Code No: 123AN **R15**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech II Year I Semester Examinations, March - 2017

PROBABILITY AND STATISTICS

(Common to ME, CSE, IT, MCT, AME, MIE, MSNT)

Time: 3 Hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit.
Each question carries 10 marks and may have a, b, c as sub questions.

PART- A

(25 Marks)

1.a) The distribution function of random variable X is given by

$$F(X) = \begin{cases} 0, & x < 2 \\ k(x-2), & 2 \le x \le 6 \\ 1, & x > 6 \end{cases}$$ find the value of k. [2]

b) Find moment generating function. Deduce moment generating function of Poisson distribution. [3]

c) The Probability density function of two-dimensional random variable is

$$f(x,y) = \begin{cases} \frac{8}{9}xy, & 1 < x < y < 2 \\ 0, & \text{Other wise} \end{cases}$$

Compute marginal density function of X. [2]

d) If $\sigma_x = \sigma_y = \sigma$ and angle between two regression lines is $\tan^{-1}\left(\frac{4}{3}\right)$, compute r. [3]

e) If we can assert with 99% confidence that the maximum error is 0.05 and P = 0.2. deduce the size of the sample. [2]

f) State the properties of F-distribution. [3]

g) Define transient and study states in queueing model. [2]

h) Explain Customer behaviour in the queue. [3]

i) Differentiate random variable and random process. [2]

j) Compose steady-state distribution of the Markov chain $\begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$. [3]

PART-B

(50 Marks)

2.a) A coin is tossed until a head appears. Expect the number of tosses required?

b) If the random variable X takes the values 1,2,3 and 4 such that
$2P(X=1) = 3P(X=2) = p(X=3) = 5P(X=4)$,
derive the probability distribution function and cumulative distribution function of X. [5+5]

OR

3.a) A machine manufacturing bolts is known to produce 5% defective. In a random sample of 10 bolts, compute the probability that there are (i) exactly 3 defective bolts (ii) not more than 3 defective bolts.

b) In Normal distribution, 7% of items under 35 and 89% under 63. Compute mean and variance of the distribution. [5+5]

4.a) The joint probability mass function of X and Y is given by

$$f(x,y) = \begin{cases} \frac{x+y}{21}, x = 1,2,3\,; y = 1,2 \\ 0, Other\ wise \end{cases}.$$

Compute covariance of (x, y).

b) Using the formula $r = \frac{\sigma_x^2+\sigma_y^2-\sigma_{x-y}^2}{2\sigma_x\sigma_y}$, compute r from the following data. [5+5]

X	92	89	87	86	83	77	71	63	53	50
Y	86	88	91	77	68	85	52	82	37	57

OR

5.a) In a partially destroyed laboratory record of an analysis of correlation data, the following results only are legible: variance of X is 9, regression equations are $8x - 10y + 66 = 0, 40x - 18y = 214$

Compute (i) the mean values of X and Y.

(ii) coefficient of correlation between X and Y.

(iii) the standard deviation of Y.

b) From the data relating to the yield of dry bark (X_1), height (X_2) and girth (X_3) for 18 cinchona plants, the following correlations were obtained:

$r_{12} = 0.77, r_{13} = 0.72$ and $r_{23} = 0.52$.

Compute (i) $r_{12.3}$ (ii) $R_{1.23}$. [5+5]

6.a) A coin was tossed 400 times and head turned up 216 times. Test the hypothesis that the coin is unbiased at 5% level of significance.

b) A sample of 100 electric bulbs produced by manufacturer A showed a mean life time of 1190 hours and a standard deviation of 90 hours. A sample of 75 bulbs produced by manufacturer by B showed a mean life time of 1230 hours, with a standard deviation of 120 hours. Is there a difference between the mean life time of two brands at a significance level of (i) 0.05 (ii) 0.01. [5+5]

OR

7.a) Eleven school boys were given a test in drawing. They were given a month's further tuition and a second test of equal difficulty was held at the end of it. Do the marks given evidence that the students have benefitted by extra coaching?

Boys	1	2	3	4	5	6	7	8	9	10	11
Marks I test	23	20	19	21	18	20	18	17	23	16	19
Marks II test	24	19	22	18	20	22	20	20	23	20	17

b) A set of 5 similar coins is tossed 320 times and the result is

No. of heads	0	1	2	3	4	5
Frequency	6	27	72	112	71	32

Test the hypothesis that the data follow a binomial distribution. [5+5]

8. Customers arrive at a one-man barber shop according to a Poisson process with mean inter-arrival time of 12 minutes. Customers spend an average of 10 minutes in barber chair.

a) compute the expected number of customers in the barber shop?

b) compute the percentage of time an arrival can walk straight into the barber's chair without having to wait.

c) compute the average time customers spend in the queue?

d) compute the probability that more than 3 customers in the system? [10]

OR

9. In a single-server queuing system with Poisson input and exponential service times, if the mean arrival rate is 3calling units per hour, the expected service time is 0.25 hour, and the maximum possible number of calling units in the system is 2, compute (a) $P_n (n \geq 0)$.

(b) the average number of calling units in the system.

(c) the average waiting in the queue. [10]

10.a) A stochastic(random) process is described by $X(t) = A \sin t + B\ cost$ where A and B are independent random variables with zero means and equal standard deviation. Show that the process is stationary of second order.

b) Three advocates A,B,C have 400,500 and 600 clients respectively at t=0. During one year though no new client has been added, migration from one to the other have taken places as given below:

From A 50 have gone to B and 25 to C

From B 50 have gone to A and 100 to C

From C 25 have gone to A

Prepare the transition probability matrix and estimate the number of clients associated with A, B, C after one year. [5+5]

OR

11. The transition probability matrix of a Markov chain $\{X_n\}$; n=1,2,3,.... having 3 states 1,2,and 3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is $P^{(0)} = (0.7, 0.2, 0.1)$

Compute a) $P(X_2 = 3)$

b) $P(X_3 = 2, X_1 = 3, X_0 = 2)$. [10]

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