Urban OR Quiz 1
October 20, 2003

"We blew it."
--Henry Fonda (Easy Rider)

Don't blow it: Just answer all questions, showing all work.   Good luck.

1. Suppose that x_1 and x_2 are independent and both U(0,a), and let G(a) be the
    expected value of the *cube* of the larger of the two variables.   Use Crofton's
    method to determine G(a).

2.   Suppose that vehicles are distributed over a large city under a spatial Poisson
    process with parameter λ, and let v_i be the straight-line distance to a given point
    from the ith nearest vehicle to that point.   Find the probability that $v_4 > 4v_1$ *and*
    also that $v_4 > 2v_2$. (We are referring to the joint event involving v_1 and v_2 , not
    two separate events.)

3. Mendel arrives at a random time to ride the Lemon Line, on which the intervals
    between trains are equally like to be 4, 5, and 6 minutes.

    (i)   What is the probability that he waits between 4 and 5 minutes for the
               next train? (HINT: What is the conditional probability that he waits
               between 4 and 5 minutes if he arrives in an interval of length X?)

    (ii)   Would the answer to (i) increase or decrease if the Lemon Line
               became less variable and all intervals between trains were exactly 5
               minutes?   Intuitively, why does the answer move in that direction?

4. Consider a small factory that has 3 machines that are all subject to breaking down.
    Machines break down independently of each other. Whenever a machine breaks
    down, it is sent to the factory's repair shop for service. The repair shop has two
    parallel and identical repair stations. Machines at the shop are repaired in a first
    come, first served order. It has been observed that the time needed to repair a
    machine at a repair station has an exponential pdf with an expected value of 2
    hours. The time until a repaired machine breaks down again (and has to go back
    to the repair shop) also has an exponential pdf with an expected value of 9 hours.

     Find the expected number of machines that are operating at this factory in steady
    state. A numerical answer is expected. *Please explain your work clearly.*