Code No: 121AB **R15**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech I Year Examinations, August/September - 2017

MATHEMATICS-I

(Common to CE, EEE, ME, ECE, CSE, EIE, IT, MCT, MMT, AE, AME, MIE, PTE, CEE, MSNT)

Time: 3 hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART- A

(25 Marks)

1.a) Determine whether the vectors (1, 2, 3), (2, 3, 4), (3, 4, 5) are linearly dependent or not. [2]

b) Define Skew-Hermitian matrix and show that the matrix $A = \begin{pmatrix} 3i & 2+i \\ -2+i & -i \end{pmatrix}$ is Skew-Hermitian. [3]

c) If $u = 2xy,\ v = x^2 - y^2,\ x = r\cos\theta$ and $y = r\sin\theta$, find $\dfrac{\partial(u,v)}{\partial(r,\theta)}$. [2]

d) Using Lagrange's mean value theorem, prove that $|\sin b - \sin a| \le |b - a|$. [3]

e) Change the order of integration in the integral $\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$. [2]

f) Evaluate $\int_0^\infty e^{-x^2} dx$ using Gamma function. [3]

g) Find particular integral of $(D^2 - 2D + 1)y = \dfrac{e^x}{x}$. [2]

h) Find an integrating factor of $(x^2 + y^2)dx - 2xy\,dy = 0$. [3]

i) State first and second shifting theorems. [2]

j) Obtain the Laplace transform of $f(t) = \dfrac{\sin t}{t}$. [3]

PART-B

(50 Marks)

2.a) Reduce the matrix $A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 5 \\ 1 & 5 & 5 & 7 \\ 8 & 1 & 14 & 17 \end{pmatrix}$ to echelon form and hence find its rank.

b) Verify Cayley-Hamilton theorem for $A = \begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$ and hence find A^{-1}. [5+5]

OR

3. Reduce the quadratic form $Q = 2(xy + yz + zx)$ to canonical form and find its rank, nature, index and signature. [10]

4.a) Discuss the applicability of Rolle's theorem for the function $f(x) = \frac{\ln(x^2+6)}{5x}$ in $[2,3]$.

b) State Cauchy's mean value theorem and verify the same for the functions $f(x) = e^{-x}, g(x) = e^{x}$ in $[2,6]$. [5+5]

OR

5.a) Determine whether the functions $u = \frac{x^2 - y^2}{x^2 + y^2}$, $v = \frac{2xy}{x^2 + y^2}$ are functionally dependent. If so, find the relation between them.

b) Find the point on the paraboloid $z = x^2 + y^2$ which is closest to the point (3, - 6, 4). [5+5]

6.a) State and prove the relation between Beta and Gamma functions.

b) Prove that $\int_0^1 x^m (\ln x)^n \, dx = \frac{(-1)^n \, n!}{(m+1)^{n+1}}$, where n is a positive integer and $m > -1$. [5+5]

OR

7.a) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\, dy$ by changing to polar coordinates.

b) Find the volume of the solid in the first octant bounded by the paraboloid $z = 9 - x^2 - 4y^2$. [5+5]

8.a) Solve $y(8x - 9y)dx + 2x(x - 3y)dy = 0$.

b) If the air is maintained at 30^0 C and the temperature of the body cools from 80^0 C to 60^0 C in 12 minutes, find the temperature of the body after 24 minutes. [5+5]

OR

9.a) Solve $(D^2 - 4D + 4)y = 2x^2 + e^x + \cos(2x+3)$.

b) Find the general solution of the differential equation $y'' + y = x \sin x$ by the method of variation of parameters. [5+5]

10.a) Find the Laplace transform of $f(t) = t\, e^{-t} \sin 2t \cos 2t$.

b) Find $L^{-1}\left\{\ln\left(1 + \frac{1}{s^2}\right)\right\}$. [5+5]

OR

11.a) Find the Laplace transform of the periodic function

$$f(t) = \begin{cases} 1, & 0 < t < a \\ -1, & a \le t < 2a \end{cases}, \quad f(t+2a) = f(t).$$

b) Using Laplace transforms, solve $\frac{d^2x}{dt^2} + 9x = \cos 2t$, $x(0) = 1$, $x\left(\frac{\pi}{2}\right) = -1$. [5+5]

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