1.053J/2.003J Dynamics and Control I
Fall 2007

Final Exam
18th December, 2007

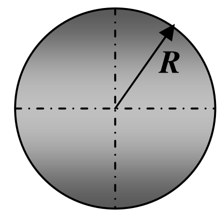
Important Notes:

1. You are allowed to use three letter-size sheets (two-sides each) of notes.
2. There are **five (5) problems** on the exam, and seven pages in total (including this page). Please make sure you are not missing any pages.
3. Each problem is worth 20 points.
4. You have <u>**three hours**</u> to solve the exam.

GOOD LUCK!

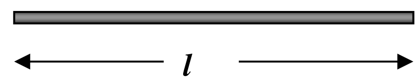
Formulae for the moments of inertia ***I*** of some two-dimensional bodies <u>about their center of mass:</u>

Disk with mass *m* and radius *R*:



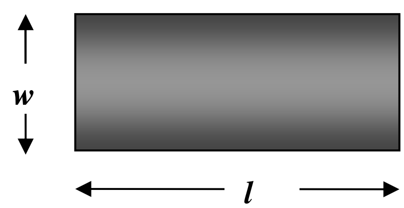
$$I = \frac{1}{2} m R^2$$

Thin rod with mass *m* and length *l*:



$$I = \frac{1}{12} m l^2$$

Rectangular block with mass *m*, length *l,* and width *w*:



$$I = \frac{1}{12} m (l^2 + w^2)$$

Problem 1

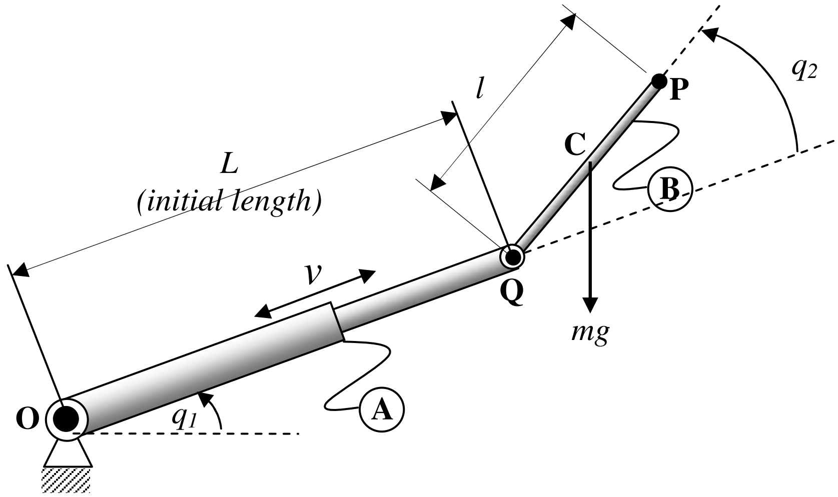


Figure for Problem 1

A robotic arm shown in the figure above consists of link A and link B. Link A rotates at constant angular speed $\dot{q}_1 = \Omega$ (with respect to an inertial frame), driven by a motor at point O. Link A is also telescoping, *i.e.*, increasing in length at a constant rate v from an initial length L. Link B is driven by a motor at point Q and rotates at constant angular speed $\dot{q}_2 = \lambda$ *with respect to link* A. Link B can be taken as a rigid thin rod of constant length l. Note that gravity acts, the mass of link B is m, and the center of mass of link B is point C.

The motor at point Q must clearly apply a torque (a moment couple, in fact) to link B to make it move the way it does. We seek to calculate this torque for motor and bearing sizing purposes. The sub-parts below break this up into two steps.

Using the Newtonian approach:

a. **(10 points)** Find the acceleration of point C.

b. **(10 points)** Now find the torque exerted on link B by the motor at point Q.

Problem 2

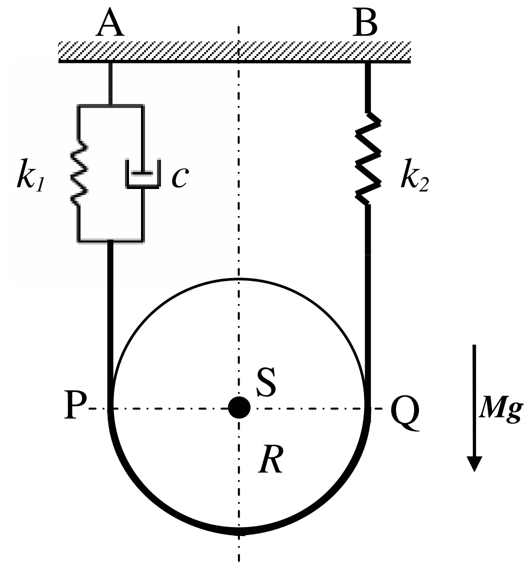


Figure for Problem 2

We present the model of the lower-sheave of a crane – the part the hook hangs from. A pulley of mass M and radius R is connected to a spring-damper system, with spring constant k_1 and damping constant c, on one side, and to a spring of spring constant k_2 on the other side, as shown in the figure above. The pulley (which can be approximated as a disk) rolls without slipping about the cable, and it can also translate upwards and downwards along the vertical axis, but we assume that the pulley <u>can't move to the left or right</u>, so that the springs and dashpot remain vertical at all times. Note that gravity acts. We wish to examine the behavior of the pulley and need the equations of motion, say when the pulley is pulled down (with some rotation due to unequal extensions of cable-lengths AP and BQ) and released. <u>Broad redundant hint: The system has 2 dof's!</u>

(20 points) Using the Lagrangian approach, find the equation(s) of motion of the system.

Problem 3

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Photo of skateboard half-pipe.

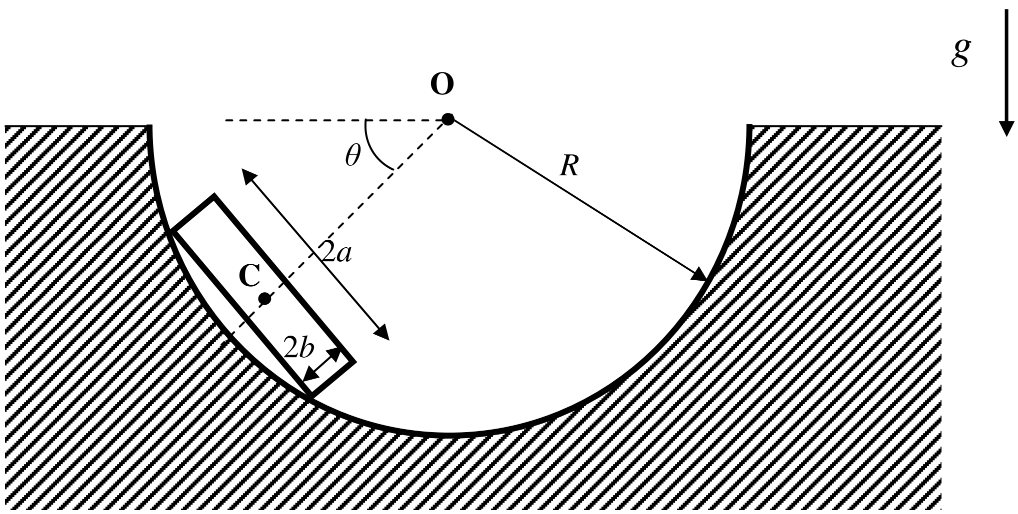

Figure for Problem 3

We seek to model the behavior of skateboard in a half-pipe. A rectangular block of center *C*, mass *m*, length 2*a*, and width 2*b*, slides along a semicircular surface of center O and radius *R*, as shown in Figure. The block and surface are both smooth so that frictional forces can be neglected. Note that gravity acts. Ignore any friction or air-drag.
Hint: We recommend using the angle θ shown in the figure as one of the generalized coordinates to simplify the math later. Do you need others?

Using the Lagrangian approach:

a. **(2 points)** Gimme question: How many degrees of freedom does the system have? Define the corresponding generalized coordinate(s).

b. **(7 points)** Find the equation(s) of motion of the system.

c. **(4 points)** Find the equilibrium position(s) of the system in the physically meaningful range of $0 \leq \theta \leq \pi$

d. **(7 points)** Linearize the equation(s) of motion about the equilibrium position(s) found in (c), and study the stability of each equilibrium position.

Problem 4

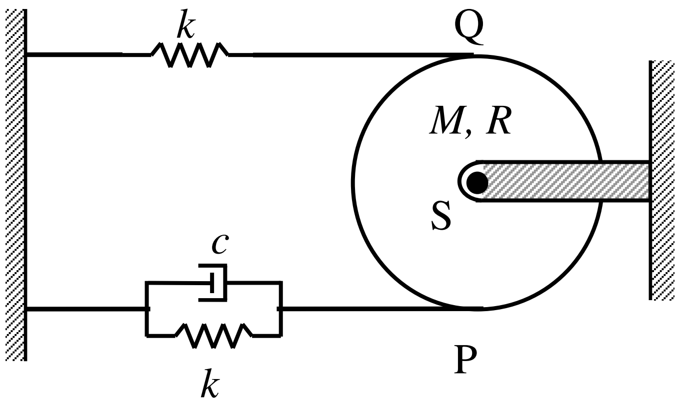

Figure for Problem 4

We are studying the behavior of a winch where we worry about vibration. A pulley of mass M and radius R has a cable wound around it. The cable is attached to an immovable, effectively fixed, load on the other side. The cable can be modeled as having on one side a spring constant k and damping constant c, and on the other side a spring constant k, as shown in the figure above. The pulley (which can be approximated as a disk) can rotate about its fixed spindle with axis through point S. The cable winds around the pulley and the pulley rotates without slipping. The springs and dashpot remain horizontal at all times. The system is un-stretched but taut in the stated configuration. The pulley is rotated by a small amount and then released.

Using any approach you prefer:

a. **(15 points)** Find k and c so that the system has a damped natural frequency η (a known value) and a damping coefficient $\zeta = 0.8$.

b. **(5 points)** Keeping the springs and dashpot with the values that were found in *a*, assume now that the pulley is replaced with a larger pulley of radius $2R$ but having the same mass M. How will the system's damping factor ζ change?

Problem 5

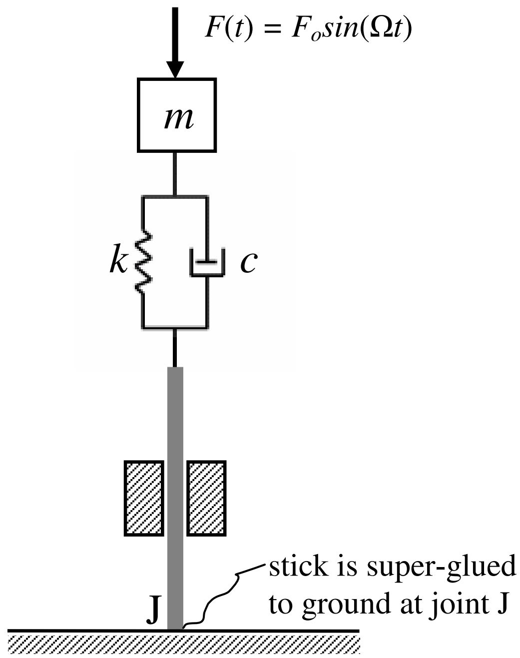

Figure for Problem 5

In this problem we will study the dynamics of a pogo-stick. An odd friend of yours has super-glued a pogo-stick to the ground and is now jumping on it. To model this, assume that the pogo-stick and your friend can be treated as a lumped mass m, connected to a spring dashpot system with spring constant k and damping constant c, as shown in the figure. The jumping action can be modeled as exerting a force $F(t) = F_o sin(\Omega t)$. Ignore lateral motion, and assume the spring and dashpot remain vertical at all times. Remember that in parts (a) and (b), we will assume that the lower end of the pogo-stick is fixed to the ground at joint J by using super-glue. **Gravity can be neglected** in this problem.

Using any approach you prefer:

a. **(2 points)** Define the generalized coordinate(s) and find the equation(s) of motion of the system.

b. **(8 points)** Derive the particular solution and the complete solution for the displacement of the mass. Assume the system is under-damped. Leave any unknown constants as unknowns because we haven't given you initial conditions.

c. **(10 points)** In this part we assume that enough time has passed so that we only have to worry about the particular solution. We worry, in fact even hope, that the super-glue joint is going to fail, much to the merriment of all. For this we need to calculate the vertical force on the joint J. Please derive that force. What is the maximum value that this force can have?