II B.Tech I Semester Examinations,May/June 2012
MATHEMATICAL FOUNDATION OF COMPUTER SCIENCE
Common to Information Technology, Computer Science And Engineering
Time: 3 hours   Max Marks: 75
Answer any FIVE Questions
All Questions carry equal marks
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1. Show that T → S is a valid conclusion from the given premises
   $(P \Lambda Q) V (R \to S)$, T → R, ¬PΛQ)   [15]

2. (a) Verify Euler's formula for the following graph. Figure 1

   (b) What do you mean by a spanning tree? Explain BFS method for finding a
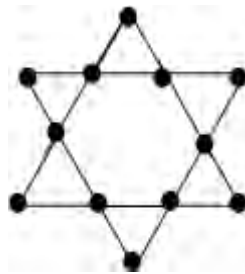   spanning tree for the graph.   [15]

Figure 1:

3. Find a general expression for a solution to the recurrence relation
   $a_n - 5a_{n-1} + 6a_{n-2} = 2^n$ for n ≥ 2.   [15]

4. (a) Prove that the intersection of two submonoids of a monoid is a monoid.

   (b) Define subgroup, semigroup and monoid. Give an example of a monoid which
       is not a group.   [8+7]

5. (a) Find the CNF of QV(PΛ¬Q)V(¬ P Λ¬ Q).

   (b) Show that the following statements are logically equivalent without using truth
       table.
       $(P \to Q) \Leftrightarrow (P \Lambda Q) V (\neg P \Lambda \neg Q)$.   [15]

6. (a) Use the multinomial theorem to expand $(x_1 + x_2 + x_3 + x_4)^4$

   (b)Show that C(n,1)+3C(n,3)+5C(n,5)+....=2C(n,2)+4C(n,4)+...= $n \, 2^{n-2}$ [15]

7. (a) Let R be the set of real numbers & let f:R → R be defined by $f(x) = x^2$. Is
       'f' invertible?

   (b) Let R = {(1,2), (3,4), (2,2)} and S = {(4,2), (2,5), (3,1), (1,3)}. Find RoS,
       SoR, Ro(SoR), (RoS)oR, RoR, SoS, RoRoR, SoSoS   [15]

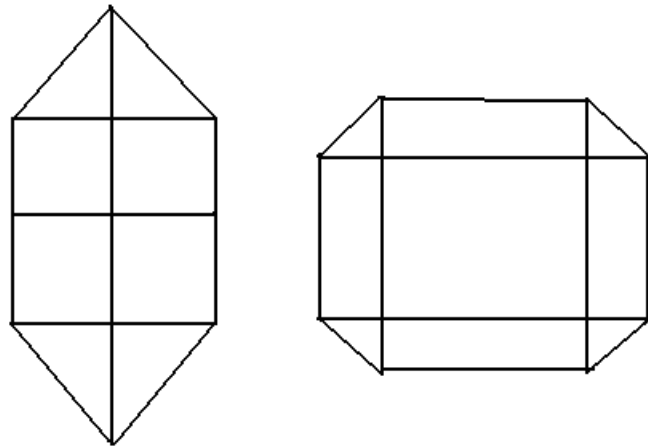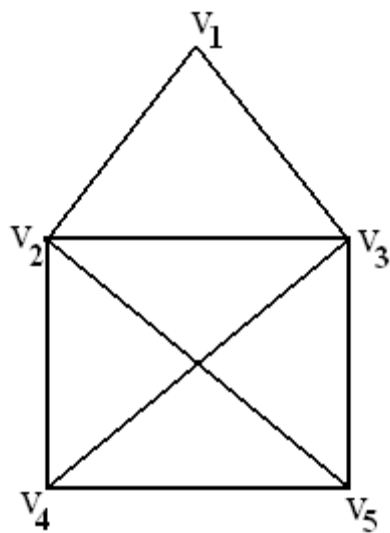8. (a) Prove that the following graphs (figure 2) are not isomorphic.

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Figure 2:

Figure 3:

(b) Find the chromatic polynormal of following (figure 3) graph. [15]

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1. (a) Using DFS method construct a spanning tree for the graph G = ({a,b,c,d,e,f,g,h,i},R),
where R = {(a,b),(a,c),(b,c),(b,d),(b,g),(c,e),(c,f),(d,f),(e,g),(f,g),(f,h),(f,i),(g,h),(g,i),(h,i)}

   (b) If the adjacency matrix is given by $\begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 \\ 1 & 2 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix}$. Find the loops and the
   multi edges of the graph and draw the graph.　　　　　　　　[15]

2. (a) Prove or disprove the validity of the following arguments using the rules of
   inference.
   All men are fallible
   All kings are men
   Therefore, all kings are fallible

   (b) Show that $(\exists x)\,(p\,(x)\,\Lambda Q\,(x)) \Rightarrow (\exists x)\,(p\,(x)\,\Lambda \exists\,(x)\,Q\,(x))$.　　　　[15]

3. Lt U = {1, ... , 9} A = {1,2,4,6,8}, B = {2,4,5,9}, C = {x/x is a '+' ve integer
   and $x^2 \leq 10$}. Compute the following.

   (a) A - B

   (b) B - A

   (c) A - C

   (d) $(A \cup B) - C$

   (e) $A \cup (B - C)$

   (f) (A - B) - C

   (g) $(A \cup B) - (A \cup C)$

   (h) $(A \cup B) - (B \cap C)$　　　　　　　　　　　　　　　[15]

4. Write the generating function for $a_r = 5^r + (-1)^r\,3^r + 8c(r+3, r)$　　　[15]

5. Given the algebraic system <N, +> and <Z_4, $+_4$>, where N is the set of natural
   numbers and + is the addition operation on N and Z_4 denote the set of equivalence
   classes generated as Z_4 = { [0],[1], [2], [3]} AND $+_4$ define an operation on Z_4
   given by [i] $+_4$ [j] = [(i+j) (mod 4)] for all I,j = 0,1,2,3. Show that there exists a
   homomorphism from <N,+> to <Z_4,$+_4$>.　　　　　　　　[15]

6. (a) Find the number of words that can be formed by scrambling the letters of the
   word SCRAMBLE.

(b) A salesperson at a computer store would like to display six models of personal computers, five models of computer monitors, and four models of keyboards. In how many different ways can we arrange them in a row if the items of the same family are to be next to each other? [15]

7. (a) Show that $\neg(P \leftrightarrow Q) \leftrightarrow (P \wedge \neg Q) V (\neg P V Q)$.

   (b) Find PDNF of $(\neg P \leftrightarrow R) \wedge (Q \leftrightarrow P)$ [15]

8. (a) Prove that S_5 is isomorphic to K_{14}.

   (b) Prove that every planer graph G has $\chi(G) \leq 6$. [15]

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1. (a) Show that if a group $<G,*>$ is of even order. Then there must bge an element $a \in G$, such that $a \neq e$ and $a*a = e$.

   (b) Determine all the proper subgroups of the symmetric group $<S_3, \diamondsuit>$ described in the table   [7+8]

$\diamondsuit$	P_1	P_2	P_3	P_4	P_5	P_6
P_1	P_1	P_2	P_3	P_4	P_5	P_6
P_2	P_2	P_1	P_5	P_6	P_3	P_4
P_3	P_3	P_6	P_1	P_5	P_4	P_2
P_4	P_4	P_5	P_6	P_1	P_2	P_3
P_5	P_5	P_4	P_2	P_3	P_6	P_1
P_6	P_6	P_3	P_4	P_2	P_1	P_5

2. (a) Construct the truth table for the following statement
      $(P \rightarrow Q) \vee (\neg P \rightarrow R)$

   (b) Show that the following statements are logically equivalent without using truth table.
      $(P \rightarrow R) \wedge (Q \rightarrow R) \Leftrightarrow (PVQ) \rightarrow R$ [15]

3. (a) Let Q(x,y) denote the statement "x = y + 2", what are the truth values of the propositions Q(1,2) and Q(2,0).

   (b) If S={-2.-1,0,1,2} determine the truth value of $\left|x\right|^2 \leq 3\left|x\right| - 2,\ x \in S$ [15]

4. (a) A function f:(Z×Z) → Z is defined by f(x,y) = 4x - 5y. Prove that f is not one-to-one, but onto

   (b) If A,B,C are three sets such that A ⊆ B. Show that $(A \times C) \subseteq (B \times C)$

   (c) If A = {1,2,3}, B = {4,5}. Find
      i. A×B
      ii. B×A   [15]

5. (a) Find the generating function of n^2-2.

    (b) Solve $a_n = a_{n-1} + n$, where $a_0 = 2$ by substitution   [15]

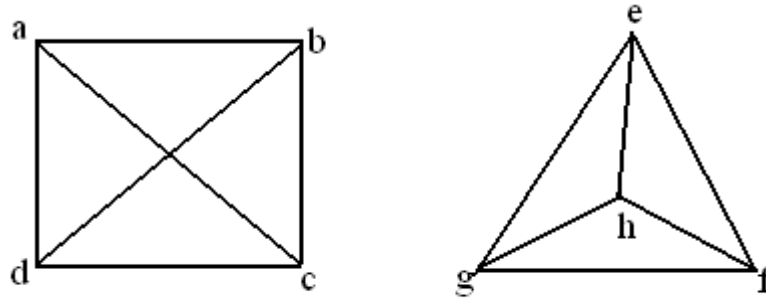6. Find whether the following graphs (figure 4) are isomorphic.   [15]

Figure 4:

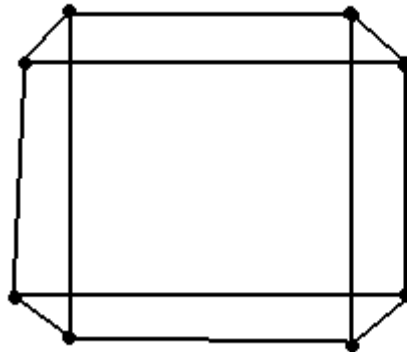7. (a) Verify Euler's formula for following graph (figure 5).

Figure 5:

    (b) Use Krushal's algorithm to find the minimum spanning tree for the following graph (figure 6).   [15]

8. (a) Find the number of arrangements of four books from 12 on a shelf.

    (b) Find the number of distinct triples (x_1, x_2, x_3) of non-negative integers satisfying $x_1 + x_2 + x_3 < 15$.   [15]
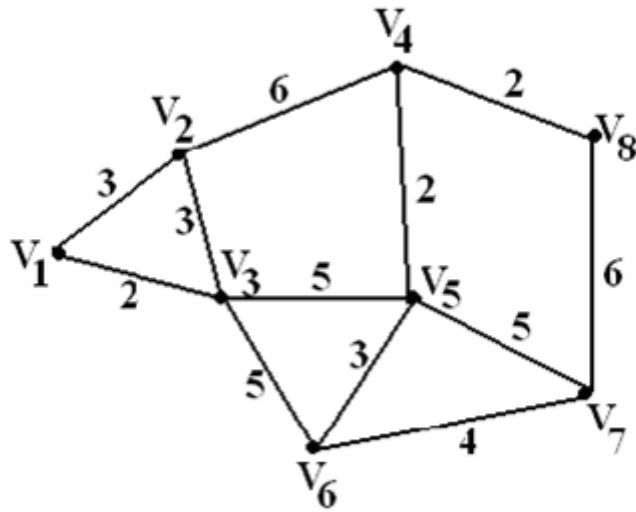
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Figure 6

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1. (a) Let A = {1,2,3}. Determine all partitions of A.

   (b) Let x = {1,2,3} anf f,g,h & s are the functions from X to X given by
   f = {< 1, 2 >, < 2, 3 >, < 3, 1 >}
   g = {< 1, 2 >, < 2, 1 >, < 3, 3 >}
   h = {< 1, 1 >, < 2, 2 >, < 3, 1 >}
   s = {< 1, 1 >, < 2, 2 >, < 3, 3 >}
   Find fog, gof, fohog, sog, gos, sos, fos, foh, fosoh, fof.   [15]

2. (a) Find how many solutions are there to the given equation that satisfies the given condition $x_1 + x_2 + x_3 = 20$, where each x_i is a positive integer.

   (b) How many ways can the letters of the word ALGORITHM be arranged in a row if A and L must remain together as a unit?   [15]

3. (a) Give the adjacency matrix of the digraph G = ({a,b,c,d}, R), where R = {(a,b),(b,c),(d,c),(d,a),}

   (b) Find the indegree and outdegree of each vertex of the following graph (figure 7).   [15]

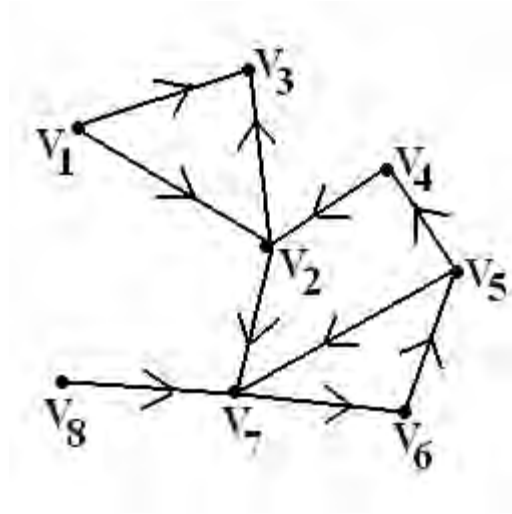4. (a) Construct the truth table for the following statement (PV¬Q) → (PΛQ).

Figure 7:

(b) Show that the following statements are logically equivalent without using truth table. $P(V(Q \Lambda R) \Leftrightarrow (PVQ) \Lambda (PVR)$ [15]

5. Find the general solution for the recurrence relation $a_n - a_{n-1} = 4(n + n^3)$, where n ≥ 1, and $a_0 = 5$ [15]

6. Determine each of the following complete bipartite graph $K_{m,n}$ is Hamiltonian or not. If a graph is not Hamiltonian, does it contain a Hamiltonian path?

   (a) $K_{2,3}$

   (b) $K_{3,3}$

   (c) $K_{2,4}$

   (d) $K_{3,4}$ [15]

7. (a) Is the following conclusion validly derivable from the premises given? Verify. If (x)(P(x) → Q(x)), (exists y)P(y) then (exists z)Q(z).

   (b) Prove that (x)(H(x) → A(x)) ⇒ (x)((∃ y)(H(y) Λ N(x, y)) → (∃ y)(A(y) Λ N(x, y)). [15]

8. (a) Let A = NxN and let * be a binary operation on A defined by (a,b) * (c,d) = (ad + bc, bd) for all (a,b), (c,d) ∈ NxN. Show that A has no identity element.

   (b) Show that the set of all roots of the equation $x^4 = 1$ forms a group under multiplication. [8+7]

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