1.053J/2.003J Dynamics and Control I
Fall 2006

Exam 1
30th October, 2006

Important Notes:
1. You are allowed to use one letter-size sheet (two-sides) of notes.
2. There are three problems on the exam. You have 80 minutes to solve them.

1. A space shuttle orbiter and an astronaut

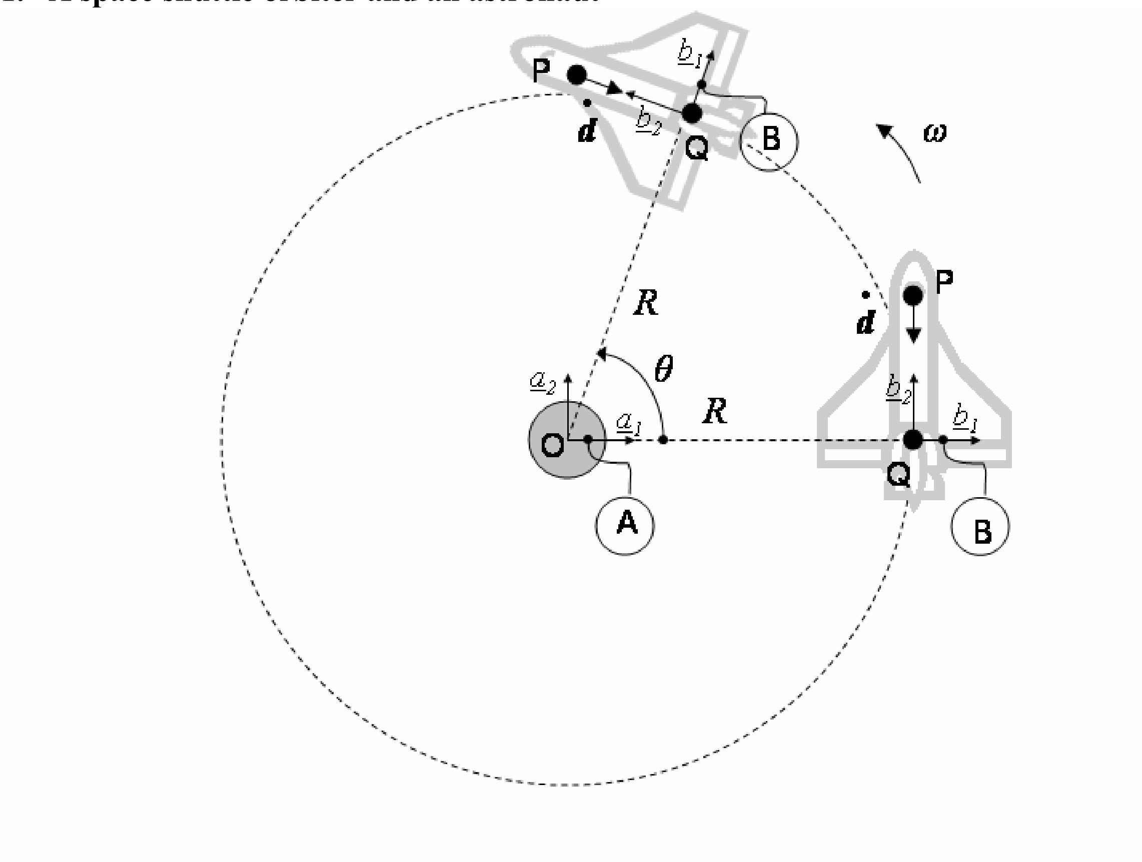

Figure 1

A space shuttle is orbiting the earth in a circular orbit with a *constant* angular speed ω as shown in Figure 1. A point Q fixed on the space shuttle is at distance R from the origin O. Frame A is fixed. Frame B is attached to the space shuttle. The space shuttle orbits such that the basis vector $\underline{b}_2$ always remains tangential to the orbit as shown in the figure. Clearly, $\dot{\theta} = \omega$.

a. Find the acceleration of point Q with respect to frame A as a function of θ and $\dot{\theta}$.

b. An astronaut represented by point P begins to walk with a *constant* speed $\dot{d}$ with respect to the space shuttle along $-\underline{b}_2$, *i.e.* the negative $\underline{b}_2$ direction from the cockpit towards point Q as shown in the figure. Find the acceleration of P with respect to frame A if P is at distance d from point Q.

c. Find a relationship between $\dot{d}$ and ω such that at a certain location on her path, her acceleration with respect to frame A is exactly zero. Find that location as well relative to the space shuttle.

2. L-shape

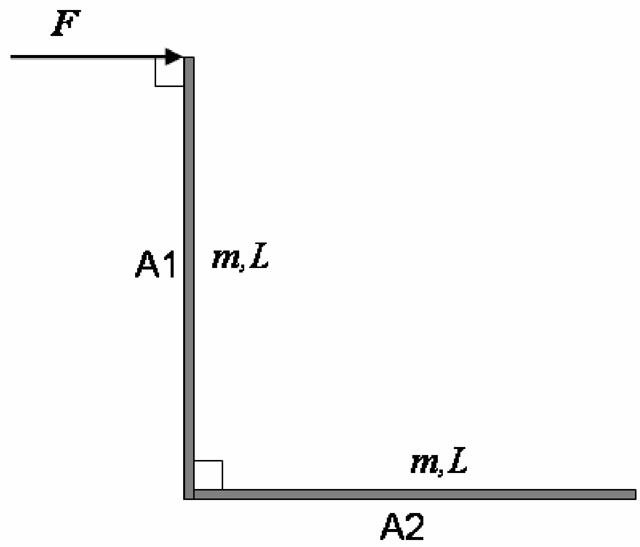

Figure 2

Consider the L-shaped structure, shown in Figure 2, which consists of two slender rods A1 and A2 that are glued together. Each has mass m and length L. A massless(!) rocket is attached to one end of rod A1. At $t = 0$, the rocket is ignited and it produces a thrust of magnitude F on the L-shape. Throughout the motion, the direction of force F remains perpendicular to rod A1 as shown in the figure. Ignore gravity and assume that the motion is planer.

a. Find the center of mass of the L-shaped object.
b. Assume that the L-shaped object behaves as a rigid body in 2D. Define convenient frames, identify generalized co-ordinates of the L-shaped object and find the equations of motion of the system.

3. System of pulleys

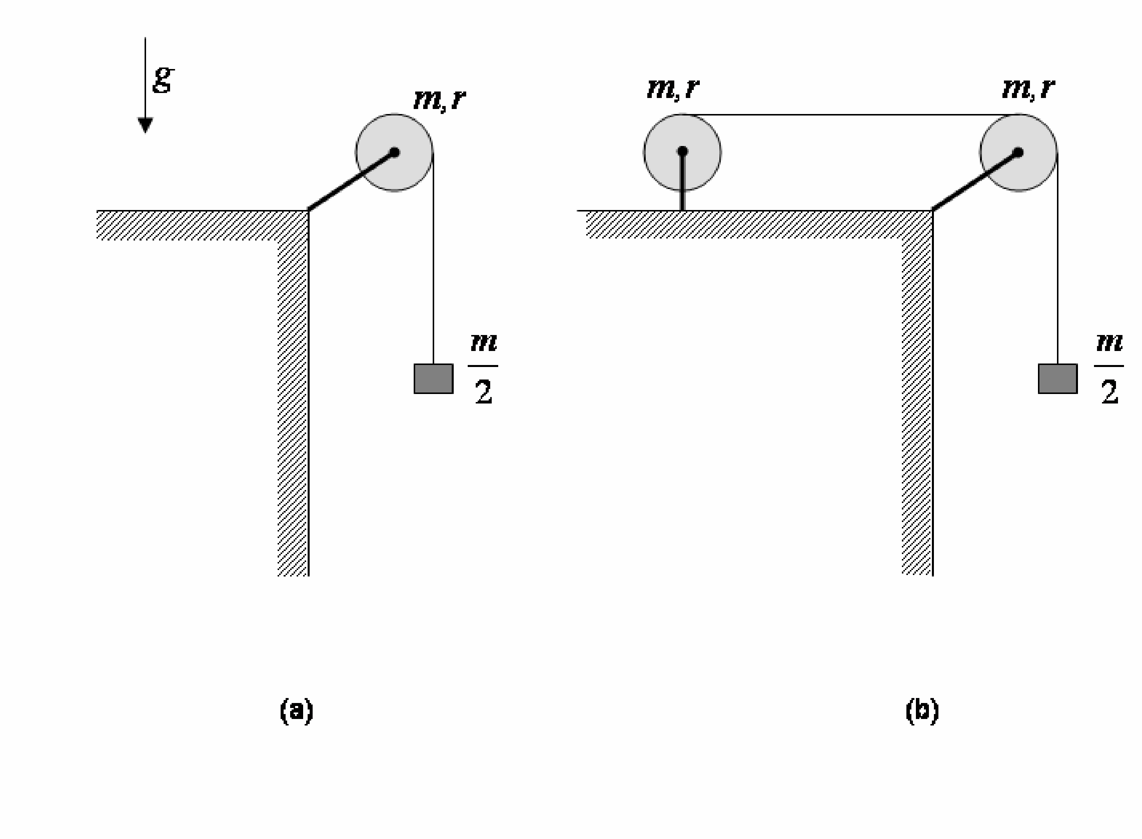

Figure 3

a. A point mass of mass $\frac{m}{2}$ is hanging from a thin, massless and inextensible string that winds several turns around a pulley of mass m and radius r as shown in Figure 3(a). The pulley is free to rotate about the pivot supported from the table and accordingly, the string winds/unwinds and the point mass moves up and down. This system can be imagined as a spool. Initially the point mass is supported such that it is hanging vertically and suddenly the support is removed. Using the work-energy principle, find the acceleration of the point mass. Note that gravity acts.

b. Instead of one pulley, now suppose that the same point mass is hanging from a string that goes over two identical pulleys, each of mass m and radius r, as shown in Figure 3(b). The left-most pulley acts like a spool. Find the acceleration of the point mass in this case.

c. Observe the pattern of your answers in part a and part b. Based on this, how many identical pulleys you need to design a system such that the acceleration of the point mass is $\frac{g}{5}$, where g is the acceleration due to gravity.