Number Systems and Codes

philosophy of Number Systems

- Number System is a basis for counting Various items
  We are familiar with decimal number System with its
  10 digits : 0, 1, 2, 3, 4, ..., 9.
- But modern computers Communicate and operate with binary numbers which use only 0 and 1.

  Ex: 18 — 10010

  In decimal it is represented by two digits where as in binary 5 digits.

  ∴ If decimal quanties are represented in binary form they take more digits.
- For large decimal numbers people have to deal with very large binary strings and therefore, they do not like working with binary numbers. This fact give rise to these new number Systems.

  octal, Hexadecimal and Binary Coded decimal
- These number Systems represent binary number in a Compressed form.

  ∴ these number Systems are now widely used to compress long strings of binary numbers.

1.1 Decimal number Systems

- In this we can express any number in units, hundred, thousand and so on.

Ex: 5678.9

: 5000+600 + 70 + 8 + 0.9 = 5678.9

Can also be written as 5678.9_{10} where 10 subscript indicates the radix or base.

- In power of 10 we can write

$$5\times10^3 + 6\times10^2 + 7\times10^1 + 8\times10^0 + 9\times10^{-1}$$

- The left most bit which has the greatest weight is called most significant bit (MSB)

- The Right most bit which has the least weight is called least significant bit (LSB)

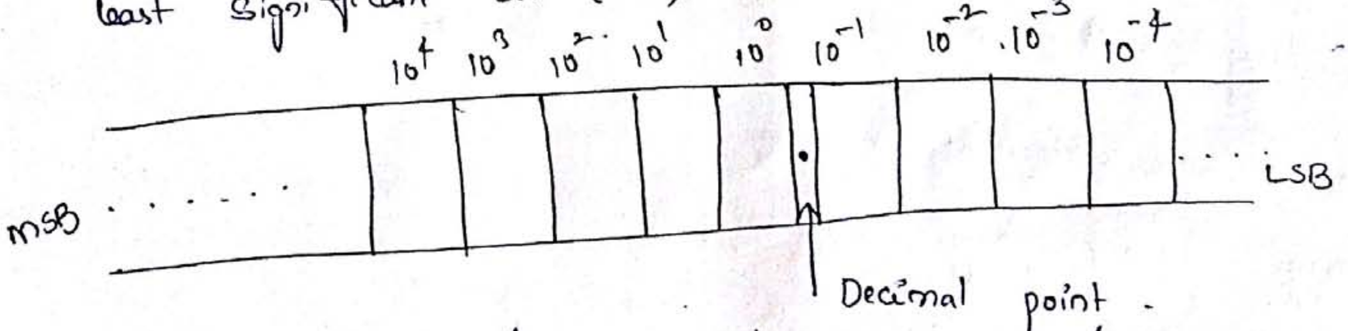

fig:- Decimal position values as powers of 10

Ex:- Represent decimal number 98.72 in power of 10

$$9\times10^1 + 8\times10^0 + 7\times10^{-1} + 2\times10^{-2}$$

for 9 power of 10 is 1
for 8 power of 10 is 0
for 7 power of 1/10 is -1
for 2 power of 1/100 is -2

Binary Number System :-

Binary system with its two digits is a base-two system. The two binary digits are 1 and 0. Each binary digits are known as bit.

- In binary system weight is expressed as power of 2.

Ex: Represent binary number 1101.101 in power of 2 and find its decimal equivalent

$$N = 1\times 2^3 + 1\times 2^2 + 0\times 2^1 + 1\times 2^0 + 1\times 2^{-1} + 0\times 2^{-2} + 1\times 2^{-3}$$

$$= 8+4+0+1+\frac{1}{2}+0+\frac{1}{8}$$

$$= (13.625)_{10}$$

1.3 Octal Number System :-

It uses first eight digits of decimal number system 0, 1, 2, 3, 4, 5, 6, and 7. As it uses 8 digits, its base is 8

Ex: Represent octal number 567 in power of 8 and find its decimal equivalent

$$5\times 8^2 + 6\times 8^1 + 7\times 8^0 = 5\times 64 + 6\times 8 + 7\times 1 = (375)_{10}$$

1.4 Hexadecimal Number System :-

- Its base is 16, it is having 16 digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F. Since its base is a power of 2, (2^4) it is easy to convert hexadecimal to binary numbers and vice versa.

Ex: It is useful for human communications with a computer

- Each hexadecimal digit represents a group of four binary digits called nibbles.

Relation b/n decimal, binary, Hexadecimal, octal:-

Decimal	Binary	Hexadecimal	Octal
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	8	18
9	1001	9	19
10	1010	A	12
11	1011	B	13
12	1100	C	14
13	1101	D	15
14	1110	E	16
15	1111	F	17

Represent
%

Represent hexadecimal number 3FD in power of 16 and find its decimal Equivalent (5)

$3 \times 16^2 + F \times 16^1 + D \times 16^0$

↓ ↓ ↓

3 F D

$3 \times 256 + 15 \times 16 + 13 \times 1$

$= 768 + 240 + 13 = (1021)_{10}$.

1.5. Counting in Radix (Base):—

— In general, a number represented in radix r, has r characters in its set and r can be any value. They are 0, 1, ... r−1.

Radix	characters in set
2	0, 1
3	0, 1, 2
4	0, 1, 3, 3
⋮	
7	0, 1, 2, 3, 4, 5, 6, −
8 (octal)	0, 1, 2, 3, 4, 5, 6, 7
⋮	⋮
(10) (Decimal)	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
⋮	
(16) (Hexadecimal)	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

Ex: find decimal Equivalent of $(231.23)_4$

$= 2\times 4^2 + 3\times 4^1 + 1\times 4^0 + 2\times 4^{-1} + 3\times 4^{-2}$

$= 32 + 12 + 1 + 0.5 + 0.1875$

$= (45.6875)_{10}$

Ex: Count from 0 to 9 in Radix 5

Sol Radix 5 has 5 characters.

0, 1, 2, 3, 4, 5 10, 11, 12, 13, 14

5 in radix $(10)_5$ —— $(5)_{10}$

$1\times 5^1 + 0\times 5^0 = 5+0 = 5_{10}$

$(11)_5$ —— $(6)_{10}$

$(11)_5 = 1\times 5^1 + 1\times 5^0 = 5+1 = 6_{10}$

$(7)_{10}$ —— $(12)_5 \Rightarrow 1\times 5^1 + 2\times 5^0 = 7_{10}$

$(8)_{10}$ —— $(13)_5 \Rightarrow 1\times 5^1 + 3\times 5^0 = 8_{10}$

$(9)_{10}$ —— $(14)_5 \Rightarrow 1\times 5^1 + 4\times 5^0 = 9_{10}$

Ex: Represent $(13)_{10}$ in octal

$(13)_{10}$ —— $(?)_8$

8 | 13
  1 - 5

$= (15)_8$

check $(15)_8$ —— $(?)_{10}$

$1\times 8^1 + 5\times 8^0$

$= 8+5 = (13)_{10}$

Number System Conversion

1 Binary to octal :-

— for octal numbers base is 8 and the base for binary is 2. i.e the base for octal number is the third power of base for binary numbers

∴ By grouping of 3 digits of binary numbers and then converting Each group digits to its octal Equivalent

Ex: octal to Binary Convert $(\underline{111}\ \underline{101}\ \underline{100})_2$ to octal

— $(7\ 5\ 4)_8$.

2.2 Octal to Binary

— Each digit of the octal number is individually converted to its binary Equivalent to get octal to binary conversion of the number

Ex: $(634)_8$ — $(?)_2$

$= (110\ 011\ 100)_2$

Ex: $(725.63)_8$ — $(?)_2$

$(111010101 . 110011)_2$

2.3 Binary to Hexadecimal

— Base for hexadecimal is 16 and the base for binary is 2. The base for hexadecimal number is the fourth power of the base for binary numbers

∴ By grouping 4 digits of binary number & converting each group digit to its hexadecimal equivalent, we can convert binary number to its hexadecimal equivalent.

Ex: Convert $(1101\ 1000\ 1001\ 1011)_2$ to hexadecimal equivalent

$(D\ 8\ 9\ B)_H$

2.4 Hexadecimal to Binary Conversion:

— Each digit of the hexadecimal number is individually converted to its binary equivalent to get hexadecimal to binary conversion of the number.

Ex: Convert $(5A9.B4)_H$ to binary

$(0101\ 1010\ 1001.\ 1011\ 0100)_2$

Ex: Convert $(3FD)_H$ to binary

$(0011\ 1111\ 1101)_2$.

2.5 octal to hexadecimal:

1. convert octal to binary number
2. Convert binary number to its hexadecimal Equivalent.

Ex: $(615)_8$ ——— $(\ )_{16}$

(i) $(110\ 001\ 101)_2 \rightarrow$ (ii) $= (18D)_H$

Hexadecimal to octal Conversion

(i) Convert hexadecimal number to its binary Equivalent

(ii) Convert binary number to its octal Equivalent.

Ex: $(25B)_H$ —— $(?)_8$

$(0010 \quad 0101 \quad 1011)_2$.

001 001 011 011

1 1 3 3 = $(1133)_8$

Converting any Radix to Decimal

– In general numbers can be represented as

$$N = A_{n-1}r^{n-1} + A_{n-2}r^{n-2} + \dots + A_1 r^1 + A_0 r^0$$
$$+ A_{-1}r^{-1} + A_{-2}r^{-2} + \dots + A_{-m}r^{-m}$$

N = Number in Decimal

A = Digit

r = Radix

n = The no. of digits in the integer portion of the number

m = The no. of digits in the fractional portion of number

Ex: $(1101.1)_2$ —— $(?)$

$= 1\times 2^3 + 1\times 2^2 + 0\times 2^1 + 1\times 2^0 + 1\times 2^{-1}$

$= 8 + 4 + 0 + 1 + 0.5$

$= (13.5)_{10}$

Ex: $(475.25)_8 \longrightarrow (?)_{10}$

$N = 4\times8^2 + 7\times8^1 + 5\times8^0 + 2\times8^{-1} + 2\times8^{-2}$

$= 256 + 56 + 5 + 0.25 + 0.078125$

$= (317.32813)_{10}$

Ex: $(3102.12)_4 \longrightarrow (?)_{10}$

$N = 3\times4^3 + 1\times4^2 + 0\times4^1 + 2\times4^0 + 1\times4^{-1} + 2\times4^{-2}$

$= 192 + 16 + 0 + 2 + 0.25 + 0.125$

$= (210.375)_{10}$

Ex: $(614.15)_7 \longrightarrow (?)_{10}$

$= 6\times7^2 + 1\times7^1 + 4\times7^0 + 1\times7^{-1} + 5\times7^{-2}$

$= 294 + 7 + 4 + 0.142857 + 0.102$

$= (305.24468)_{10}$

Conversion of Decimal Numbers to any Radix Number

Two Steps:

Step 1: Convert integer part. — This is done by Successive division method.

Step 2: Convert fractional part — This is done by Successive multiplication method.

(i) Successive Division for integer part:—

Ex: $(37)_{10}$ —— $(?)_2$

$$\begin{array}{l|l} 2 & 37 \\ 2 & 18 - 1 \\ 2 & 9 - 0 \\ 2 & 4 - 1 \\ 2 & 2 - 0 \\ & 1 - 0 \end{array} \uparrow = (100101)_2$$

$2^5\ 2^2\ 2^0$; $32 + 4 + 1 = 37$

Ex: $(3509)_{10}$ —— $(\ )_{16}$ – $(\ )_H$

$$\begin{array}{l|l} 16 & 3509 \\ 16 & 219 - 5 \\ & 13 - 11 \end{array} \uparrow = (DB5)_{16}$$

Ex: $(54)_{10}$ — $(?)_4$

$$\begin{array}{l|l} 4 & 54 \\ 4 & 13 - 2 \\ & 3 - 1 \end{array} \uparrow = (312)_4$$

(ii) Successive multiplication for fractional part:

- The number to be converted is multiplied by radix of new number, producing a product that an integer part and a fractional part.
- The integer part of the product becomes a numeral in the new radix

— The fractional part is again multiplied by the radix & the process is repeated until fractional part reaches 0 or the new radix number is carried out to sufficient digits. The integer part of each product is read downward to represent the new radix number.

Ex: $(0.8125)_{10}$ —— $(?)_2$

Fraction	Radix	Result	Recorded	carry
0.8125	× 2	= 1.625	= 0.625	1 (MSB)
0.625	× 2	= 1.25	= 0.25	1
0.25	× 2	= 0.5	= 0.5	0
0.5	× 2	= 1.0	= 0.0	1 (LSB)

$(0.1101)_2$

$(0.1101)_2$ — $(0.8125)_{10}$

Ex: $(0.95)_{10}$ —— $(?)_2$

0.95×2 = 1.9 = 0.9	1	
0.9×2 = 1.8 = 0.8	1	
0.8×2 = 1.6 = 0.6	1	
0.6×2 = 1.2 = 0.2	1	
0.2×2 = 0.4 = 0.4	0	
0.4×2 = 0.8 = 0.8	0	
0.8×2 = 1.6 = 0.6	1	

— 0.8 is repeated and if we multiply further, we will get repeated sequence. If we stop here, we get 7 binary digits (0.1111001)

$(0.1289062)_{10}$ — $(?)_{16}$

0.1289062 × 16 — 2.0625	2
0.0625 × 16 — 1.0	1

$(0.21)_{16}$

Ex: Convert $(24.6)_{10}$ — $(?)_2$

Step1: Seperate out integer and fraction part.

Integer part : 24 , fraction part : 0.6

Step2: find Equivalent binary number for Integer part.

2	24	
2	12	0
2	6	0
2	3	0
	1	1

$(11000)_2$

Step3: find Equivalent binary number for fractional

0.6×2	=1.2	0.2	1
0.2×2	=0.4	0.4	0
0.4×2	=0.8	0.8	0
0.8×2	=1.6	0.6	1
0.6×2	=1.2	0.2	1

$(0.10011)_2$.

∴ $(11000.10011)_2$

Questions

1) Convert $(725.25)_8$ to dec, bin, hex dec.

Sol (i) $= 7\times8^2 + 2\times8^1 + 5\times8^0 + 2\times8^{-1} + 5\times8^{-2}$

$= 448 + 16\times5 + \frac{1}{4} + \frac{5}{64}$

$= 469 + 0.25 + 0.078125$

$= (469.328125)_{10}$

(ii) $(725.25)_{10}$ — $(\ \ )_2$

$(111010101.010101)_2$

(iii) $\underline{0001}\ \underline{1101}\ \underline{0101}\ .\ \underline{0101}\ \underline{0100}$

$(1\ D\ 5\ .\ 5\ 4)_{16}$

Weighted Codes:

In this Each digit position of the number represents a Specific weight.

Ex: If number is 567 then wt of 5 is 100
wt of 6 is 10
wt of 7 is 1

Ex: Binary, BCD

– BCD is an abbreviation for binary coded Decimal. BCD is a numeric code in which Each digit of a decimal number is represented by a separate group of bits.

...plement Representation of negative numbers. ①

In digital Computers, to Simply the Subtraction operation and for logical manipulation Complements are used.

There are two types of Complements for Each radix System: The radix complement and diminished radix complement. The first is referred to as the r's complement and Second as the (r-1)'s complement.

for Example, in binary System we Substitute base value 2 in place of r to refer Complements as 2's complement and 1's complement. In decimal number system, we substitute base value 10 in place of r to refer complements as 10's Complement and 9's complement.

1's complement Representation:

The In 1's complement Representation, the number changes all 1's to zero's and all zero's to 1's.

find 1's complement of $(1101)_2$

1101 ← number
0010 ← 1's complement.

find 1's Complement of 10111001.

10111001 ← number
01000110 ← 1's complement.

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2's complement Representation.

The 2's complement is the binary number that results add 1 to the 1's complement. It is given as

2's complement = 1's complement + 1

The 2's complement form is used to represent negative numbers.

find 2's complement of $(1001)_2$

```
 1 0 0 1   ← number
 0 1 1 0   ← 1's complement
 +     1
 -------
 0 1 1 1   - 2's complement.
```

find 2's complement of $(1010001)_2$

```
 1 0 1 0 0 0 1  ← number
 0 1 0 1 1 1 0  — 1's complement
            +1
 -------------
 0 1 0 1 1 1 1
```

1's complement Subtraction

Subtraction of binary numbers can be accomplished by the direct method by using the 1's complement method, which allows to perform subtraction using only addition.

for subtraction of two numbers we have two cases.

- Subtraction of smaller number from larger number and
- subtraction of larger number from smaller number.

Subtraction of smaller number from larger number.

method:

1. Determine the 1's complement of the smaller number.
2. Add the 1's complement to the larger number.
3. Remove the carry and add it to the result.

1. Subtract $(101011)_2$ from $(111001)_2$ using the 1's complement.

```
  111001
  010100   — 1's complement.        57
 -------                            43
 ①001101                           ----
  |___________+1                    14
 -------
  001110   → final answer.
 -------
```

Subtraction of larger number from smaller number.

method:

1. Determine 1's complement of larger number.
2. Add the 1's complement to the smaller number.
3. Answer is in 1's complement form. To get the answer in true form take the 1's complement and assign negative sign to the answer.

Subtract $(111001)_2$ from $(101011)_2$ using the 1's complement method.

```
  101011
  000110   — 1's complement of 111001
 -------
  110001   — Answer in 1's complement form.
 -------
 -001110   — Answer in true form.
```


2's Complement Subtraction

In 2's complement subtraction, the subtraction is accomplished by addition. let us see the methods for 2's complement subtraction.

→ Subtraction of smaller number from larger number.

method:

1. Determine the 2's complement of a smaller number.
2. Add the 2's complement to the larger number.
3. Discard the carry.

Subtract $(101011)_2$ from $(111001)_2$ using the 2's complement method.

```
                 1 1 1 0 0 1          0 1 0 1 0 0
2's comp →       0 1 0 1 0 1                   +1
                ------------         ------------
               (1)0 0 1 1 1 0         0 1 0 1 0 1
                ↓                    ------------
             Discard
```

0 0 1 1 1 0 — final answer.

Subtraction of larger number from smaller number:

method:

1. Determine the 2's complement of larger number.
2. Add the 2's complement to the smaller number.
3. Answer is in the 2's complement form. To get the answer in the true form take the 2's complement and assign negative sign to the answer.

(3)

...btract $(111001)_2$ from $(101011)_2$ using 2's complement method.

```
  1 0 1 0 1 1
  0 0 0 1 1 1
 -------------
  1 1 0 0 1 0
```

```
  000110
      +1
 --------
  000111   - 2's complement.
```

↳ Answer in 2's complement form

```
   0 0 1 1 0 1
          +1
 -------------
 - 0 0 1 1 1 0
```

Decimal	Signed magnitude	1's complement	2's complement.
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
-0	1000	1111	0000
-1	1001	1110	1111
-2	1010	1101	1110
-3	1011	1100	1101
-4	1100	1011	1100
-5	1101	1010	1011
-6	1110	1001	1010
-7	1111	1000	1001

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Signed Binary Numbers

Signed Binary Numbers mainly used for represent Sign of the number.

However, because of hardware limitations, in computers both positive and negative numbers are represented with only binary digits.

The left most bit (sign bit) in the number represents sign of the number.

The sign bit is 0 positive numbers

The sign bit is 1 negative numbers.

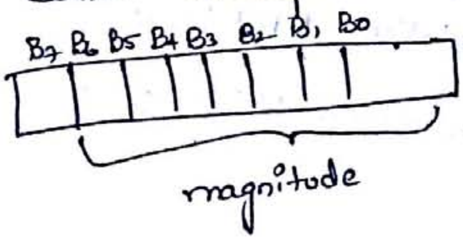
These numbers are represented by the signed magnitude format.

Below fig shows the sign magnitude format for 8-bit signed number.

Here, the most significant bit (msB) represents sign of the number.

if msB is 1, number is negative and if msB is 0, number is +ve, The remaining bits represent magnitude of the numbers.

Here some Examples for sign-magnitude numbers.



+6

2 | 6
2 | 3 − 0
  1 − 1

00000110

−14

2 | 14
2 | 7 − 0
2 | 3 − 1
  1 − 1

10001110

+24 00011000

-64 1100 0000

In case of unsigned 8-bit binary numbers the decimal is 0 to 255..

- For signed magnitude 8-bit binary numbers the largest magnitude is reduced from 255 to 127.

because we need to represent both positive and negative numbers.

maximum +ve number 0111 1111 = +127

maximum -ve number 1111 1111 = -128

010
101

000

We have seen +ve and -ve number representation in the signed magnitude format.

This is the only way to represent +ve numbers!

because there are two more way represent negative numbers:

Signed - 1's Complement representation.

Signed 2's complement representation

1001 9
0011 3
0110 6

Let us see how -6 represents in three formats:

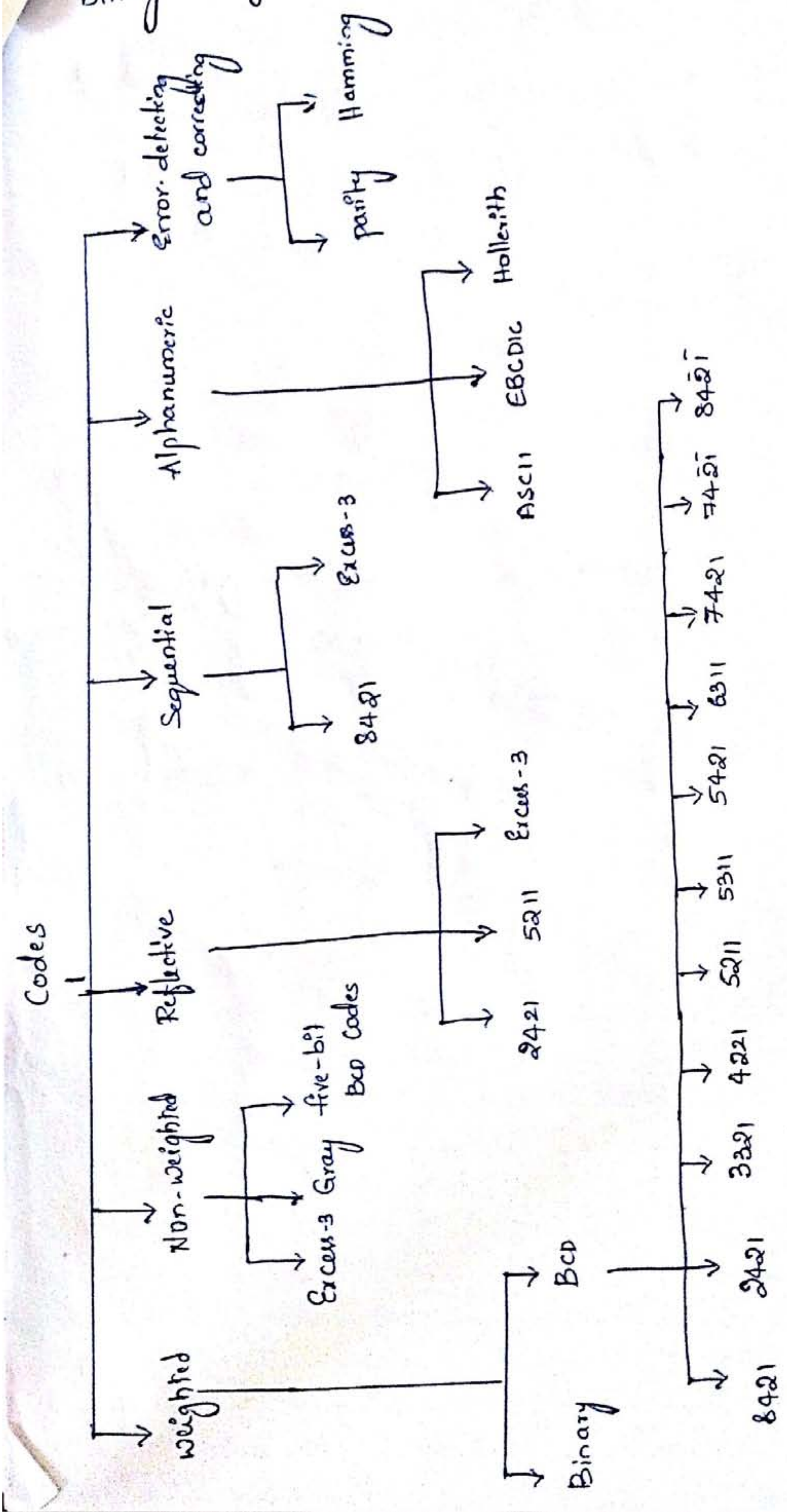
Signed - magnitude representation 10000110

Signed 1's complement representation 11111001

Signed 2's complement representation 11111010

y codes:-

The digital data is transmitted, stored and represented as groups of binary digits (bits). The group of bits, also known as binary code,



classification of various binary codes.

Alphanumeric codes : The code which consists of both alphabets and numbers.

ASCII code:- It is a 7-bit code which includes 0-9-bit code upper case alphabets, lower case alphabets, and other Symbols.

it is preceded by 011 for decimal no.

0 → 011 0000
9 → 011 1001

A — 65 → 100 0001
Z — 90 → 10 11010

a — 97 → 110 0001
z — 122 — 111 1010.

2 | 90
2 | 45 — 0
2 | 22 — 1
2 | 11 — 0
2 | 5 — 1
2 | 2 — 1
1 — 0

EBCDIC:- It is a 8-bit code. It is preceded by 1111 for decimal digits

a — 129
z — 154.

Decimal Digit	BCD Code 8 4 2 1
0	0 0 0 0
1	0 0 0 1
2	0 0 1 0
3	0 0 1 1
4	0 1 0 0
5	0 1 0 1
6	0 1 1 0
7	0 1 1 1
8	1 0 0 0
9	1 0 0 1

— In multidigit coding, Each decimal digit is individual coded with 8421 BCD Code

Ex: $(58)_{10} \xrightarrow{8421}$ 0101 1000.

— If we want to represent Same 58 in binary the Equivalent is $(111010)_2$ i.e we require only 6 digits that means 8421 BCD is less efficient than pure binary number.

— Advantage: It is easy to convert between it and deci...

— Disadvantage: low efficiency, the arithmetic operations are complex than they are in pure binary.

2421 code:

Decimal Digit	2421
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1110
9	1111

2: Non weighted Codes: These are not assigned with any weight to each digit position. Ex:

Excess-3 Code:

It is a modified form of a BCD number. It can be derived from the natural BCD code by adding 3 to each coded number

Ex: 12 in BCD as 0001 0010

Now adding 3 to each digit we get Excess-3 code as

```
  0001    0010
    11      11
  ------------
  0100    0101  — Excess-3 code for (12)10
```

BCD Addition:

The addition of two BCD numbers can be best understood by considering the three cases that occur when two BCD digits are added.

Sum Equal to 9 or less with carry 0.

let us Consider additions of 3 and 6 in BCD.

```
  6      0110  <- BCD for 6
 +3      0011  <- BCD for 3
 --      ----
  9      1001  <- BCD for 9.
```

```
  9
+ 3
---
 12
```

Sum greater than 9 with Carry 0.

```
  6      0110
  8      1000
 --      ----
 14      1110  <- invalid BCD (1110) >9.
```

Whenever invalid BCD no. occurs, the sum has to be corrected by the addition of six (0110) in the invalid BCD no. as shown below.

```
  6      0110
  8      1000
 --      ----
 14      1110  -- invalid BCD no.
         0110
     ---------
     0001 0100  <- BCD for 14.
       1    4
```

Sum Equals 9 or less with carry 1.

let us consider addition of 8 and 9 in BCD.

```
 8          1000
 9          1001
--      ---------
17      0001 0001   — incorrect BCD result.
        ---------
        0001 0001   — add 6 for correction
             0110
        ---------
        0001 0111
        ---------
           1    7   — BCD for 17.
```

```
24      0010     0100
18      0001     1000
        ----     ----
        0011     1100    invalid BCD no.
                              1100>9.
                 0110    add 6 for correction.
        -------------
        0100     0010    — propagate carry to
        -------------       next higher digit.
```

```
589     0101    1000    1001
199     0001    1001    1001
---     --------------------
788     0111    0010    0010
                0110    0110
        --------------------
        0111    1000    1000   — corrected sum.
        --------------------
```

99
26
53

0111 1001 99
0111 0011 73 - 9's complement for BCD 26. 26 / 73

1110 1100

0110 1100 > 9 add 6 to result.

1111 0010 1111 > 9.

0110

10101 0010
→ 1

0101 0011 BCD for 53.

89
- 54

99
54
45

1000 1001
0100 0101

1100 1110 1110 > 9 So add 6.

0110

1101 0100 1101 > 9 add 6.

0110

10011 0100
→ 1

0011 0101 BCD for 35.

175	0001	0111	0101	
326	0011	0010	0110	
501	0100	1001	1011	1011 > 9
			0110	add 6 to 1011 for correction.
	0100	1010	0001	1010 > 9. add 6.
		0110		
	0101	0000	0001	propagate carry to next higher digit.

BCD Subtraction :

Addition of Signed BCD numbers Can be performed by using 9's and (or) 10's Complement methods.

A negative BCD number Can be Expressed by taking the 9's or 10's complement.

$$\begin{array}{r} 8 \\ -2 \end{array} \qquad \begin{array}{r} 9 \\ 2 \\ \hline 7 \end{array} \qquad \begin{array}{r} 8 \\ +7 \\ \hline 15 \\ \hookrightarrow 1 \\ \hline 6 \end{array}$$

79 − 26. perform the BCD Subtraction using 9's complement method.

Subtraction using 10's complement.

Regular Subtraction   10's complement Subtraction.

```
  8            9            8
- 2            2            8
----          ---          ----
  6            7           16.
              +1
              ---
               8
```

```
  8        1000
+ 8        1000
----      ------
 16       10000
           0110
          ------
           0110  - 6.
          ------
```

- find the 10's Complement of negative numbers.
- Add two numbers using BCD addition
- if carry is not generated find the 10's complement of the result otherwise find the 9's complement of the result.

```
→  79      99      79      0111  1001
 - 26      26     +73      0111  0100
 ----     ----    ----    -----------
   53      73              1110  1101     1101 > 9.
  ----     +1                    0110
                          -----------
                           1111  0011     1111 > 9
                           0110
                          -----------
                 discard (1) 0101  0011
                              5     3
```

Excess-3 :- It is a modified form of BCD & add in Decimal code add 3 to BCD of each digit.

	BCD		Excess-3	
12 –	0001	0010	0100	0101

It is a Self Complementing code.

Excess-3 addition:

Ex:- $(592)_{10}$ – 1000 1100 0101

9's Comp (592) 0111 0011 1010

Ex:

	BCD	
98	1001	000
Excess	1100	101
	0011	0100

99
98
01

Excess-3 addition

To perform Excess-3 addition we have to

- Add two Excess-3 numbers.
- if carry = 1 → add 3 to the Sum of two digits
  = 0 → Subtract 3

```
  8      1011
+ 6      1001
 14   00010100
      00110011
      01000111
        1   4
```

```
  1    0100 — Excess-3
 +2    0101 — Excess-3
  3    1001
Sub3 - 0011
       0110 — Excess-3 for 3.
```

Decimal Digit	Excess-3 code
0	0011
1	0100
2	0101
3	0110
4	0111
5	1000
6	1001
7	1010
8	1011
9	1100

- In Excess-3 code, we get 9's complement of a number by just complementing each bit. Due to this Excess-3 code is called self-complementing code.

Ex: 592 in Excess-3 1000 1100 0101 9's complement of 592 is

```
 999
 592
-----
 407
```

407 in Excess-3 0111 0011 1010

i.e nothing but just complementing the 592 in Excess-3 you will get 9's complement of 592.

Ex: 403 in Excess-3 0111 0011 0110

9's complement of 403 is 1000 1100 1001

$$\begin{array}{r} 999 \\ -403 \\ \hline 596 \end{array}$$

596 in Excess-3 is 1000 1100 1001

Gray Code:

It is a special case of unit-distance code. In this bit patterns for two consecutive numbers differ in only one bit position. It is also called as cyclic code.

Decimal Code	Gray code
0	0000
1	0001
2	0011
3	0010
4	0110
5	0111
6	0101
7	0100
8	1100
9	1101
10	1111
11	1110
12	1010

Decimal	Gray
13	1011
14	1001
15	1000

gray position. Signi

gray code any two adjacent code groups differ only in one bit position. It is also called as reflected code, because two least Significant bits for 4 through 7 are the mirror images of those for 0 to 3.

- Similarly, the three LSB for 8 to 15 are the mirror images of are from 0 to 7.

Binary to Gray conversion.

Let us represent binary numbers as $B_1, B_2, B_3, B_4, \dots B_n$ and its Equivalent gray code as $G_1, G_2, G_3 \dots G_n$

$G_1 = B_1$

$G_2 = B_1 \oplus B_2$

$G_3 = B_2 \oplus B_3$

$G_4 = B_3 \oplus B_4$

$\vdots$

$G_n = B_{n-1} \oplus B_n$

Ex: 1) $(10110110)_2 \xrightarrow{\text{Gray}} (11101101)$

2) $(1010101010110)_2 \xrightarrow{\text{Gray}} (1111111111101)$

3) $(110010)_2 \xrightarrow{\text{Gray}} (101011)$

Gray to Binary Conversion

Steps

1. The MSB of the binary number is the Same as the MSB of Gray code number.

2. To obtain the next binary digit, perform an Ex-OR between the bit just written down and the next gray code bit. Write down the result.

3. Repeat Step 2 until all gray code bits have been Ex-ORed with binary digits. The Sequence of bits that have been written down is the binary Equivalent of gray code

Ex: 1) Gray Code : 1 0 1 0 1 1

1 1 0 0 1 0 (Binary)

2) Gray code : 1 1 1 1 1 1 1 1 1 0 1

$(1 0 1 0 1 0 1 0 1 1 0)_2$

3) Gray code 1 1 1 0 1 1 0 1

$(1 0 1 1 0 1 1 0)_2$

→ Alphanumeric codes: It includes numbers, letters, and other Symbols.

ASCII (American standard code for information interchange) — 011

EBCDIC (Extended binary coded decimal Interchange code) — 1111

Detecting and Correcting Codes

- When the digital information in the binary form is transmitted from one circuit to another an error may occur.
- To maintain data integrity between transmitter and receiver extra bits are added in the data, these allow the detection and sometimes correction of errors.

Parity Bit:

- It is used for the purpose of detecting errors during transmission.
- A parity bit is an extra bit included with a binary message to make the number of 1s either odd or even.
- The message, including the parity bit is transmitted and then checked at the receiving end for errors.
- An error is detected if the checked parity does not correspond with the one transmitted.
- The ckt that generates the parity bit in the Tx is called as parity Generator.
- The ckt that checks the parity in the Rx is called as parity checker.

Decoding the Received Codewords (Detecting the Error) :-

At the receiving end the receiver doesnot know the transmitted word. However it knows A matrix used for generation of code words. Its function is to check the message bits using check bits.

Steps:

1. form the matrix H as $H = [A^T \vdots I_r]$

   where A^T of sub-matrix A

   I_r = identity matrix of order of r (no. of check bits)

   matrix H is called parity check matrix.

2. Now if $H \cdot R^T = 0$ — Received word is correct

   $H \cdot R^T \neq 0$ Error in received code.

Ex: For a (4,2) block code received code is 10 11

If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, find received code is correct or wrong.

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} ; \quad A^T = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

n – msg. bits

k – check bits

$$H = [A^T \vdots I_r]$$

$$= \begin{bmatrix} 1 & 0 & \vdots & 1 & 0 \\ 1 & 1 & \vdots & 0 & 1 \end{bmatrix}$$

WKT,

$$R = [1 \; 0 \; 1 \; 1]$$

$$R^T = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

$$H \cdot R^T = \left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right] \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\cdot1 \oplus 0\cdot0 \oplus 1\cdot1 \oplus 0\cdot1 \\ 1\cdot1 \oplus 1\cdot0 \oplus 0\cdot1 \oplus 1\cdot1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

for Even no. of 1's of Ex-OR = 0
odd no. of 1's of Ex-OR = 1

$\therefore H \cdot R^T = 0$, received code is correct.

Ex: ~~for~~ a (4,2) block code received code is 1010 If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, find received code is correct or wrong

$$H = [A^T \,\vdots\, I_r] = \left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right]$$

$$R^T = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$H \cdot R^T = \left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right] \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1\cdot1 \oplus 0\cdot1 \oplus 1\cdot1 \oplus 0\cdot0 \\ 1\cdot1 \oplus 1\cdot0 \oplus 0\cdot1 \oplus 1\cdot0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$\therefore H \cdot R^T \neq 0$, received code is wrong

Error Correction:

- In order to correct the codeword we multiply received codeword with transpose of parity check matrix to get syndrome.
- Syndrome is compared with the row of transpose of parity check matrix. (H^T)
- matching row number is the number of bits in error. Error bit is then inverted to get the correct code.

Steps :-

1. find $S = RH^T$

   $R \rightarrow$ Received code $H^T \rightarrow$ transpose of H.

   $S \rightarrow [S_1, S_2, S_3 \ldots]$ is called Syndrome.

2. Match the result, i.e S, with row of H^T, The no. of row where the match occur gives the no. of bit in error. This bit is inverted to correct the error.

Ex:- for a (6,3) block code received code is 1 1 0 1 0 0

If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ find the received code is correct/wrong

If wrong correct it

$$A^T = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$H = [A^T \vdots I_r] = \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

ived
code
rrone.

$R = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \end{bmatrix}$

$R^T = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

$$H \cdot R^T = \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1.1 \oplus 1.1 \oplus 1.0 \oplus 1.1 \oplus 0.0 \oplus 0.0 \\ 0.1 \oplus 0.1 \oplus 1.0 \oplus 0.1 \oplus 1.0 \oplus 0.0 \\ 0.1 \oplus 1.1 \oplus 1.0 \oplus 0.1 \oplus 0.0 \oplus 1.0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$\because$ $HR^T \neq 0$, received code is wrong.

let us find $S = R \cdot H^T$

$$S = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \end{bmatrix}_{1\times 6} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{6\times 3}$$

$1.1 \oplus 1.1 \oplus 0.1 \oplus 1.1 \oplus 0.0 \oplus 0.0 \quad 1.0 \oplus 1.0 \oplus 0.1 \oplus 1.0 \oplus 0.1 + 0.0$

$1.1 \oplus 1.1 \oplus 0.1 \oplus 1.0 \oplus 0.0 \oplus 0.1]$

$= \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$

$S = [1\ 0\ 1]$ matches with second row of H^T

∴ Second bit of received code is in error. By inverting second we get,

correct code $= R_c = [1\ 0\ 0\ 1\ 0\ 0]$

Hamming Code

- It provides the detection of a bit error, also identifies which bit is in error so that it can be corrected. So it is called as error detecting and correcting code.

Number of parity bits

- No. of parity bits depend on the no. of information bits. If no. of parity bits is p, then the relation is

$$2^p \geq x + p + 1$$

$2^1 \geq 4+1+1$

$2^2 \geq 4+2+1$

$4 \geq 7$

$2^3 \geq 4+3+1$

$8 \geq 8$

Ex: If $x = 4$, then p is formed by trial

let $p = 2$

$2^2 \geq 4+2+1$

$4 \geq 7 \longrightarrow$ not satisfied.

So, let $p = 3$

$2^3 \geq 4+3+1$

$8 \geq 8$

∴ Three parity bits are required to provide single error.

Correction for four information bits.

Seco... ion of parity bits in the code :-

- These are located in the positions that are numbered corresponding to ascending powers of two (1, 2, 4, 8, ...)..

- So, for 4-bit information total no. of bits are 7 as $D_7, D_6, D_5, D_4, D_3, D_2, D_1$

Assigning Values to parity Bits

- In hamming code, Each parity bit provides a check on certain other bits in the total code.

Bit designation	D_7	D_6	D_5	D_4	D_3	P_2	P_1
Bit location	7	6	5	4	3	2	1
Binary location number	111	110	101	100	011	010	001

Assigning of P_1: P_1 has a 1 for its right most digit. This checks all bit locations, including itself, that have 1's in the same location in the binary number location.

$\therefore P_1$ checks 1, 3, 5, 7 and assigns P_1 according to Even or odd parity.

- for Even parity hamming code, it assigns P_1 such that bit locations 1, 3, 5 and 7 will have Even parity.

Assigning of P_2 :- P_2 has 1 for its middle bit. It checks all bit locations, including itself that have 1s in the middle bit.

$\therefore P_2 \rightarrow$ ~~P_2~~, 3, 6 and 7.

Assigning of P_4 :- P_4 has 1 for its left most digit

$\therefore P_4 \rightarrow$ 4, 5, 6 and 7

Ex: Encode the binary word 1011 into seven bit even parity Hamming code.

Step 1: No. of parity bits is

$2^p = 2^3 = 8$

$x+p+1 = 4+3+1 = 8$

Step 2: Construct a bit location table

Bit designation	D_7	D_6	D_5	D_4	D_3	P_2	P_1
Bit location	7	6	5	4	3	2	1
Bit location number	111	110	101	100	011	010	001
Information bits	1	0	1		1		
parity bits				0		0	1

$\therefore$ code 1 0 1 0 1 0 1.

Ex: Determine the single error correcting code for the following information code 10111 for odd parity.

No. of parity bits is $2^p \geq x+p+1$

$2^4 \geq 5+4+1$

$16 \geq 10$

Total bits $= 5+4 = 9$

Bit Designation	D_9	P_8	D_7	D_6	D_5	P_4	D_3	P_2	P_1
Bit location	9	8	7	6	5	4	3	2	1
Binary location number	1001	1000	0111	0110	0101	0100	0011	0010	0001

	1001	1000	0111	0110	0101	0100	0011	0010	0001
position bits	1		0	1	1		1		
Even parity bits		0				1		1	0

∴ code word = 1 00 111 11 0

Detecting and Correcting an Error :

— In this each parity bit, along with its corresponding group of bits must be checked for proper parity.

The correct result of individual parity check is marked as '0' wrong as '1'. After all parity checks, binary word is formed taking resulting bit for P_1 as LSB. This word gives bits location where error has occured.

If all bits are zero, then no error.

Assume that the even parity Hamming code is 0110011 is transmitted and that 0100011 is received. The receiver doesnot know what was transmitted. Determine bit location where error has occured using received code.

step1 : construct bit location table

Bit designation	D_7	D_6	D_5	P_4	D_3	P_2	P_1
Bit location	7	6	5	4	3	2	1
Binary location number	111	110	101	100	011	010	001
Received Code	0	1	0	0	0	1	1

Step2 : check for parity bits

- for P_1 : P_1 checks 1, 3, 5 and 7

There is one 1 in the group
∴ parity check for even parity is wrong — 1 (LSB)

for P_2 : P_2 checks 2, 3, 6 and 7

There are two 1s in the group.

∴ parity check is correct → 0

for P_4 : P_4 checks 4, 5, 6 and 7

There is one 1 in the group

∴ parity check is wrong → 1

The resultant word is 101. It says 5 location is in Error. It is 0 and should be 1. ∴ Correct code is 0110011

Ex: A Seven bit hamming code is received as 1111101 what is the correct code. (Assume as Even parity).

Step1

Bit designation	D_7	D_6	D_5	P_4	D_3	P_2	P_1
Bit Location	7	6	5	4	3	2	1
Bit Location number	111	110	101	100	011	010	001
Received code	1	1	1	1	1	0	1

Step2 : Check for parity

P_1 : 1, 3, 5, 7 → 1111 → (Even) → 0 LSB

P_2 : 2, 3, 6, 7 → 0111 → (odd) → 1

P_3 : 4, 5, 6, 7 → 1111 → (Even) → 0

010 → 2 bit position

∴ Correct code is 1111111

message below has been coded in the even parity hamming. Decode message assuming that at most a single error occured in each code word.

a) 1 001 001

b) 0 111 001

c) 1110110

d) 0011001

(i) 1001001

Bit designation	D_7	D_6	D_5	P_4	D_3	P_2	P_1	Error code
Bit location	7	6	5	4	3	2	1	
Hamming code message	1	0	0	1	0	0	1	
1, 3, 5 and 7 → P_1							1	0
2, 3, 6, 7 → P_2						0		1
4, 5, 6, 7 → P_4				1				0

010 → Error in bit position 2

∴ Correct code is 1 001 0 11 and message is 1000

(ii) 0111 001

Bit designation	D_7	D_6	D_5	P_4	D_3	P_2	P_1	Error code
Bit location	7	6	5	4	3	2	1	
Hamming code message	0	1	1	1	0	0	1	0
1, 3, 5, 7 → P_1							1	1
2, 3, 6, 7 → P_2						0		1

110 → Error in 6 position

Correct Code is 0011 001, message is 0010.

(iii) 1110110

Bit designation	D_7	D_6	D_5	P_4	D_3	P_2	P_1	Error code
Bit location								
Hamming coded message	1	1	1	0	1	1	0	
1, 3, 5, 7 → P_1							0	1 LSB
2, 3, 6, 7 → P_2						1		0
4, 5, 6, 7 → P_4				0				1

∴ 101 → 5 bit position Error

∴ correct code is 1100110

message bits 1101

(iv) 0011011

Bit designation	D_7	D_6	D_5	P_4	D_3	P_2	P_1	Error code
Bit location	7	6	5	4	3	2	1	
Hamming coded message	0	0	1	1	0	1	1	
1, 3, 5, 7 → P_1							1	0 (LSB)
2, 3, 6, 7 → P_2						1		1
4, 5, 6, 7 → P_4				1				0

∴ 010 → Error 2bit

∴ Correct Code is 0011001

∴ message is 0010