Map Method:

for simplification of boolean Expressions by boolean algebra, so we need better understanding of boolean laws, rules and theorems. During the process of simplification we have to predict each successive step. for these reasons we can never simplified expressions by boolean algebra alone.

Map method gives us systematic approach for simplifying a boolean Expression.

The map method, first proposed by Veitch and modified by Karnaugh. hence it is known as the Veitch diagram or the Karnaugh map.

One Variable, two - Variable, Three - Variable, and four - Variable maps.

The basis of this method is a graphical chart known as Karnaugh map (K-map). it contains boxes called cells. Each of the cell represents one of the 2^n possible products that can be formed from n Variables.

A 2-Variable map contains $2^2=4$ cells, a 3-Variable map contains $2^3=8$ cells, below fig shows the 1, 2, 3, and 4 variable maps.

A	0
0	m_0
1	m_1

1-variable (2 cells)

A \ B	0	1
0	m_0	m_1
1	m_2	m_3

2-Variable map (4 cells)

A \ BC	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

3-Variable map (8 cells)

→ Gray code Sequence

AB \ CD	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

4-Variable map (16 cells)

plotting a K-map.

logic function can be represented in various forms truth table, SOP boolean Expression and POS boolean Expression.

Representation of truth table on Karnaugh map.

Representation of truth table with 2, 3 and 4 variables.

K-map plotted from truth tables with the terms which are having output 1, have the corresponding cells marked with 1's. The other cells are marked with zeros.

Ex:

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

A \ B	0	1
0	0	1
1	1	0

OR

A \ B	$\bar{B}$	B
$\bar{A}$	0	1
A	1	0

Representation of 2-variable truth table on K-map.

Sum terms Representation on Karnaugh map.

A Boolean Expression with sum terms (sum of product form) can be plotted with the K-map by placing a 1 in each cell corresponding to a term (minterm) in the SOP Expression. Remaing cells are filled with zeros. This is illustrated in the following Examples.

Example: plot Boolean Expression $Y = AB\bar{C} + ABC + \bar{A}\bar{B}C$ on the K-map.

Solution: The Expression has 3 Variables and hence it can be plotted using 3-Variable map as shown below.

$A'B'C$

A \ BC	00	01	11	10
0	(0)	1 (1)	(3)	(2)
1	(4)	(5)	1 (7)	1 (6)

ABC ABC'

OR

$A'B'C$

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0 (0)	1 (1)	0 (3)	0 (2)
A	0 (4)	(5)	1 (7)	1 (6)

ABC ABC'

Example: plot Boolean Expression.

$Y = A'BC'D' + AB'CD' + A'BCD' + AB'CD + ABCD'$ on the Karnaugh map.

The Expression has 4-Variables and hence it can be plotted using Variable map as shown below.

$A'BCD'$

$A'BC'D'$

AB \ CD	00	01	11	10
00	(0)	(1)	(3)	(2)
01	1 (4)	(5)	(7)	1 (6)
11	(12)	1 (13)	(15)	(14)
10	(8)	(9)	1 (11)	1 (10)

ABC'D AB'CD AB'CD'

OR

$A'BC'D'$ $A'BCD'$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0 (0)	0 (1)	0 (3)	0 (2)
$\bar{A}B$	1 (4)	0 (5)	(7)	1 (6)
AB	0 (12)	1 (13)	0 (15)	0 (14)
$A\bar{B}$	0 (8)	0 (9)	1 (11)	1 (10)

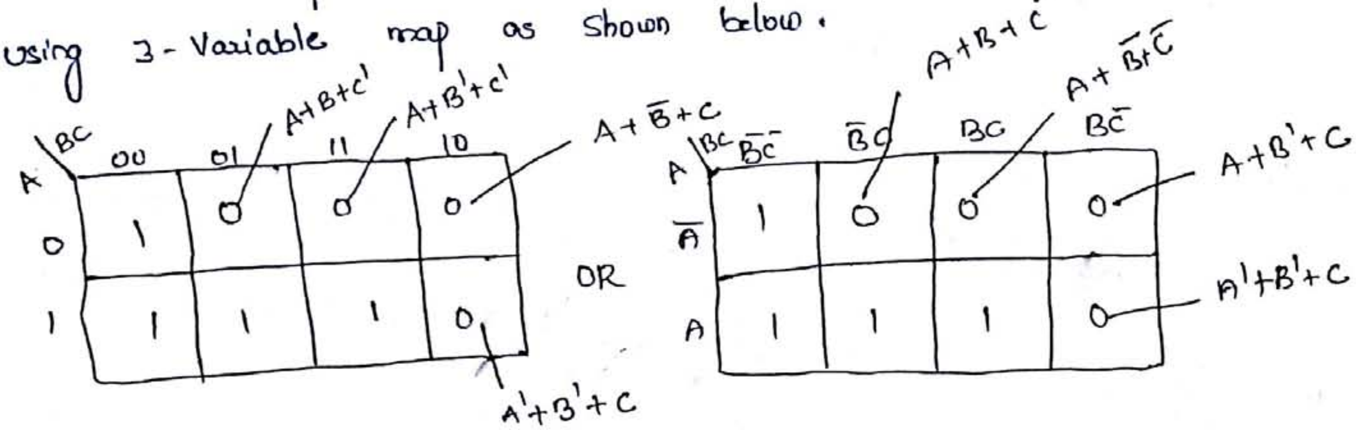
ABC'D AB'CD AB'CD'

product terms Representation on Karnaugh maps :

A Boolean Expression with product terms (the product of Sums) can be plotted on the Karnaugh map by placing a 0 in each cell corresponding to a term (maxterm) in Expression. Remaining cells are filled ones. This is illustrated in the following Examples.

Example: plot Boolean Expression $Y = (A+B+\bar{C})(A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+C)(A+B\ldots$ on the Karnaugh map.

Solution: The Expression has 3 Variables and hence it can be plotted using 3-Variable map as shown below.



$m_2 = A+\bar{B}+C$ $\quad m_3 = A+\bar{B}+\bar{C}$, $\quad m_6 = \bar{A}+\bar{B}+C$ $\quad m_1 = A+B+\bar{C}$

Using K-maps to obtain minimal Expressions for complete Boolean function

Once the boolean function is plotted on the Karnaugh map we have to use grouping technique to simplify the boolean function.

Grouping nothing but a Combining terms in adjacent cells. Two cells are said to be adjacent if they conform single change Rule.

When adjacent 1's are grouped then we get result in the SOP form otherwise we get result in the POS form.

us See Various grouping rules.

Grouping two Adjacent ones (pair).

Ex: $Y = \bar{A}\bar{B}C + \bar{A}BC$

A \ BC	00	01	11	10
0		1	1	
1				

The K-map contains a pair of 1's that are horizontally adjacent to each other, the first represents $\bar{A}\bar{B}C$ and second $\bar{A}BC$. In this two terms only the B variable appears in both normal and complemented form. So, by combining these two we can eliminate the B term and the result is $\bar{A}C$.

Ex: $A'BC + ABC = BC$

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	0	0	1	0
A 1	0	0	1	0

- In K-map we can combine two vertically adjacent 1's
- In a K-map leftmost column and right most column are considered to be adjacent.

Thus, two 1's in these columns with a common row can be combined to eliminate one variable.

- Ex: $Y = AB'C' + ABC'$

A \ BC	$B'C'$ 00	$B'C$ 01	BC 11	BC' 10
0	0	0	0	0
1	1	0	0	1

$= AC'$

In a K-map the leftmost column and rightmost column are considered to be adjacent. Thus, the two 1's in these columns with a common row can be combined to eliminate one variable.

Ex: $Y = A'B'C'D + AB'C'D.$

$= B'C'D\,(A'+A)$

$= B'C'D.$

AB \ CD	C'D' 00	C'D 01	CD 11	CD' 10
A'B' 00		1		
A'B 01				
AB 11				
AB' 10		1		

$B'C'D$

Let us see this Example, 1s from top row and bottom of some column are combined ... Eliminate Variable A, Since in a map the top row and bottom row are considered to be adjacent.

Ex: $A'B'C + A'BC + ABC$

$= A'B'C + A'BC + A'BC + ABC$ Rule $[A+A = A]$

$= A'B'C + A'BC + A'BC + ABC$

$= A'C(B+B') + BC(A+A')$

$= A'C + BC.$

A \ BC	00	01	11	10
0		1	1	
1			1	

This shows a K-map that has two overlapping pair of 1's. This shows that we can share one term between two pairs that

→ Another Example

$Y = A'B'C + A'BC + ABC + ABC'$

$= A'C(B'+B) + AB(C+C')$

$= A'C + AB$ $[A'+A = 1]$.

pair not Required

A \ BC	00	01	11	10
0		1	1	
1			1	1

This fig shows a K-map where three group of pairs can be formed. But only two pairs are Enough to include all 1s present ...

Grouping of 4 adjacent ones (Quad)

– In a k-map we can group four adjacent 1's. The resultant group is called Quad.

– four 1's may be horizontally adjacent or vertically adjacent.

A \ BC	00	01	11	10
0				
1	1	1	1	1

Horizontally adjacent

$Y = A$

AB \ CD	00	01	11	10
00			1	
01			1	
11			1	
10			1	

Vertically adjacent.

$Y = CD$

If a K-map contains four 1's in a Square. and they are Considered as adjacent to each other. [fig ①]

AB \ CD	00	01	11	10
00				
01		1	1	
11		1	1	
10				

fig ① $Y = BD$

AB \ CD	00	01	11	10
00				
01				
11	1			1
10	1			1

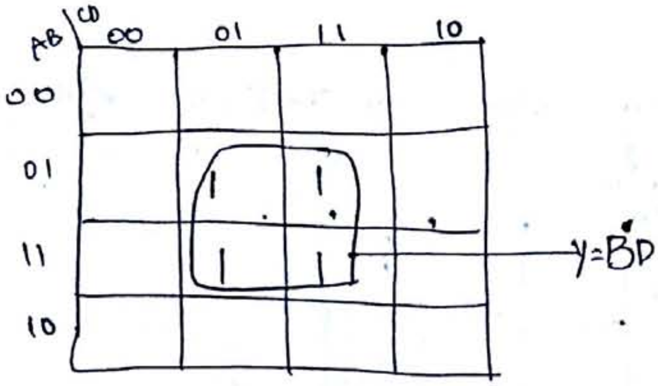
fig ②

$Y = AD.$

– The four 1's in fig ② are also adjacent, because the top and bottom rows are considered to adjacent to each other and the leftmost and rightmost columns are also adjacent to each other.

Ex: $Y = \bar{A}B\bar{C}D + \bar{A}BCD + AB\bar{C}D + ABCD$.

Solution: As there are 4-Variable total no. of cells $= 2^4 = 16$.



3. Grouping Eight adjacent ones (octet):

— Grouping of eight adjacent 1's is called as octet.

— either you can group horizontally, vertically.

— when an octet is combined in a four variable map, three of four variables are eliminated because only one variable remains unchanged.

Simplification of SOP Expression:

- The combination of pairs, quads, and octets on a karnaugh map can be used to obtain a simplified expression.

— A pair of 1's eliminates one variable.

— A quad of 1's eliminates two variables.

— An octet of 1's eliminates three variables.

— In general, when a variable appears in both complemented and uncomplemented form within a group. that variable eliminated from a resultant expression.

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Generalised procedure to Simplify boolean Expression

1. plot the K-map and place 1's in those cells corresponding to the 1s in the truth table or Sum of product Expression. place 0s in other cells.
2. check the K-map for adjacent 1s and then circle those 1's which are not adjacent to any other 1s. These are called isolated 1s.
3. check for those 1s which are adjacent to only one other 1 and Encircle Such pairs.
4. Check for quards and octets of adjacent 1s Even if it contains Some 1s that have already been Encircled, while doing this make Sure that there are minimum number of groups.
5. Combine any pairs necessary to include any 1's that have not yet been grouped.
6. from the Simplified Expression by Summing product terms of all the groups.

Example: minimize the Expression $y = A\bar{B}C + \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$

A \ BC	00	01	11	10
0	1	1	1	
1	1	1		

1. first See for isolated ones. In this there are no. isolated ones.
2. See for adjacent pair of 1's — 1
3. See for Quad — 1

$$\boxed{Y = \bar{B} + \bar{A}C}$$

2) minimize the Exp $y = \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + AB\bar{C}\bar{D} + AB\bar{C}D + A\bar{B}CD + \bar{A}\bar{B}C\bar{D}$

AB \ CD	00	01	11	10
00				1
01	1	1		
11	1	1		
10		1		

$y = BC + AB'C + A'B'CD'$

3) minimize the Exp $y = A'B'C'D' + ABCD' + AB'C'D' + AB'C'D' + ABC'D' + A'B'CD + A'B'CD'$

AB \ CD	00	01	11	10
00	1		1	1
01				
11	1			1
10	1		1	1

$y = B'D' + \cancel{A}B'C + AD'$

4) Reduce the following fn to its minimum SOP form.

$y = A'B'C'D + A'BC'D + A'BCD + A'BCD' + ABC'D' + ABC'D + ABCD + AB'CD$

AB \ CD	00	01	11	10
00		1		
01		1	1	1
11	1	1	1	
10			1	

$A'C'D$. $A'BC$ ABC' ACD

— There are no isolated 1's

— There is no octet, but there is a quad. However, all 1's in the quad have already been grouped. Therefore this quad is ignored.

$y = A'C'D + A'BC + ABC' + ACD$.

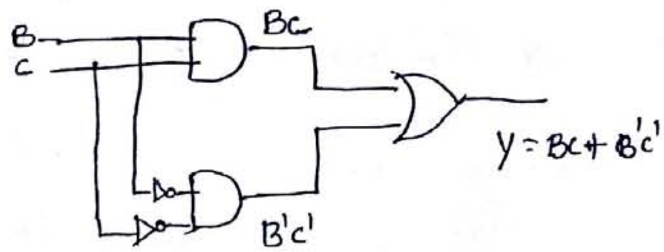
Implement the logic function specified by the truth table (Below) using the Karnaugh map method. Y is the output variable and A, B and C are the input variables.

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

A \ BC	00	01	11	10
0	1		1	
1	1		1	

$B'C' + BC$.

The boolean expression can be implemented as follows



Simplification of POS Expression:-

Once the expression is plotted on the K-map instead of making the group of ones, we have to make groups of zero.

Steps

1. plot the k-map and place 0's in those cells corresponding to the 0's in the truth table or maxterms in the products of sum expression
2. check the k-map for adjacent 0's and encircle those 0's which are not adjacent to any other 0's. These are called isolated as
3. check for those 0's which are adjacent to only one other 0 and encircle such pairs.

4. Check for quads and octets of adjacent 0's Even if it contains that have already been Encircled. While doing this make sure there are minimum number of groups.

5. Combine any pairs necessary to include any 0's that have not been grouped.

6. From the Simplified SOP Expression for $\bar{F}$ by Summing product terms of all the groups.

Note: The Simplified SOP necessary to include any 0s Expression is in the complemented form because we have grouped 0s to simplify the Expression.

7. Use Demorgan's theorem on $\bar{F}$ to produce the Simplified Expression in POS form.

Ex: minimize the Expression

$$Y = (A+B+\bar{C})(A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+\bar{C})(\bar{A}+B+C)(A+B+C) \cdot$$ (POS) (maxterms

Solution: $(A+B+\bar{C}) = m_1$, $(A+\bar{B}+\bar{C}) = m_3$, $(\bar{A}+\bar{B}+\bar{C}) = m_7$

$(\bar{A}+B+C) = m_4$, $(A+B+C) = m_0$.

A \ BC	00	01	11	10
0	0	0	0	
1	0		0	

$$\bar{Y} = B'C' + BC + A'C$$

$$Y = \bar{\bar{Y}} = \overline{\bar{B}\bar{C} + BC + \bar{A}C}$$

$$= (\bar{\bar{B}}+\bar{\bar{C}})(\bar{B}+\bar{C})(\bar{\bar{A}}+\bar{C})$$

$$= (B+C)(\bar{B}+\bar{C})(A+\bar{C})$$

Minimize the following Exp in POS form.

$$Y = (\bar{A}+\bar{B}+C+D)(\bar{A}+\bar{B}+\bar{C}+D)(\bar{A}+\bar{B}+\bar{C}+\bar{D})(\bar{A}+B+C+D)$$
$$(A+\bar{B}+\bar{C}+D)(A+\bar{B}+\bar{C}+\bar{D})(A+B+C+D)(\bar{A}+\bar{B}+C+\bar{D})$$

Solution: $(\bar{A}+\bar{B}+C+D) = M_{12}$ $(\bar{A}+\bar{B}+\bar{C}+D) = M_{14}$ $(\bar{A}+\bar{B}+\bar{C}+\bar{D}) = M_{15}$

$(\bar{A}+B+C+D) = M_8$, $(A+\bar{B}+\bar{C}+D) = M_6$, $(A+\bar{B}+\bar{C}+\bar{D}) = M_7$

$(A+B+C+D) = M_0$ and $(\bar{A}+\bar{B}+C+\bar{D}) = M_{13}$

AB \ CD	00	01	11	10
00	0			
01			0	0
11	0	0	0	0
10	0			

There is no isolated 0's.

$$\bar{Y} = BC + AB + B'C'D'$$

$$Y = \bar{\bar{Y}} = \overline{BC + AB + B'C'D'}$$
$$= (\bar{B}+\bar{C})(\bar{A}+\bar{B})(\bar{B'}+\bar{C'}+\bar{D'})$$
$$= (\bar{B}+\bar{C})(\bar{A}+\bar{B})(B+C+D)$$

Ex: Reduce the following function using K-map technique.

$$f(A,B,C,D) = \pi m(0,2,3,8,9,12,13,15)$$

AB \ CD	00	01	11	10
00	0		0	0
01				
11	0	0	0	
10	0	0		

$$\bar{F} = AC' + ABD + A'B'C + A'B'D'$$

$$\bar{\bar{F}} = \overline{AC' + ABD + A'B'C + A'B'D'}$$
$$= (A'+C)(A'+B'+D')(A+B+C')(A+B+D)$$

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Don't Care Conditions

In some logic ckts, certain input conditions never ... i.e. the corresponding o/p never appears. In such cases the o/p is not designed, defined, it can be either High or low. These output levels are indicated by 'x' or 'd' in the truth ... are called don't care o/p's.

- A ckt designer is free to make the o/p for any "don't care" condition either a '0' or '1' in order to produce the simplest o/p Expression.
- It is important to decide which don't cares to change to and which to 1 to produce the best K-map grouping.

Ex: find the reduced SOP form of the following function

$$f(A,B,C,D) = \Sigma m(1,3,7,11,15) + \Sigma d(0,2,4)$$

AB \ CD	00	01	11	10
00	X	1	1	X
01	X	0	1	0
11	0	0	1	0
10	0	0	1	0

To form a quad of cells 0, 1, 2, and 4 the don't care conditions 0 and 2 are replaced by 0 since it is not required to form any group.

AB \ CD	00	01	11	10
00	1	1	1	1
01	0	0	1	0
11	0	0	1	0
10	0	0	1	0

$= A'B' + CD.$

Ex: Reduce the following fn using K-map.

$$f(A,B,C,D) = \Sigma m(5,6,7,12,13) + \Sigma d(4,9,14,15)$$

AB \ CD	00	01	11	10
00	0	0	0	0
01	X	1	X	X
11	1	1	X	X
10	0	X	0	0

To form a octet of cells 4, 5, 6, 7, 12, 13, 14 and 15 the don't care conditions 4, 14, and 15 are replaced by 1s. The remaining don't care condition 9 is replaced by 0 to get simplified fn.

AB \ CD	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	1	1	1	1
10	0	0	0	0

$f(A,B,C,D) = B$

Ex:- Implement the following using logic gates.

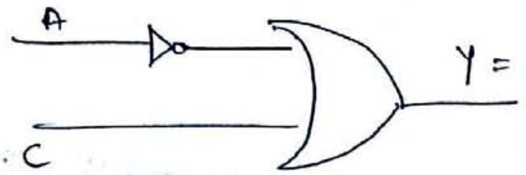
$$f(A,B,C) = \sum(0,1,3,7) + \sum d(2,5)$$

A \ BC	00	01	11	10
0	1	1	1	X
1	0	X	1	0

To form two quads both don't care conditions are replaced by 1s and we get.

1	1	1	1
	1	1	

$= \bar{A} + C$



Five Variable K-map:-

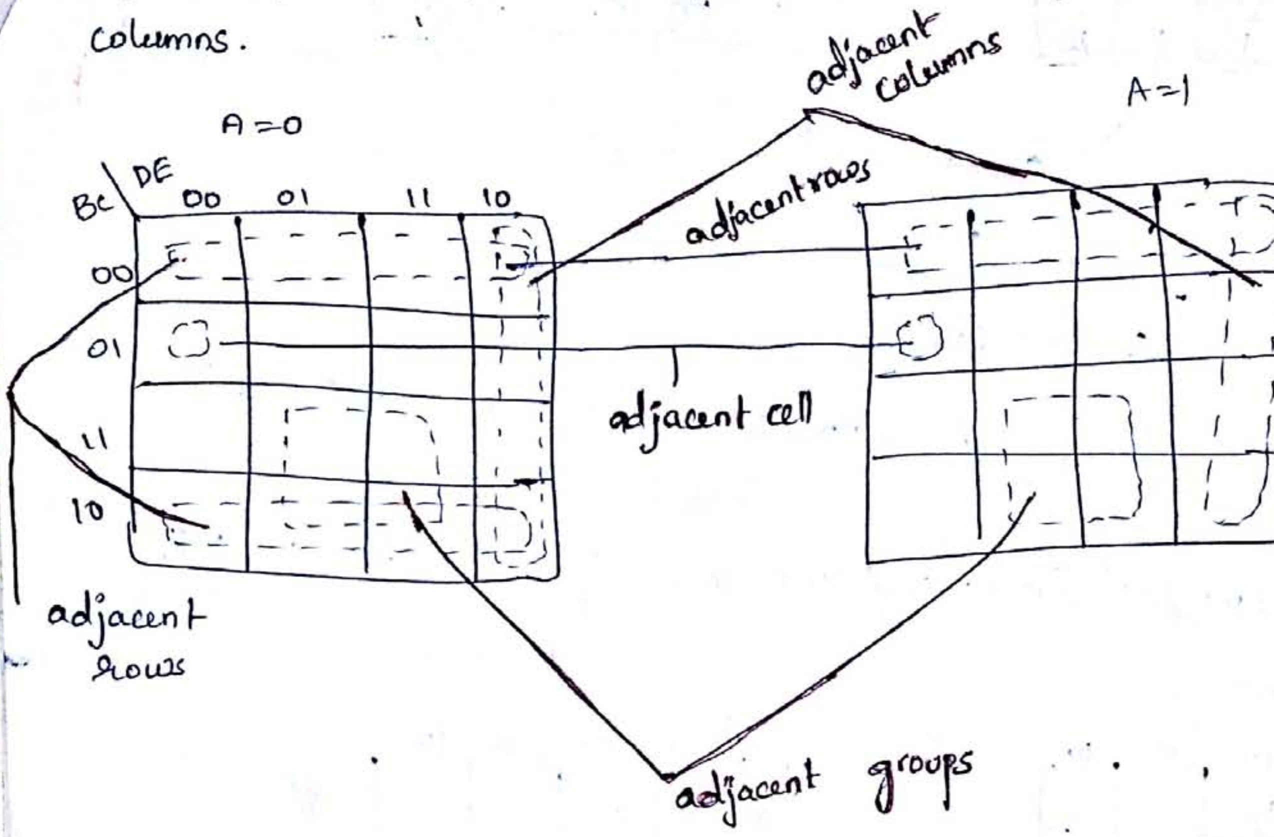
- A 5-Variable K-map require $2^5 = 32$ cells, but adjacent are difficult to identify on a single 32-cell map.

  ∴ Two 16-bit K-maps are used. If the variables A, B, C, D and E two identical 16-cell maps containing B, C, D, E can be constructed. One map is used for A and other for $\bar{A}$.

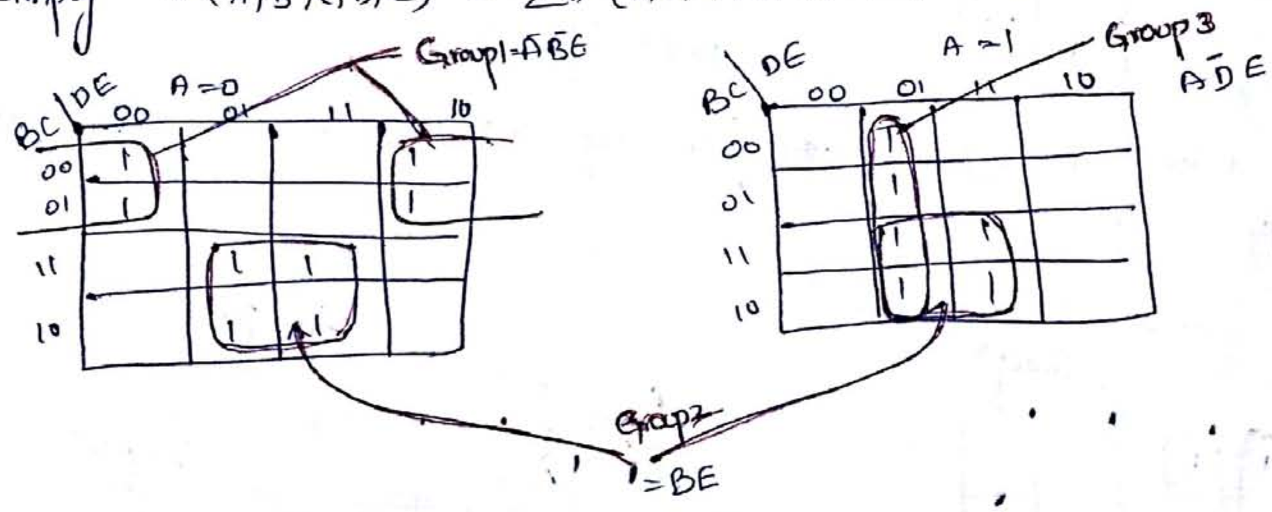
- In order to identify the adjacent grouping in the five variable we must imagine the two maps superimposed on one another.
- Every cell in one map is adjacent to the corresponding cell in the other map, because only one variable changes between such corresponding cells.
- Every row on one map is adjacent to the corresponding row
- The rightmost and leftmost columns within each 16-cell map are

…cent, just as they are in any 16-cell map, as are the top … bottom rows.

However, the rightmost column of one map is not adjacent to the leftmost column of other map. Since those are not corresponding columns.



Simply $f(A,B,C,D,E) = \sum m(0,2,4,6,9,11,13,15,17,21,25,27,29,31)$



$f(A,B,C,D,E) = \bar{A}\bar{B}\bar{E} + BE + A\bar{D}E$

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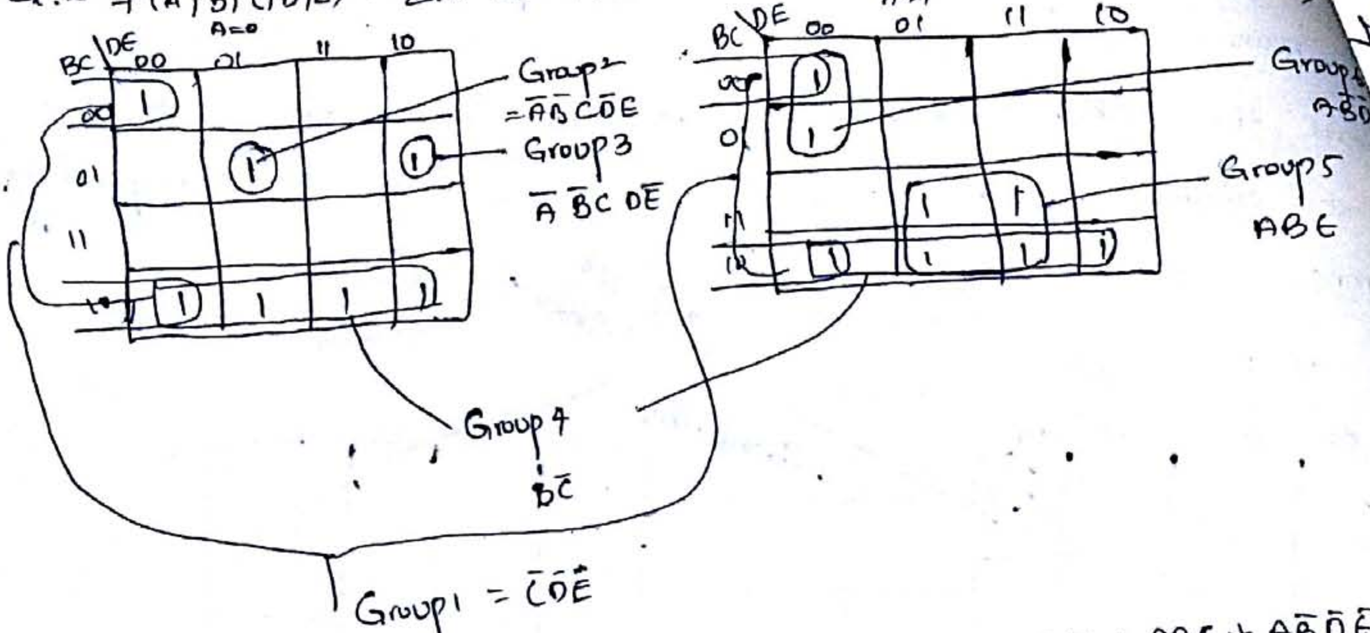
Ex:- $f(A,B,C,D,E) = \Sigma m\,(0,5,6,8,9,10,11,16,20,24,25,26,27,29,31)$



$$f(A,B,C,D,E) = \bar C\bar D\bar E + \bar A\bar B C\bar D E + \bar A\bar B C D\bar E + B\bar C + ABE + A\bar B\bar D\bar E.$$

Ex: $f(A,B,C,D,E) = \Sigma m\,(1,4,8,10,11,20,22,24,25,26) + d\,(0,12,16,17)$

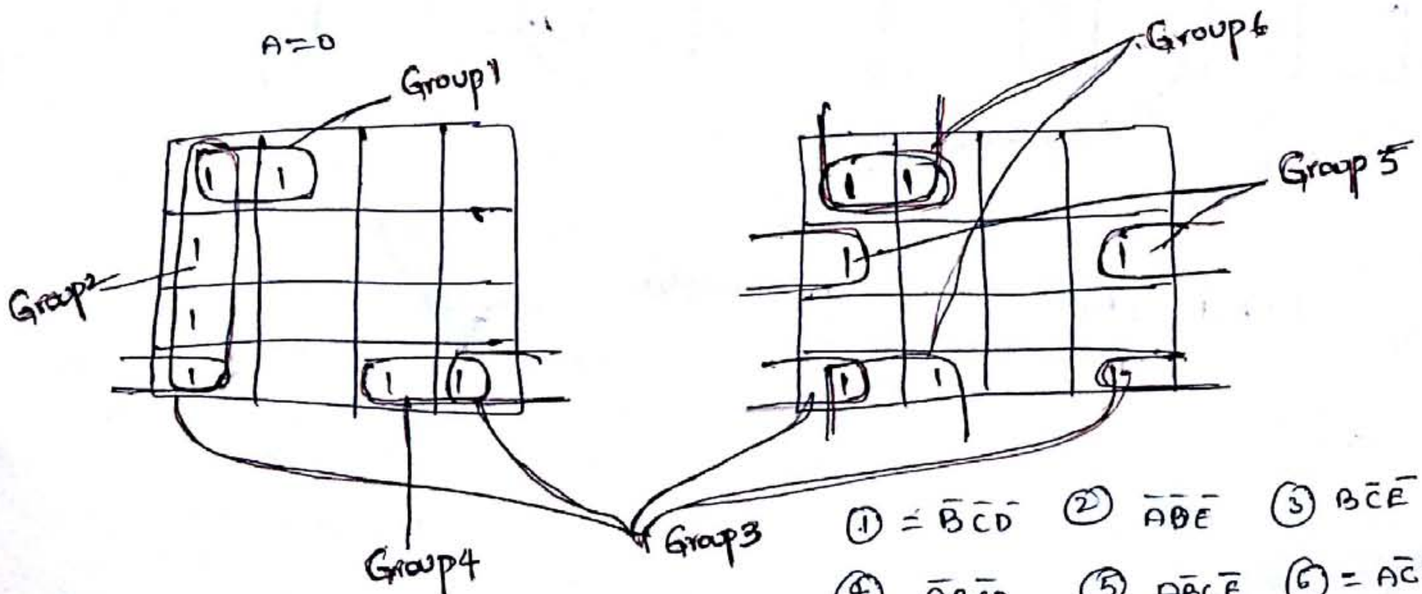
A=0

BC\DE	00	01	11	10
00	X	1		
01	1			
11	X			
10	1		1	1

A=1

BC\DE	00	01	11	10
00	X	X		
01	1			1
11				
10	1	1		1

In order to minimize the given Exp all don't care has to be taken as 1's.

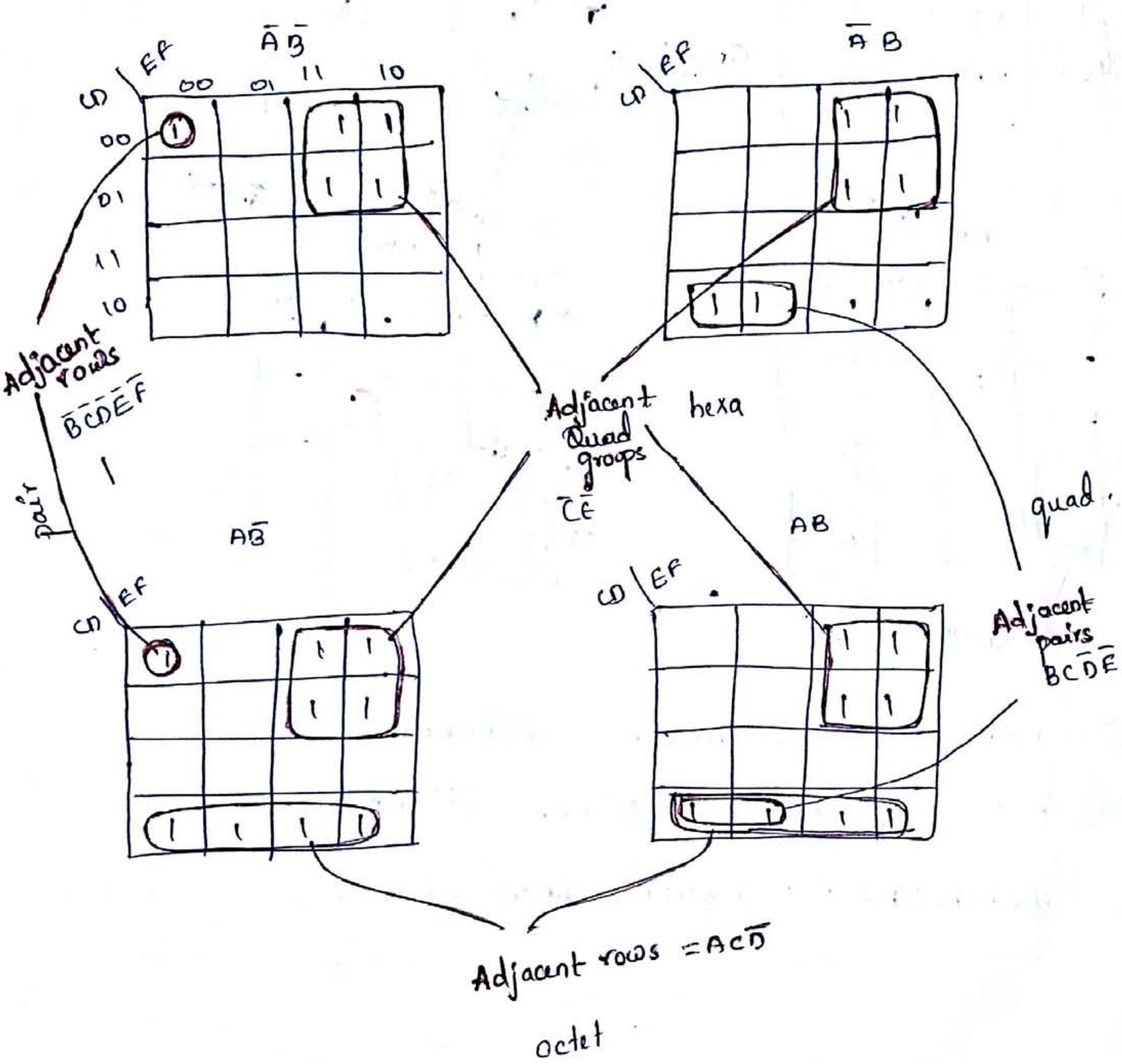


① $= \bar B\bar C\bar D$ ② $\bar A\bar D\bar E$ ③ $B\bar C\bar E$

④ $\bar A B\bar C D$ ⑤ $A\bar B C\bar E$ ⑥ $= A\bar C\bar D$

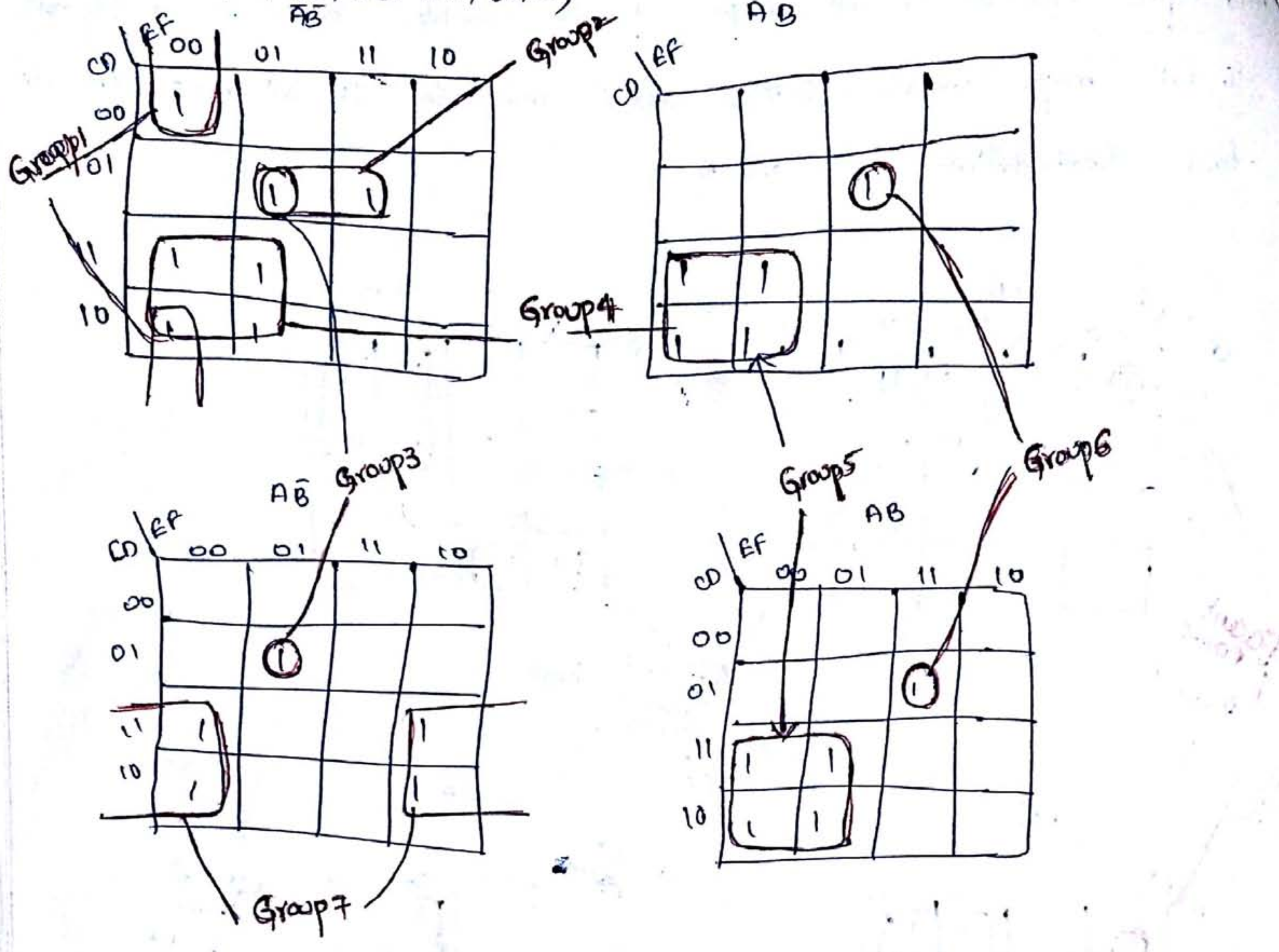
6 Variable K-maps:-

A 6-Variable K-map requires $2^6 = 64$ cells. These cells are divided into four identical 16-cell maps. If the variables are A, B, C, D, E and F 16-cell maps contain C, D, E and F and each 16-cell map represents four combinations of A and B.



Ex: Simplify the Boolean fn

$f(A,B,C,D,E,F) = \sum m(0,5,7,8,9,12,13,23,24,25,28,29,37,40,46,55,56,57,60,61)$



① = $\bar{A}\bar{B}\bar{D}\bar{E}\bar{F}$ ② $\bar{A}\bar{B}\bar{C}DF$ ③ $\bar{B}CD\bar{E}F$

④ $\bar{A}C\bar{E}$ ⑤ $BC\bar{E}$ ⑥ $B\bar{C}DEF$ ⑦ $A\bar{B}C\bar{F}$

$f(A,B,C,D,E,F) = \bar{A}\bar{B}\bar{D}\bar{E}\bar{F} + \bar{A}\bar{B}\bar{C}DF + BCD\bar{E}F$

Quine – Mc Cluskey Method :–

The minimization of Boolean Expressions using K-maps is usually to a maximum of six variables.

The Quine – mc cluskey method, also known as the tabular method. It is a more systematic method of minimizing Expressions of even larger number of variables.

Principle :– The minterms whose binary Equivalents differ only in one place can be combined to reduce the minterms.

Example: obtain the set of prime implicants for the boolean Expression $f = \sum m(0,1,6,7,8,9,13,14,15)$ using the tabular method.

Step 1: list all minterms in binary form

minterm	Binary Representation
m_0	0000
m_1	0001
m_6	0110
m_7	0111
m_8	1000
m_9	1001
m_{13}	1101
m_{14}	1110
m_{15}	1111

Step2: Arrange minterms according to no. of 1's.

minterm	Binary Rep.
m0	0000 ✓
m1	0001 ✓
m8	1000 ✓
m6	0110 ✓
m9	1001 ✓
m7	0111 ✓
m13	1101 ✓
m14	1110 ✓
m15	1111 ✓

Step3: Compare Each term in Each group with next higher group terms. If they differ in Single variable, mark that position with '–'

minterm	Binary Rep.
0, 1	000– ✓
0, 8	–000 ✓
1, 9	–001 ✓
8, 9	100– ✓
6, 7	011– ✓
6, 14	–110 ✓
9, 13	1–01
7, 15	–111 ✓
13, 15	11–1
14, 15	111– ✓

Step4: Repeat step 3

minterm	Binary Rep.
0, 1, 8, 9	–00–
0, 8, 1, 9	–00–
6, 7, 14, 15	–11–
6, 14, 7, 15	–11–

p5: List P.I

P.I	Binary Rep.
9, 13 (1-01)	1-01
13, 15 (11-1)	11-1
0, 1, 8, 9 (-00-)	-00-
6, 7, 14, 15 (-11-)	-11-

Step 6: EPI Chart (essential prime implicant)

EPI / minterms	m_0	m_1	m_6	m_7	m_8	m_9	m_{13}	m_{14}	m_{15}
✓ 9, 13 either or						⊙	⊙		
13, 15							⊙		⊙
✓ 0, 1, 8, 9	⊙	⊙			⊙	⊙			
✓ 6, 7, 14, 15			⊙	⊙				⊙	⊙

Check for single dot in a column:- 0, 1, 8, 14, 6, ~~7~~ which are not cover all minterms, so check for double dot

double dot : 9, 13 (1-01)

$$Y = A\bar{C}D + \bar{B}\bar{C} + BC.$$

(2) $F(A,B,C,D,E) = \sum m(0,2,4,5,6,7,8,10,14,17,18,21,29,31) +$

$\sum_d (11,20,22)$

* When a switching function has don't cares, we set them equal to 1 and find P.I.

* In determining E.P.I and minimal cover of the function we set all don't cares equal to 0. That is the don't care decimal number donot appear in P.I table.

Sol: Step1: List all minterms with binary equivalents.

Step2: Arrange the minterms according to no. of 1's.

minterm	Binary Rep.	
0	00000	✓
2	00010	✓
4	00100	✓
8	01000	✓
5	00101	✓
6	00110	✓
10	01010	✓
17	10001	
18	10010	✓
20	10100	✓
7	00111	✓
11	01011	✓
14	01110	✓
21	10101	✓
22	10110	✓
29	11101	✓

3) Compare each term in each group with next higher order term.

minterm	Binary representation	
0, 2	0 0 0 – 0	✓
0, 4	0 0 – 0 0	✓
0, 8	0 – 0 0 0	✓
2, 6	0 0 – 1 0	✓
2, 10	0 – 0 1 0	✓
2, 18	– 0 0 1 0	✓
4, 5	0 0 1 0 –	✓
4, 6	0 0 1 – 0	✓
4, 20	– 0 1 0 0	✓
8, 10	0 1 0 – 0	✓
5, 7	0 0 1 – 1	✓
5, 21	– 0 1 0 1	✓
6, 7	0 0 1 1 –	✓
6, 14	0 – 1 1 0	✓
6, 22	– 0 1 1 0	✓
10, 11	0 1 0 1 –	
10, 14	0 1 – 1 0	✓
17, 21	1 0 – 0 1	
18, 22	1 0 – 1 0	✓
20, 21	1 0 1 0 –	✓
20, 22	1 0 1 – 0	✓
21, 29	1 – 1 0 1	
29, 31	1 1 1 – 1	

Step4: Repeat step3

minterms	Binary Rep.
0, 2, 4, 6	0 0 – – 0
0, 2, 8, 10	0 – 0 – 0
2, 6, 10, 14	0 – – 1 0
2, 6, 18, 22	– 0 – 1 0
4, 5, 6, 7	0 0 1 – –
4, 5, 20, 21	– 0 1 0 –
4, 6, 20, 22	– 0 1 – 0

Step5: List all P.I.

P.I	Binary Rep.
10, 11	0 1 0 1 –
17, 21	1 0 – 0 1
21, 29	1 – 1 0 1
29, 31	1 1 1 – 1
0, 2, 4, 6	0 0 – – 0
0, 2, 8, 10	0 – 0 – 0
2, 6, 10, 14	0 – – 1 0
2, 6, 18, 22	– 0 – 1 0
4, 5, 6, 7	0 0 1 – –
4, 5, 20, 21	– 0 1 0 –
4, 6, 20, 22	– 0 1 – 0

Step6: EPI Chart

PI / minterms	m_0	m_2	m_4	m_5	m_6	m_7	m_8	m_{10}	m_{14}	m_{17}	m_{18}	m_{21}	m_{29}	m_{31}	d_{11}	d_{20}	d_{22}
10, 11								⊙							⊙		
✓ 17, 21										⊙		⊙					
21, 29												⊙	⊙				
✓ 29, 31													⊙	⊙			
0, 2, 4, 6	⊙	⊙	⊙		⊙												
✓ 0, 2, 8, 10	⊙	⊙					⊙	⊙									
✓ 2, 6, 10, 14		⊙			⊙			⊙	⊙								
✓ 2, 6, 18, 22		⊙			⊙						⊙						⊙
✓ 4, 5, 6, 7			⊙	⊙	⊙	⊙											
4, 5, 20, 21			⊙	⊙								⊙				⊙	
4, 6, 20, 22			⊙		⊙											⊙	⊙

$Y = A + B + C$

$Y = 17, 21 + 29, 31 + 0, 2, 8, 10 + 2, 6, 10, 14 + 2, 6, 18, 22 + 4, 5, 6, 7$

$= A\bar{B}\bar{D}E + ABCE + \bar{A}\bar{C}\bar{E} + \bar{A}D\bar{E} + \bar{B}D\bar{E} + \bar{A}\bar{B}C$

Problems on K-map

3-var:-

1. $F(x, y, z) = \Sigma m(2, 3, 4, 5) = x \oplus y$
2. 〃 $= \Sigma m(3, 4, 6, 7) = yz + xz'$
3. 〃 $= \Sigma m(0, 2, 4, 5, 6) = z' + xy'$
4. 〃 $= \Sigma m(1, 2, 3, 5, 7) = C + A'B$
5. 〃 $= \Sigma m(0, 2, 6, 7) = xy + \bar{x}\bar{z}$
6. 〃 $= \Sigma m(0, 2, 3, 4, 6)$
7. 〃 $= \Sigma m(0, 1, 2, 3, 7) = \bar{a} + bc$
8. 〃 $= \Sigma m(3, 5, 6, 7) = xy + xz + yz$
9. $xy + x'y'z' + x'yz' = \Sigma m(0, 2, 6, 7) = xy + \bar{x}\bar{z}$
10. $x'y' + yz + x'yz' = \Sigma m(0, 1, 2, 3, 7) = \bar{x} + yz$

4-var:-

1. $F(w, x, y, z) = \Sigma m(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$
   $= y' + w'z' + xz'$
2. 〃 $= \Sigma m(0, 1, 2, 6, 8, 9, 10) = \bar{B}\bar{D} + \bar{B}\bar{C} + \bar{A}C\bar{D}$

ime Implicant:

It is a smallest possible product term removing of any literal from which is not possible. The product must be an implicant of a given fn for it to be a prime implicant.

→ The bunch of 1's on the K-map which form a 2-square, 4-square, etc. is called PI or Subcube.

Ex: $f(A,B,C) = \Sigma m(1,5,7,8,10,12,13,15)$

AB \ CD	00	01	11	10
00		1		
01		1	1	
11	1	1	1	
10	1			1

$= A\bar{B}\bar{D} + AB\bar{C} + \bar{A}\bar{C}D + BD$

$h_1 = A\bar{B}\bar{D}$

if A is removed $\bar{B}\bar{D} \rightarrow (0, 1, 8, 10)$ is not an implicant

$\bar{B}$ $\qquad A\bar{D} \rightarrow (8, 10, 12, 14)$ " " " .

if $\bar{D}$ is removed $A\bar{B} \rightarrow (8, 9, 10, 13)$ " " " .

$\therefore h_1$ is a prime implicant

Essential prime Implicant:

It is the prime Implicant, It must cover at least one minterm which is not covered by any other P.I.

$EPI = \{\bar{A}\bar{C}D,\ BD,\ A\bar{B}\bar{D},\ AB\bar{C}\}$

Ex: $f(A,B,C,D) = \Sigma m(2,6,7,9,13,15)$

AB \ CD	00	01	11	10
00				1
01			1	1
11		1	1	
10		1		

$= \bar{A}C\bar{D} + BCD + A\bar{C}D$

No. of PI $= 5 \rightarrow \underset{(2,6)}{\bar{A}C\bar{D}},\ \underset{(7,15)}{BCD},\ \underset{(9,13)}{A\bar{C}D},\ \underset{(13,15)}{ABD},\ \underset{(6,7)}{\bar{A}BC}$

No. of EPI $= 2 \Rightarrow (2,6), (9,13)$

ABD

BD

0101 − 5

0111 − 7

1101 − 13

1111 − 15

Find the prime Implicant for the following and determine which are essential.

1) $f(w,x,y,z) = \Sigma m(0,2,4,5,6,7,8,10,13,15)$

wx \ yz	00	01	11	10
00	1			1
01	1	1	1	1
11		1	1	
10	1			1

$= \bar{w}x + xz + \bar{x}\bar{z} + \bar{w}\bar{z}$

↳ prime Implicants

$\bar{x}\bar{z}$ and xz are not covered by any other cells, so those are called as the EPI.

$f(A,B,C,D) = \Sigma m(0,2,3,5,7,8,10,11,14,15)$

AB \ CD	00	01	11	10
00	1		1	1
01		1	1	
11			1	1
10	1		1	1

$= CD + \bar{B}\bar{D} + AC + \bar{A}BD + \bar{B}C$

↳ P.I.

$EPI = \bar{B}\bar{D} + \bar{A}BD + AC$

3] $f(A,B,C,D) = \Sigma m(1,5,6,7,11,12,13,15)$

AB \ CD	00	01	11	10
00		1		
01		1	1	1
11	1	1	1	
10			1	

$BD + \bar{A}\bar{C}D + AB\bar{C} + \bar{A}BC + ACD$

No. of PI = 5

$(1,5) \rightarrow \bar{A}\bar{C}D$

$(12,13) \rightarrow AB\bar{C}$

$(11,15) \rightarrow ACD$

$(5,7,13,15) \rightarrow BD$

$(6,7) \rightarrow \bar{A}BC$

$EPI = \bar{A}\bar{C}D + ACD + \bar{A}BC + AB\bar{C}$ $\{= 4\}$.

4]. $f(A,B,C,D) = \Sigma m(2,6,7,9,13,15)$

AB \ CD	00	01	11	10
00				1
01			1	1
11		1	1	
10		1		

$= \bar{A}C\bar{D} + A\bar{C}D + BCD$

no. of PI = 5 ⇒ $\underset{\bar{A}C\bar{D}}{(2,6)}$ $\underset{BCD}{(7,15)}$ $\underset{A\bar{C}D}{(9,13)}$

$\underset{ABD}{(13,15)}$ $\underset{\bar{A}BC}{(6,7)}$

no. of EPI = 2 ⇒ (2,6), (9,13)

$\bar{A}C\bar{D} + A\bar{C}D$.

5] $f(A,B,C) = \Sigma m(0,2,3,4,5,7)$

$A \backslash BC$	00	01	11	10
0	1		1	1
1	1	1	1	

$= \bar{B}\bar{C} + A\bar{B} + AC + BC + \bar{A}B + \bar{A}\bar{C}$

$\hookrightarrow$ P.I

no. of PI = 6 $\Rightarrow$ (0,2), (2,3), (3,7), (5,7), (4,5), (0,4)

$f_3 = \bar{A}\bar{C} + A\bar{B} + BC$ (or) $\bar{B}\bar{C} + AC + \bar{A}B$

no. of EPI = 0

6]. $f(A,B,C,D) = \Sigma m(0,2,3,5,7,8,12,13)$

$AB \backslash CD$	00	01	11	10
00	1		1	1
01		1	1	
11	1	1		
10	1			

$f_4 = \bar{A}\bar{B}\bar{D} + \bar{A}CD + B\bar{C}D + A\bar{C}\bar{D}$

no. of PI = 8

No. of EPI = 0.