

R16

Code No: 131AB

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY, HYDERABAD

B.Tech I Year I Semester Examinations, December - 2016

MATHEMATICS-II

(Common to CE, ME, MCT, MMT, MIE, CEE, MSNT)

Time: 3 hours

Max. Marks: 75

Note: This question paper contains two parts A and B

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART-A**(25 Marks)**

- 1.a) Find the Laplace transform of the function $f(t) = \begin{cases} t & 0 < t < a \\ -t + 2a & a < t < 2a \end{cases}$ [2]
- b) Prove that $L^{-1}\{F(s)\} = f(t)$ and $f(0) = 0$ then $L^{-1}\{sF(s)\} = \frac{df}{dt}$. [3]
- c) Evaluate $\int_0^{\infty} a^{-bx^2} dx$. [2]
- d) Show that $\beta(p, q) = \beta(p+1, q) + \beta(p, q+1)$. [3]
- e) Find the area bounded by the curves $y = x, y = x^2$. [2]
- f) Evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2 + y^2) dy dx$ by changing into polar coordinates. [3]
- g) Find the directional derivative of $xyz^2 + xz$ at $(1,1,1)$ in a direction of the normal to the surface $3xy^2 + y = z$ at $(0,1,1)$. [2]
- h) Find a unit normal vector to the surface $x^2 + y^2 + 2z^2 = 26$ at the point $(2,2,3)$. [3]
- i) Find the work done by the force $\vec{F} = 3x^2i + (2xz - y)j + zk$ along the straight line joining the points $(0,0,1)$ and $(2,1,3)$. [2]
- j) Find the circulation of \vec{F} round the curve c where $\vec{F} = (e^x \sin y)i + (e^x \cos y)j$ and c is the rectangle whose vertices are $(0,0), (1,0), (1, \frac{\pi}{2}), (0, \frac{\pi}{2})$. [3]

PART-B**(50 Marks)**

2. Solve the differential equation $\frac{d^2x}{dt^2} - 4\frac{dx}{dt} - 12x = e^{3t}$ given that $x(0) = 1$ and $x'(0) = -2$ using Laplace transforms. [10]

OR

3. Use Laplace transforms, solve $y(t) = 1 - e^{-t} + \int_0^t y(t-u) \sin u du$. [10]

- 4.a) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

- b) Prove that $\Gamma(n)\Gamma(1-n) = \frac{\pi}{\sin n\pi}$. [5+5]

OR

5.a) Using Beta and Gamma functions, evaluate the integral $\int_0^1 (1-x^2)^n dx$ where n is a positive integer.

b) If m and n are positive integers then prove that $B(m, n) = \frac{(m-1)!(n-1)!}{(m+n-1)!}$. [5+5]

6. The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the axes in A, B and C . Find the volume of the tetrahedron $OABC$. Also find its mass if the density at any point is $kxyz$. [10]

OR

7.a) Change the order of integration and solve $\int_0^c \int_{x^2/a}^{2a-x} xy^2 dy dx$.

b) Evaluate $\iiint xyz dx dy dz$ over the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$. [5+5]

8.a) Find the directional derivative of $xyz^2 + xz$ at $(1,1,1)$ in a direction of the normal to the surface $3xy^2 + y = z$ at $(0,1,1)$.

b) Prove that $\text{curl}(\vec{a} \times \vec{b}) = \vec{a} \text{div} \vec{b} - \vec{b} \text{div} \vec{a} + (\vec{b} \cdot \nabla)\vec{a} - (\vec{a} \cdot \nabla)\vec{b}$ [5+5]

OR

9. Prove that if \vec{r} is the position vector of any point in space then $r^n \vec{r}$ is irrotational and is solenoidal if $n = 3$. [10]

10. Verify divergence theorem for $2x^2yi - y^2j + 4xz^2k$ taken over the region of first octant of the cylinder $y^2 + z^2 = 9$ and $x = 2$. [10]

OR

11. If $\vec{f} = 3x^2yz^2\vec{i} + x^2z^2\vec{j} + 2x^3yzk$. Show that $\int_C \vec{f} \cdot d\vec{r}$ is independent of the path of integration. Hence evaluate the integral when C is any path joining $(0, 0, 0)$ to $(1, 2, 3)$. [10]

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