

R15

Code No: 123AH

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech II Year I Semester Examinations, March - 2017

MATHEMATICS-III

(Common to EEE, ECE, EIE, ETM)

Time: 3 Hours

Max. Marks: 75

Note: This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit.

Each question carries 10 marks and may have a, b, c as sub questions.

PART- A**(25 Marks)**

- 1.a) Find the particular integral of $x^2 \frac{d^2 y}{dx^2} - 6x \frac{dy}{dx} + 10y = x^2$. [2]
- b) Find the indicial equation of $x^2 y'' - 2xy' - (x^2 - 2)y = 0$. [3]
- c) Prove that $\int_{-R}^R P_2^2(x) dx = \frac{2}{5}$. [2]
- d) Prove that $J_1(0) = 0$. [3]
- e) Find the value of 'a' if $\cos ax \sin hy$ is harmonic. [2]
- f) Find the analytic function whose real part is xy . [3]
- g) Find the residue of $\frac{2z+3}{z^2-z-2}$ at $z = -1$. [2]
- h) Expand $\frac{1}{3-z}$ when $|z| > 3$ as Laurent series. [3]
- i) Prove that $w = C + z$ where C is a complex constant is conformal at all points. [2]
- j) Find the fixed points of $\frac{z+i}{1+iz}$. [3]

PART-B**(50 Marks)**

2. Solve the differential equation $x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + 2y = x \log x$. [10]

OR

3. Solve the differential equation in series $(1-x^2) \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0$ around $x = 0$. [10]

- 4.a) Express $x^2 + x + 1$ in terms of Legendre Polynomials. [5]

- b) Prove that $\frac{d}{dx} (x^n J_n(x)) = x^n J_{n-1}(x)$. [5+5]

OR

- 5.a) Prove that $(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + (n)P_{n-1}(x)$. [5+5]

- b) Prove that $J_4(x) = \left(\frac{48}{x^3} - \frac{8}{x}\right) J_1(x) + \left(1 - \frac{24}{x^2}\right) J_0(x)$. [5+5]

6.a) Find the analytic function whose real part is $e^{-x}(x \sin y - y \cos y)$.

b) Evaluate $\int_C \frac{dz}{(z-2)(z-4)}$ where C is $|z-3|=1/2$. [5+5]

OR

7.a) If $f(z)$ is an analytic function then show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$.

b) Evaluate $\int_C \frac{dz}{(z^2-4)(z+1)}$ where C is $|z|=3$. [5+5]

8. Evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$ using residue theorem. [10]

OR

9. Evaluate using residue theorem $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$. [10]

10.a) Under the transformation $w = \frac{z-i}{1-iz}$ find the image of the circle $|z|=1$.

b) Find the image of $|z-3i|=3$ under the mapping $w = \frac{1}{z}$. [5+5]

OR

11.a) Find the image of the region bounded by the lines $x=1$, $y=1$, $x+y=1$ under the transformation $w=z^2$.

b) Find the bilinear mapping which maps the points $z = \infty, i, 0$ into $0, i, \infty$. [5+5]

---ooOoo---