

Supplement on Flow Nets:

Flow nets are a graphical technique for predicting the magnitude of groundwater flow from a given set of boundary conditions. They are not a rigorous determination of flow, but they can give us a picture of what head looks like underground.

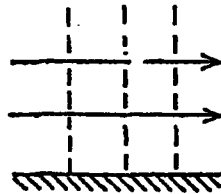
The original use of flow nets was for determining how much water was flowing under a dam. If there is a substantial amount of flow underneath a dam, it can put a lot of pressure on the sediments. Over time the groundwater can erode the sediments, and the dam can collapse – it's flood time! This is a difficult problem to solve analytically, but it turns out that flow nets can be used to get a graphical answer.

Flow nets are comprised of a set of equipotential lines (lines of constant head, h) and a set of associated flow lines (lines along which groundwater moves). Two flow lines define what is called a "stream tube." These lines are drawn within a certain set of boundary conditions (e.g. surface water bodies, impenetrable rocks). Three potential boundary conditions for a flow net that we may encounter are:

1. No-flow boundaries (e.g. a dam, low permeability bedrock, the Hancock Tower's basement).

$$\frac{\partial h}{\partial x} = 0$$

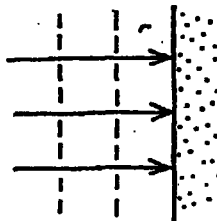
Water cannot flow across a no-flow boundary. When given a no-flow boundary, draw flow lines parallel to the boundary and head lines perpendicular to the boundary. Remember that flow lines *cannot* touch the boundary! Head lines should intersect the boundary.



2. Constant head boundaries (e.g. duck pond, Charles River, surface bodies of water).

$$h = \text{constant}$$

Flow is perpendicular to the boundary – either straight in or straight out. All head lines are parallel to the boundary.



3. Water-Table boundaries

Anywhere in an aquifer, total head is pressure head + elevation head:

$$h = z + \frac{P}{\rho g}$$

However, at the water table, the pressure head = 0. Therefore, $h = z$. Water table boundaries often define the maximum head drop for a system. If the aquifer receives no recharge from surface water or the atmosphere and maintains a steady flow, we can treat the water table as a no-flow boundary – i.e. if the aquifer never interacts with a surface water system, flow must always be below the water table. We cannot have flow *crossing* the water table!

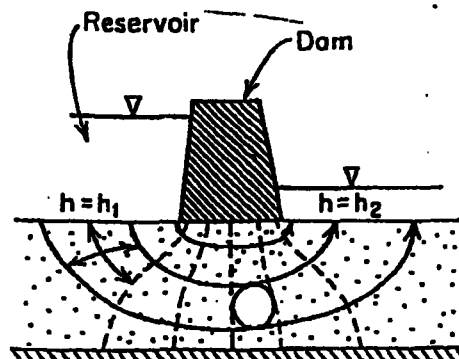
In addition to our boundary conditions, we also must remember a few simple rules in drawing flow nets:

1. Flow lines are ALWAYS perpendicular to head lines EVERYWHERE.
2. Flow lines NEVER intersect (head lines never intersect each other either!).
3. The areas between flow lines and head lines are “curvilinear squares.” That is, the aspect ratios or dimensions of the “squares” are the same, but the flow lines and/or head lines themselves can curve. We know that we have a curvilinear square if we can draw a circle inside it that is tangential to all four sides.

Further note that a flow net does not have to have finite boundaries on all sides, and that you can draw your flow net so that it has partial stream tubes along its edges.

Drawing your own flow net: Below is a drawn example of a flow net for the section of aquifer under a dam. Note the curvilinear squares. H_1 and H_2 are water table heights on either side of the dam. If I were drawing this flow net, I would :

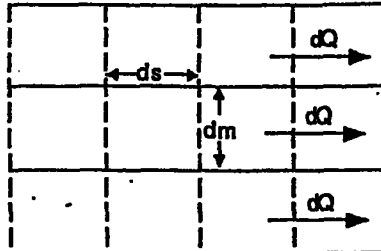
1. Draw head lines perpendicular to the dam and bedrock (no flow boundaries).
2. Draw flow lines perpendicular to the two water surfaces (i.e. curving U shapes).
3. Smooth the flow lines so that they were parallel to the bedrock.
4. Redraw lines as necessary to correct the spacing and preserve curvilinear squares. You will have to redraw lines! To draw a flow net, you just have to be courageous enough to put down the first few lines, knowing that you will have to correct the spacing later!



Calculations from flow nets:

Usually we will be interested in calculating discharge from a flow net. For a system with one recharge area and one discharge area, the following holds:

$$dQ = K * dm * dh/ds = K * dh$$



where dm is the thickness of the stream tube, ds is the distance between equipotentials, and Q is the volume discharge *per unit thickness* of aquifer (i.e. into the page).

We need $dm = ds$ for Q to stay the same. By drawing curvilinear squares, we preserve dQ along any stream tube, and thus conserve fluid mass.

We can then calculate discharge with:

$$Q = n_f * K * dH \quad \text{sums up all the stream tubes}$$

$$H = n_d * dH \quad \text{sums up all of the head drops}$$

$$Q = \frac{n_f}{n_d} * K * H$$

where n_f is the number of stream tubes, n_d is the number of head drops, K is the uniform hydraulic conductivity, and H is the total head drop over the region of flow. Note that neither n_f or n_d is required to be an integer (but it helps if one of them is an integer).

Example: You are a consultant hired by a neighborhood in Stinkwater, CA. The residents want to dam a stream (0.9 m deep) to build a swimming hole. The flow net pictured on the previous page is your plan for the dam. If the dam is built on top of an aquifer with a porosity of 0.26 and a hydraulic conductivity of 10^{-6} cm/s, and if after the dam is put in place the swimming hole is 4 m deep, how much water will flow under their dam? Assume the stream is 5 m wide.

Answer: $n_d = 6$, $n_f = 2$ (assume not much flow beyond third equipotential line)

$$H = 3.1 \text{ m}, K = 10^{-6} \text{ cm/s}$$

$$Q = (2/6) * (3.1 \text{ m}) * (10^{-6} \text{ cm/s}) * (1 \text{ m}/100 \text{ cm}) = 1.03 \times 10^{-8} \text{ m}^2/\text{s}$$

Q here is per unit thickness of aquifer. For our 5 m wide stream:

$$Q_{\text{tot}} = 5.17 \times 10^{-8} \text{ m}^3/\text{s} = 0.0047 \text{ m}^3/\text{d} = 4.5 \text{ L/d}$$

This leak rate is probably not high enough to worry about in the short term, but I would keep an eye on the dam over the course of several years.

There is a complication...ground squirrels in California are known to routinely dig holes into levies (up to forty feet long; yes, it's true!). This is difficult to model hydrologically (we won't try here) – but in the interest of your client, you should go buy some squirrel traps!