1.020 Ecology II: Engineering for Sustainability
Spring 2008

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Civil and Environmental Engineering

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Problem Set 2 – Species Competition
Due: 5PM Monday March 3, 2008

Problem Description

This problem builds on the class lectures about predator-prey ecosytems. Rather than predation, we consider two similar species competing for a common resource. We use the Lotka-Volterra competition model, which embodies the "competitive exclusion" principle:

"If two competing species coexist in the same niche in a stable environment then one species will eventually crowd out the other."

Or, as a Chinese proverb succintly states:

"One hill cannot shelter two tigers" 二虎相鬥必有一傷

The model is based on the following versions of the logistic growth equation:

$$\frac{dX_1}{dt} = r_1 X_1 \left(1 - \frac{X_1 + X_2}{K_1}\right)$$

$$\frac{dX_2}{dt} = r_2 X_2 \left(1 - \frac{X_1 + X_2}{K_2}\right) - h$$

$$h = 0 \text{ if } X_2 \le 2h_{nom}$$

$$h = h_{nom} \text{ if } X_2 > 2h_{nom}$$

X_1 and X_2 are the population biomasses (kg) of the two competing species and *h* is a harveting rate (kg day^{-1}) that represents human removal of Species 2. Species 2 could represent a commercially attractive tree or fish species. Species 1 could represent a niusance species that is excluded by the harvested species unless the harveting rate is too high. Harvesting is not feasible if $X_2 \le 2h_{nom}$.

The Lotka-Volterra competition model is based on an extension of the carrying capacity concept. The **sum** of the two populations determines the pressure on a common resource that can sustain K_1 (carrying capacity) of Species 1 or K_2 (carrying capacity) of Species 2. The remaining model inputs are the growth rate coefficients r_1 and r_2 (days^{-1}) and the initial populations.

Model Specifications
Construct a MATLAB model that simulates the two populations over the time period of 500 days. You may adjust this period if you feel it would be useful.

You should adapt one of the MATLAB codes provided for the predator-prey examples in class. In particular, use the **ode45** solver to solve the 2 population equations given above. Then your primary programming task will be to rewrite the function **rhs**.

Assume:

- $r_1 = r_2 = 0.2$ day^{-1}
- $X_1(0) = 20 \quad X_2(0) = 10$ kg (initial populations)
- $K_1 = 50$, $K_2 = 60$ kg (the resource can sustain more of Species 2)
- To start, specify simulation times to go from 0 to 500 days in steps of 0.01 days.

Using the Model

1. **Nominal case:**
   - Set $h_{nom} = 0$ (no harvesting) and show by running your program that the competition model obeys the "competitive exclusion" principle (i.e. Species 1 collapses). Why is Species 1 the one that is eliminated?
   - Derive all possible steady-state (equilibrium) populations for this nominal problem.
   - Provide time histories of the two species populations (on the same graph) and a phase plot. Make sure you plot for enough time to clearly show the extinction of Species 1.

2. **Alternative :**
   - In this case you should find the largest harvesting rate $h_{nom} = h_{\max}$ that allows the Species 2 population to dominate (i.e. any larger rate will cause Species 2 to collapse). This value of $h_{nom} = h_{\max}$ is the maximum sustainable harvesting rate. You can find $h_{\max}$ by gradually increasing h_{nom} from zero until Species 2 just collapses. What are $h_{\max}$ and the steady state values (read off the plot) for X_1 and X_2 when h_{nom} exceeds $h_{\max}$?
   - Provide time series and phase plots when h_{nom} is slightly larger than $h_{\max}$.

3. **Comments:** Do you believe that the Lotka-Volterra competition model gives a reasonable (but not necessarily perfect) description of species competition for the case where "*two competing species coexist in the same niche in a stable environment*"? Why? In either case (model reasonable or not) find a textbook or article on competition that supports your assertion with data or an appropriate species-specific example. Most Ecology texts discuss this topic. To get credit you must cite a textbook or a refereed article (not a web site).

Submit via Stellar your MATLAB source file, the requested plots (as ***.fig** files), and a text file with answers to the questions in the “Using the Model” section. Be sure to define all variables and to identify all major calculations in comments included in your MATLAB file (comments will be considered in the grade).