# 1.204 Lecture 23

## Analytic approximations
## Vehicle routing
## Transit design

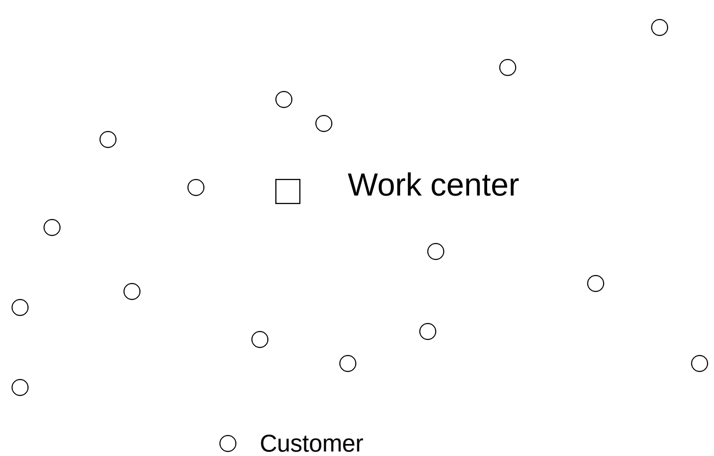
## Analytic approximations

- **First spiral in developing problem solution**
  - Assist in requirements, prototyping, initial results, review
  - Many analytic approximations are visual, unlike almost all algorithms
    - Recall role of visualization in finding roots of equations, and how poorly algorithms do without it
  - Generally allow a broader treatment of the question, with more variables, more flexible objectives and constraints
  - Provide guidance in framing heuristics
    - Many real problems do not have optimal algorithms
    - We have very few O(n) or O($n^2$) algorithms for complex problems; most are O($2^n$)

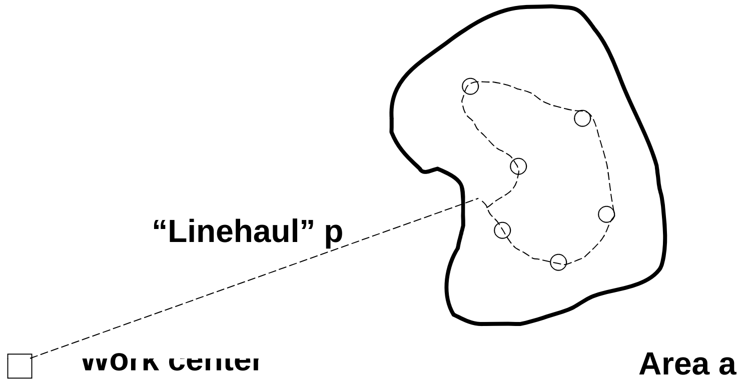
## Vehicle routing

- Variables:
  - Number of routes or employees
  - Number of customers
  - Time windows or appointments
  - Capacity of vehicle or employee
  - Whether customers are known at start of route
  - And many others…
- Objectives
  - Customer service (timeliness, appointments)
  - Cost minimization
- Constraints
  - Labor rules, …

## Dispatch routing options



## How to serve customers?

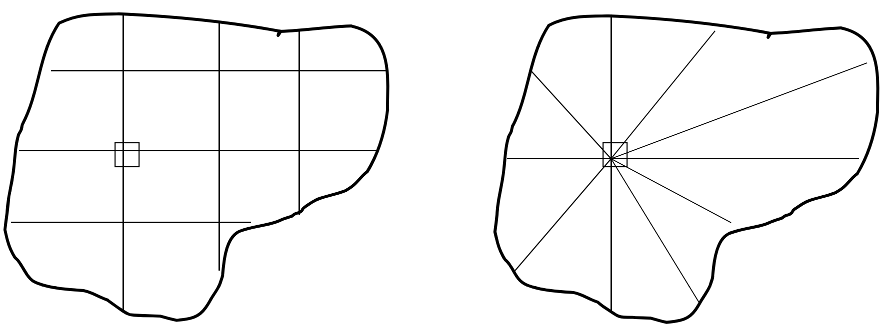



**Trip length L to visit n randomly distributed customers in area a and return:**

$$L = 2p + k\sqrt{na}$$

(Beardwood, Halton, Hammersley 1959)

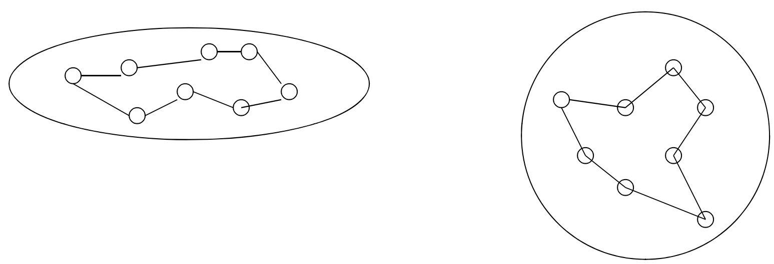
## Shape of dispatch zones



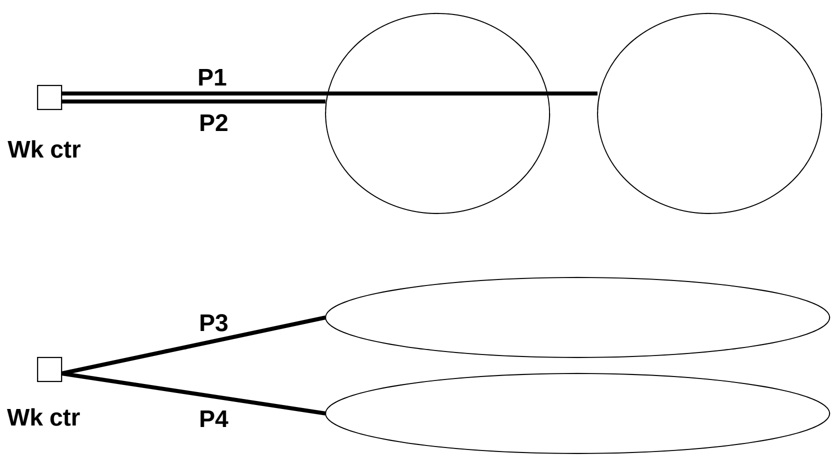
**Which shape is better?**

## Zone shape

- **Let's try to elongate them**
  - **Tour length is the same for same number of points, same area, different shapes**

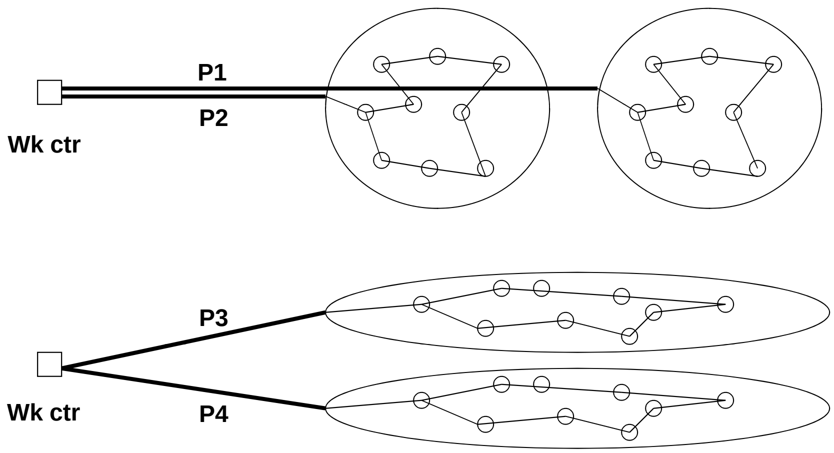


## Comparison



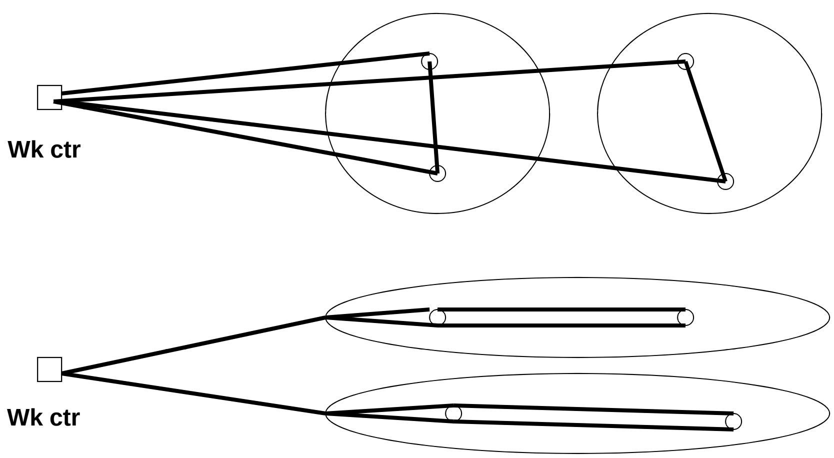

**Elongated zones have shorter driving distance if there are a lot of customers in each zone**

## Elongated zones, many customers




With many customers, elongated zones are better

## Fat zones, few customers




If there are only a few customers, shape matters more
In this case,'fat' zones are better

## Rules for building tours

- The "break point" for fat versus skinny zones is about 6 customers, based on simulation and geometric probability
  - If 6 or more customers can be served on a route:
    - Break up the area into skinny zones with the target number of points (6 or more)
    - Build tours in each zone, and fine tune
  - If 5 or fewer customers can be served on a route:
    - Break up the area into fat zones with the target number of points (5 or fewer). Only a few zones will touch the work center
    - Build tours in each zone, and fine tune
- Many dispatch systems 'cluster' jobs, which implicitly creates fat zones rather than skinny
  - Rack servicing has ~30-40 stops per day. Use skinny zones
  - Telecom dispatch has ~2-4 jobs per day. Use fat zones
  - Shared taxi has 2-4 stops per tour. Use fat zones
  - Dial-a-ride hopes to have 8-10 stops per tour. Use skinny zones

## Rules for building tours-time windows

- Build skeleton elongated or fat routes (implicit zones) based on expected customer demand
  - Non-intersecting, non-overlapping routes
- Schedule stops in the following priority:
  - Tight time windows far from work center first
  - Then tight time windows near work center
  - Then other jobs far from work center
  - Then other jobs near work center
  - Pull next day's work into today's routes as feasible
  - Don't give successive jobs with tight time windows to the same tech, if it can be avoided
- Rules determined from analysis and simulation

## Summary- dispatch analysis

- Done before writing dispatch algorithm or system
  - Understand the problem, objectives and constraints
  - Use analytical optimization, simulation, probability, …
  - Deal with broader set of issues than a single algorithm
  - Develop guidance for heuristics to be used

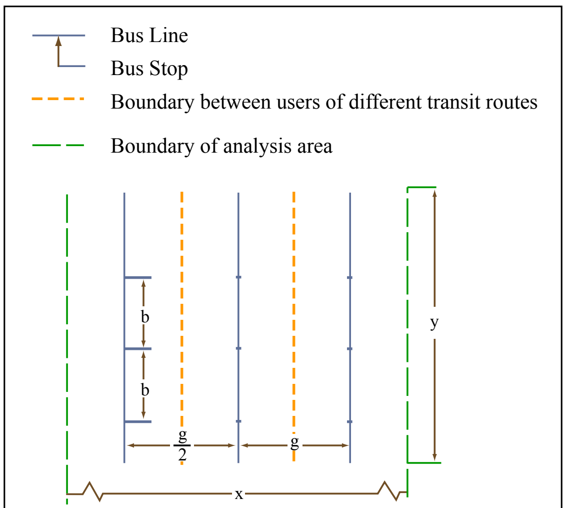
## Transit system design

- Variables for bus system design:
  - Number of routes (route spacing)
  - Headway (frequency of service)
  - Fare
  - Vehicle size
  - Route length
  - Bus stop spacing
  - Express versus local service
  - Transfer pattern

## Transit system design

- **Objectives:**
  - **Maximize ridership**
  - **Minimize deficit (or maximize profit)**
  - **Equity in service levels**
- **Constraints**
  - **Available resources (deficit limit)**
  - **Minimum service levels**
  - **System capacity**

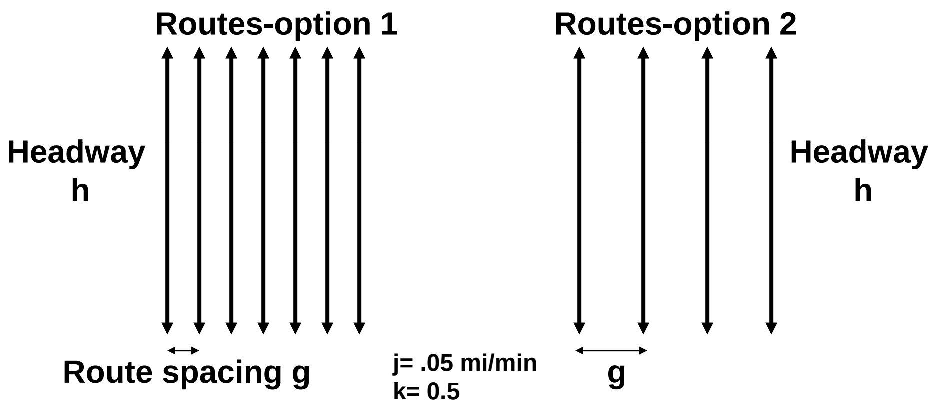
## Route spacing and headway




Spatial variable in local area analysis

Figure by MIT OpenCourseWare.

## Route spacing and headway



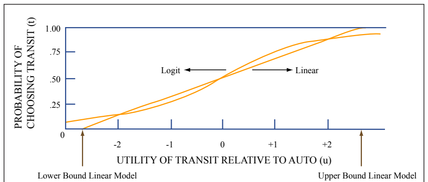

| | g= 0.5 mi<br>h= 20 min | g= 1 mi<br>h= 10 min |
|---|---|---|
| Avg walk= g/4j | 2.5 min | 5 min |
| Avg wait= kh | 10 min | 5 min |
| Total walk+wait | 12.5 min | 10 min |

## Route spacing and headway

- General result over many objectives and constraints:
  - Optimal route spacing and headway are related by
    - h*= g*/ 4jk
  - At this point, average walk time= average wait time
  - Complications:
    - Ratio of wait time/headway, k, may vary with headway
    - There may be a ‘walk refusal distance’, and demand response to walk distance may be nonlinear
    - Headway varies over the day on a route: either choose an average spacing, or have peak-only routes
  - Option 2 on previous slide is ‘optimal’
    - Same operating cost and capacity as option 1
    - Better service (lower sum of walk and wait times)
    - Higher ridership

## Fare and demand function

- **Introduce a demand function:**



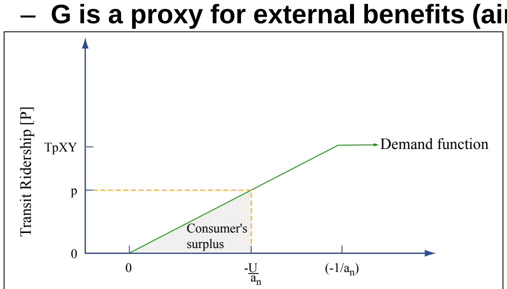

Linear mode share model

Figure by MIT OpenCourseWare.

- **Mode share $t = (a_1 + a_2(kh+(g+b)/4j) + a_3 * d/v + a_4 * f + a_5 * d)$**
  - **Where**
    - **a1..a5 are demand coefficients (a5 is auto coefficient)**
    - **d is route distance, v is bus velocity, f is bus fare**
- **Total bus ridership in area P = TpXYt**
  - **Where**
    - **X,Y are dimensions (mi),**
    - **p is trip density (trips/$mi^2$/min),**
    - **T is time period (min)**

## Cost and objective function

- **Bus operating costs:**
  - **C= 2 XYTc/ghv**
  - **There are X/g routes operating 2T/h trips of length Y/v at unit cost per minute of c**
- **Objective function: maximize net social benefits**
  - **Max consumers' surplus G + revenue R (=Pf) – cost C**
  - **Subject to a deficit constraint (C-R <= M)**
  - **G is a proxy for external benefits (air quality, GHG, …)**




Consumers' surplus measure. TpXY = total travel by all modes. $P_0$ = actual number of transit users, $U_0$ = actual utility of transit relative to auto.

Figure by MIT OpenCourseWare.

## Overall formulation

- **Max net benefit subject to (cost – revenue) <= M**

$$\max B_2 = -TpXY(a_1 + a_2(kh + (g + b)/4j) + a_3 d/v + a_4 f + a_5 d)^2/2a_4$$

subject to

$$2XTcY/ghv - TpXYf(a_1 + a_2(kh + (g + b)/4j) + a_3 d/v + a_4 f + a_5 d) - M \le 0, \quad g, h, f \ge 0.$$

- **Formulate using Lagrange multiplier**

$$\max B_1 = -TpXY(a_1 + a_2(kh + (g + b)/4j) + a_3 d/v + a_4 f + a_5 d)^2/2a_4 - y_1[2XTcY/ghv - TpXYf(a_1 + a_2(kh + (g + b)/4j) + a_3 d/v + a_4 f + a_5 d)], \quad g, h, f \ge 0.$$

## Solutions

- **Take derivatives with f, g, h and y2 to obtain 4 nonlinear equations in 4 unknowns, and solve approximately:**

$$g^* \simeq (32j^2 k a_4 c(2y_2 - 1)/vpa_2 A y_2)^{1/3}$$

$$h^* \simeq (a_4 c(2y_2 - 1)/2jk^2 vpa_2 y_2 A)^{1/3}$$

$$f^* \simeq [(1 - y_2)/(2y_2 - 1)] \cdot [A/a_4 + (4kca_2^2(2y_2 - 1)/vpja_4^2 A y_2)^{1/3}].$$

- **where**
  - **f is fare, g is route spacing, h is headway,**
  - **y2 is Lagrange multiplier or shadow price of $1 of benefit relative to $1 of deficit**
- **We vary y2 to get solutions ranging from min deficit (y2 infinite) to max social benefit (y2= 1)**
  - **We can also add a vehicle capacity constraint (y3)**

## Model summary

- Model implemented in Java code
  - Download code and documentation
- Provides framework for designing bus system:
  - Routes, headways, fares, vehicle sizes, express/local service
  - Bus stop spacing (fewer are better)
  - Route circuity (less circuity is better)
  - (Model variation used in planning Logan Express)
- Allows variation in objective and constraints
- Provides insight before addressing detailed system design with actual network and routes, using optimization algorithms and simulation
  - Most of the the term was spent on optimization algorithms for decisions and design
  - Simulation not covered, used for truly difficult/detailed issues
- We'll do analytical approximations for queuing systems in the next lecture

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