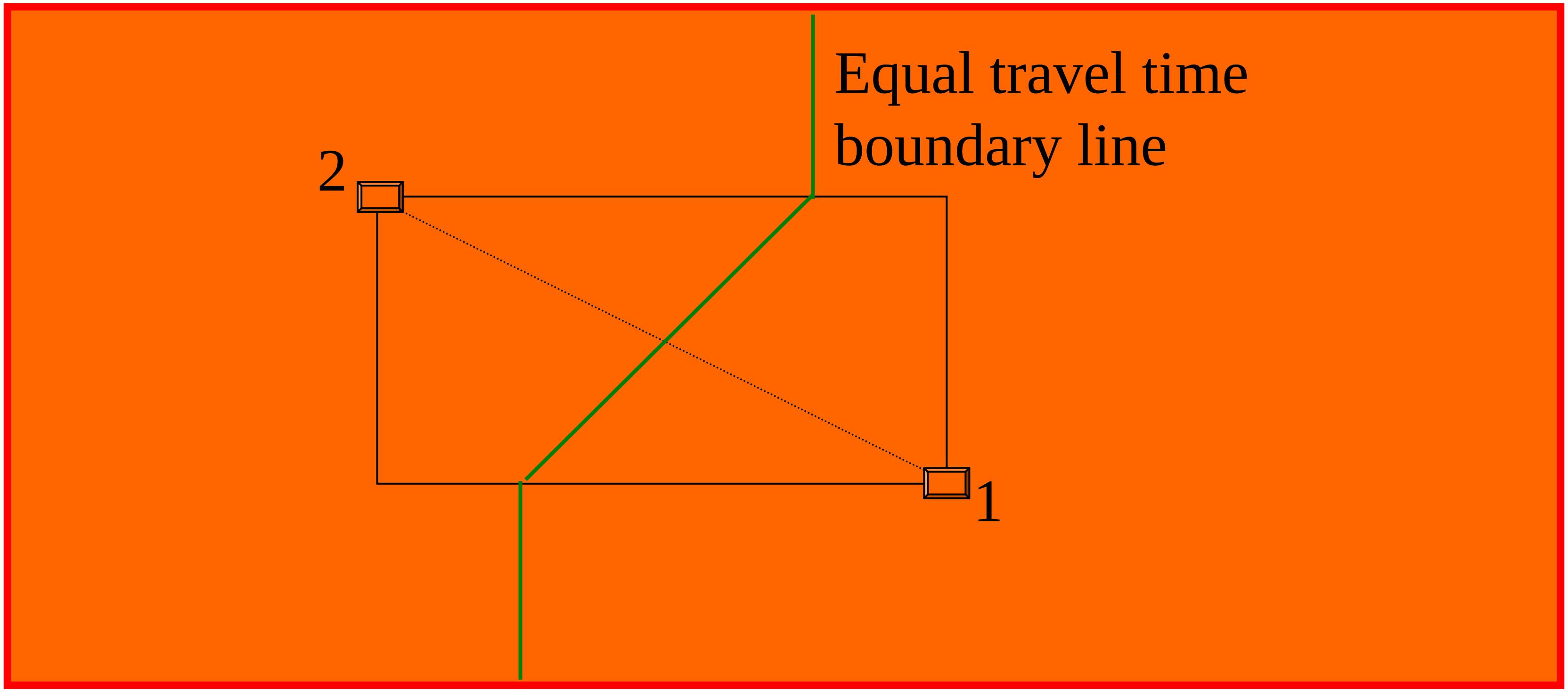


Directions
Of Travel

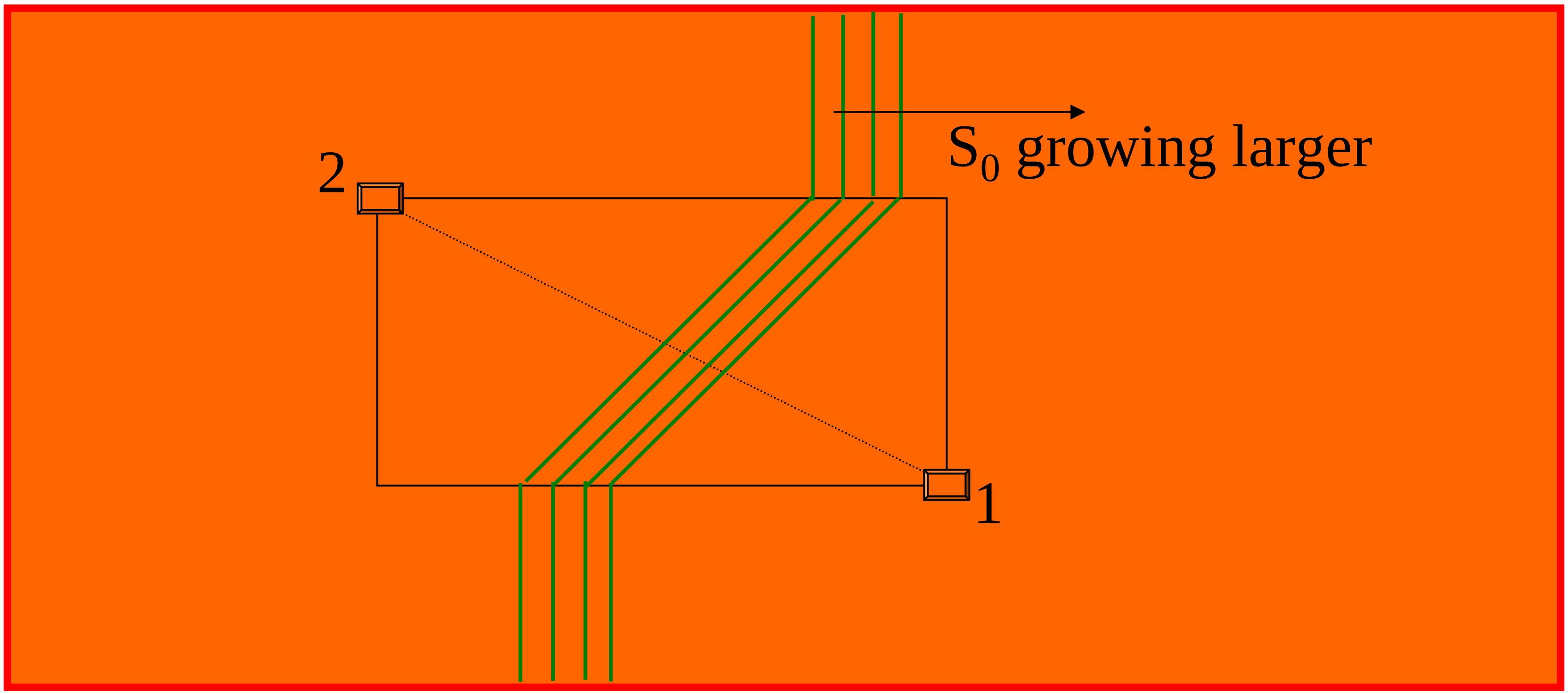




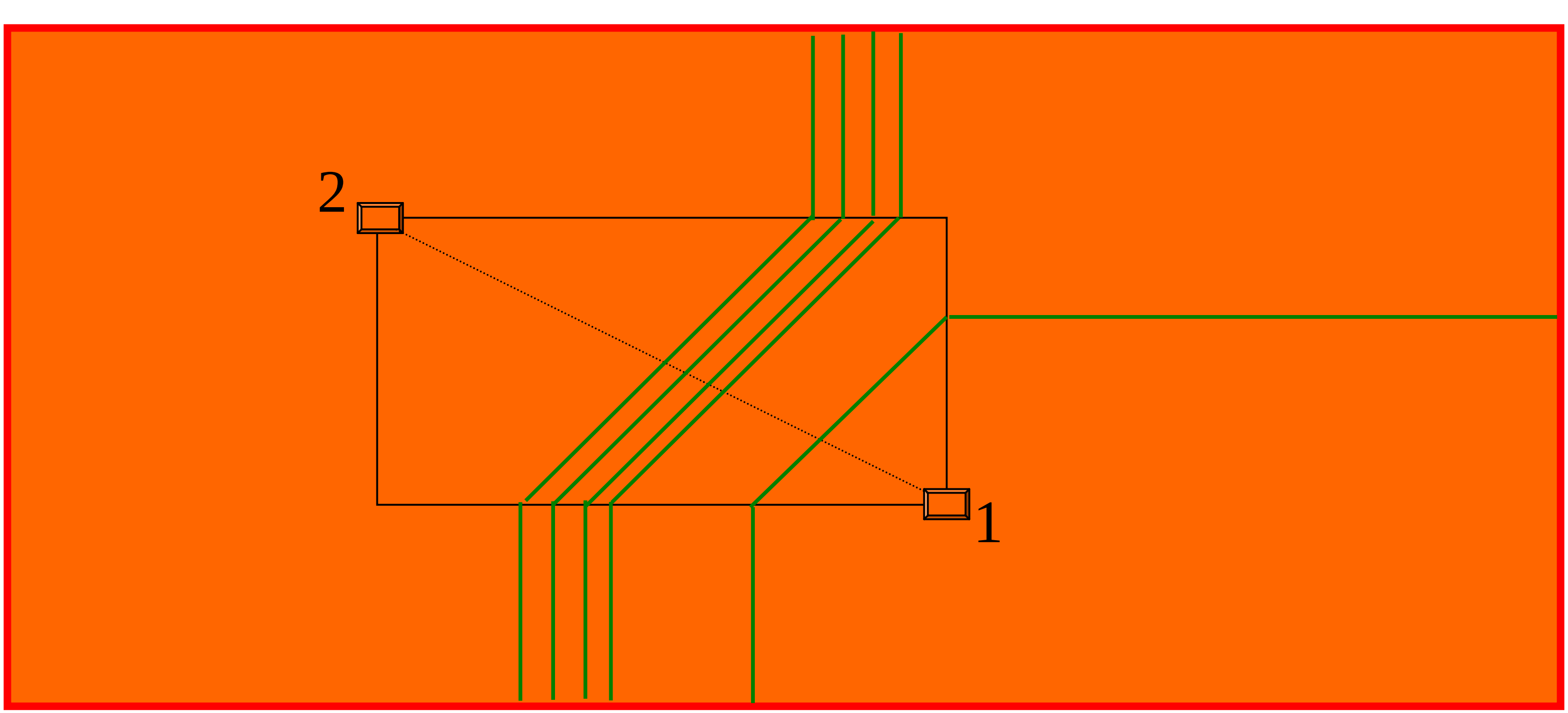
Directions
Of Travel



Directions Of Travel

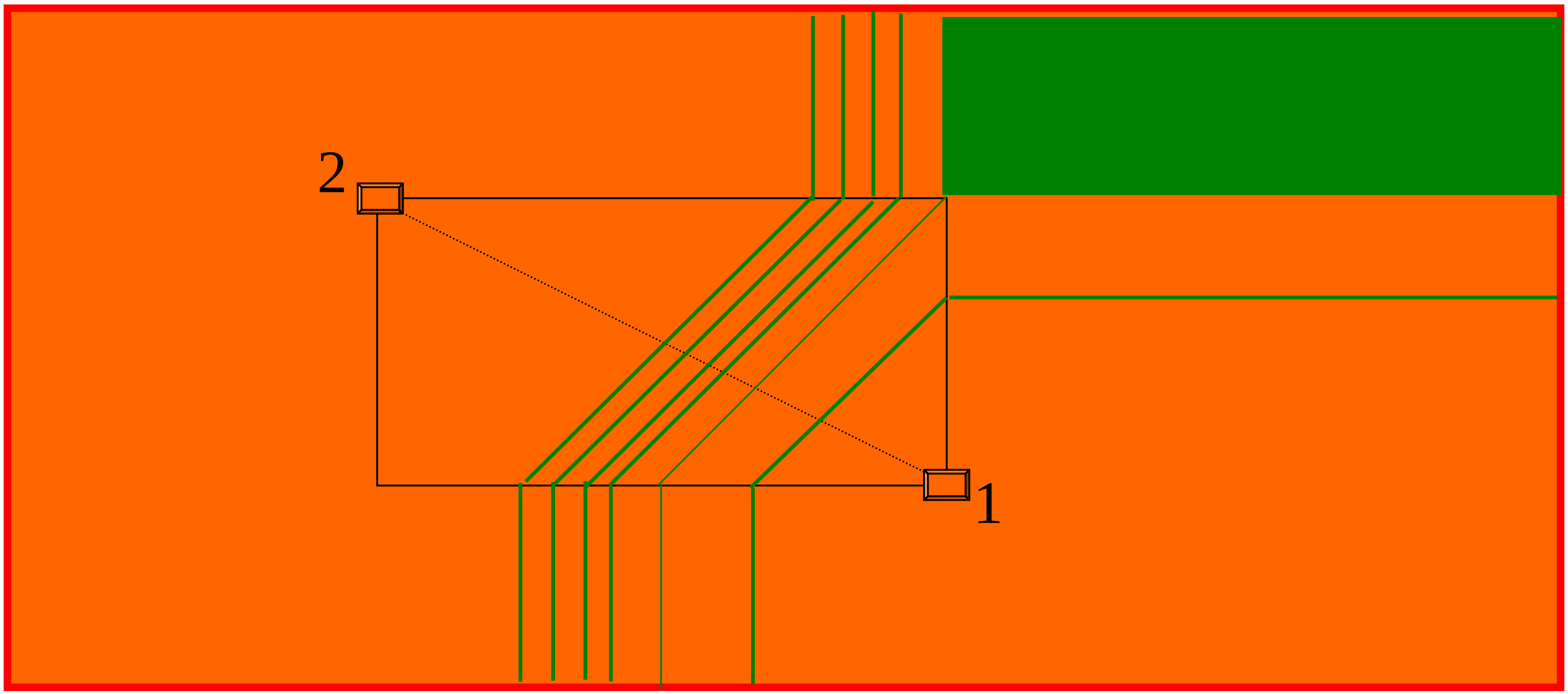



Directions
Of Travel


2
1

And what about the corner case?

Directions
Of Travel



The S_0 result is general

- Works for discrete grid
- One way streets
- General transportation network

- Rick Jarvis, in an MIT Ph.D. thesis, generalized this to N servers