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JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B.Tech II Year I Semester Examinations, March - 2017
MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE
(Common to CSE, IT)

Time: 3 Hours   Max. Marks: 75

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each
question carries 10 marks and may have a, b, c as sub questions.

PART- A
(25 Marks)

1.a) Write the converse and contrapositive of the statement: "If P is a square, then P is a rectangle". [2]

b) Rewrite the following statement informally, without quantifiers or variables.
$\forall x \in R$, if $x > 2$ then $x^2 > 4$ [3]

c) Let $I = \{0,1,2\}$ and define functions f and g from I to I as follows: For all x in I,
$$f(x) = (x^2 + x + 1) \bmod 3 \quad and \quad g(x) = (x+2)^2 \bmod 3$$
State whether $f=g$. [2]

d) Compute the transitive closure of the relation R={(1,1),(1,2),(1,3),(2,3),(3,1)} defined over a set S={1,2,3} [3]

e) State principle of inclusion. [2]

f) Give any three applications of Pigeonhole principle. [3]

g) What is a non-homogeneous recurrence relation? Give an example. [2]

h) Give any three applications of generating functions. [3]

i) What do you mean by isomorphism? Give examples of isomorphic graphs. [2]

j) What is a planar graph? Give examples of planar and non-planar graphs. [3]

PART-B
(50 Marks)

2.a) Assume x is a particular real number. Determine whether the following two statements are logically equivalent.
i) $x < 2$ or it is not the case that $1 < x < 3$
ii) $x \leq 1$ or either $x < 2$ or $x \geq 3$.

b) Translate the given statements into propositional logic using the propositions provided:
P: "The message is scanned for viruses"
Q: "The message was sent from an unknown system"
i) "The message is scanned for viruses whenever the message was sent from an unknown system."
ii) "It is necessary to scan the message for viruses whenever it was sent from an unknown system." [5+5]

OR

3.a) Using automatic theorem proving show that $(\neg Q \wedge (P \to Q)) \to \neg P$

b) Explain the following:
i) Normal Forms
ii) Free and bound variables
iii) Logical Equivalence
iv) Resolution. [5+5]

4.a) Determine whether the following functions are injective, surjective or bijective. Also describe the inverses of the bijective functions:

i) The function $f:N \to N$ with $f(a)=9$ and $f(n) = \begin{cases} 3f(n)+1 & \text{if } f(n) \text{ is odd} \\ \dfrac{1}{2}f(n) & \text{if } f(n) \text{ is even} \end{cases}$

ii) A function $f:A \to A$ satisfying $f(f(x))=f(x)$ for all $x \in A$

b) Find all homomorphisms.   [5+5]
i) $f : Z_2 \to Z_4$
ii) $f : Z_2 \to Z_5$

OR

5.a) Let A, B be finite sets with $|A| = m$ and $|B| = n$. Determine the numbers of:
i) Functions $A \to B$
ii) Injective functions $A \to B$ (provided that $m \le n$)
iii) Surjective functions $A \to B$ (provided that $m \ge n$)
iv) Bijective functions $A \to B$ (provided $m = n$)
v) Symmetric relations on A.

b) Determine all subgroups of each of the following groups: i) The additive group Z_3;
ii) The multiplicative group Z_6^*.   [5+5]

6.a) In a group of 30 people, at least how many are born in the same month?
b) In how many ways can six coupons for free lunches at different restaurants be distributed among 10 students:
i) If none is to receive more than one coupon?
ii) If there is no restriction on the number of coupons that each student can receive? [5+5]

OR

7.a) A student council consists of 15 students.
i) Suppose two members refuse to work together on projects. How many groups of seven can be chosen to work on a project?
ii) Suppose two team members insist on either working together or not at all on projects. How many groups of seven can be chosen to work on a project?

b) Count the bit-strings of length ten that:
i) Start with 01 and end with 10.
ii) Start with 01 and do not end with 10.
iii) Neither start with 01 nor end with 10.   [5+5]

8.a) Find the generating function for the following sequence:
$$\frac{1}{2}, \frac{1 \times 3}{2 \times 4}, \frac{1 \times 3 \times 5}{2 \times 4 \times 6}, \cdots \frac{1 \times 3 \times 5 \times \cdots \times (2n+1)}{2 \times 4 \times 6 \times \cdots \times (2n+2)}, \cdots$$

b) Use generating function to solve the following recurrence relation:
$a_0 = 2, a_1 = 3, a_n = 5a_{n-1} - 6a_{n-2} + 7^n$ for $n \ge 2$.   [5+5]

OR