Code No: 131AB **R16**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY, HYDERABAD

B.Tech I Year I Semester Examinations, December - 2016

MATHEMATICS-II

(Common to CE, ME, MCT, MMT, MIE, CEE, MSNT)

Time: 3 hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART- A

(25 Marks)

1.a) Find the Laplace transform of the function $f(t) = \begin{cases} t & 0 < t < a \\ -t + 2a & a < t < 2a \end{cases}$. [2]

b) Prove that $L^{-1}\{F(s)\} = f(t)$ and $f(0) = 0$ then $L^{-1}\{sF(s)\} = \frac{df}{dt}$. [3]

c) Evaluate $\int_0^{\infty} a^{-bx^2} dx$. [2]

d) Show that $\beta(p,q) = \beta(p+1,q) + \beta(p,q+1)$. [3]

e) Find the area bounded by the curves $y = x, y = x^2$. [2]

f) Evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2+y^2) dydx$ by changing into polar coordinates. [3]

g) Find the directional derivative of $xyz^2 + xz$ at (1,1,1) in a direction of the normal to the surface $3xy^2 + y = z$ at (0,1,1). [2]

h) Find a unit normal vector to the surface $x^2 + y^2 + 2z^2 = 26$ at the point (2,2,3). [3]

i) Find the work done by the force $\vec{F} = 3x^2 i + (2xz - y)j + zk$ along the straight line joining the points (0,0,1) and (2,1,3). [2]

j) Find the circulation of $\vec{F}$ round the curve c where $\vec{F} = (e^x \sin y)i + (e^x \cos y)j$ and c is the rectangle whose vertices are $(0,0), (1,0), \left(1,\frac{\pi}{2}\right), \left(0,\frac{\pi}{2}\right)$. [3]

PART-B

(50 Marks)

2. Solve the differential equation $\frac{d^2x}{dt^2} - 4\frac{dx}{dt} - 12x = e^{3t}$ given that $x(0) = 1$ and $x'(0) = -2$ using Laplace transforms. [10]

OR

3. Use Laplace transforms, solve $y(t) = 1 - e^{-t} + \int_0^t y(t-u)\, sinu\, du$. [10]

4.a) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

b) Prove that $\Gamma(n)\Gamma(1-n) = \frac{\pi}{\sin n\pi}$. [5+5]

OR

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OR

5.a) Using Beta and Gamma functions, evaluate the integral $\int_{-1}^{1}(1-x^2)^n dx$ where n is a positive integer.

b) If m and n are positive integers then prove that $B(m,n) = \frac{(m-1)!(n-1)!}{(m+n-1)!}$. [5+5]

6. The plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ meets the axes in A, B and C. Find the volume of the tetrahedron $OABC$. Also find its mass if the density at any point is $kxyz$. [10]

OR

7.a) Change the order of integration and solve $\int_0^c \int_{x^2/a}^{2a-x} xy^2 dydx$.

b) Evaluate $\iiint xyzdxdydz$ over the positive octant of the sphere $x^2+y^2+z^2=a^2$. [5+5]

8.a) Find the directional derivative of xyz^2+xz at $(1,1,1)$ in a direction of the normal to the surface $3xy^2+y=z$ at $(0,1,1)$.

b) Prove that curl $(\vec{a}\times\vec{b}) = \vec{a}\,\text{div}\,\vec{b} - \vec{b}\,\text{div}\,\vec{a} + (\vec{b}\cdot\nabla)\vec{a} - (\vec{a}\cdot\nabla)\vec{b}$ [5+5]

OR

9. Prove that if $\vec{r}$ is the position vector of any point in space then $r^n\vec{r}$ is irrotational and is solenodial if n = -3. [10]

10. Verify divergence theorem for $2x^2yi - y^2j + 4xz^2k$ taken over the region of first octant of the cylinder $y^2+z^2=9$ and $x=2$. [10]

OR

11. If $\vec{f} = 3x^2yz^2i + x^2z^2j + 2x^3yzk$. Show that $\int_C \vec{f}\cdot d\vec{r}$ is independent of the path of integration. Hence evaluate the integral when C is any path joining (0, 0, 0) to (1, 2, 3). [10]

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