

Time: 3 Hours

Max. M: 75

Note: This question paper contains two parts A and B.

Part A is compulsory which carries 25 Marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit.

Each question carries 10 Marks and may have a, b, c as sub questions.

PART - A**[25 Marks]**

- 1.a) Define a random variable. [2]
- b) State theorem of total probability. [3]
- c) Write the expression for pdf of a Poisson random variable. [2]
- d) Define Moment generating function and present generation of moments using it. [3]
- e) State central limit theorem for the case of unequal distributions. [2]
- f) Write short notes on jointly Gaussian random variables for 2 variable case. [3]
- g) What is an ergodic random process? [2]
- h) Write short notes on Poisson random process. [3]
- i) Define cross power spectral density. [2]
- j) Write the properties of power spectral density. [3]

PART - B**[50 Marks]**

- 2.a) Consider an experiment of drawing two cards at random from a bag containing four cards marked with the integers 1 through 4.
 - (i) Find the sample space S_1 of the experiment if the first card is replaced before the second is drawn.
 - (ii) Find the sample space S_2 of the experiment if the first card is not replaced.
 - b) Explain relative frequency approach of probability. [5+5]
- OR**
3. A Company producing electric relays has three manufacturing plants producing 50, 30, and 20 percent, respectively, of its product. Suppose that the probabilities that a relay manufactured by these plants is defective are 0.02, 0.06, and 0.015, respectively.
 - a) If a relay is selected at random from the output of the company, what is the probability that it is defective?
 - b) If a relay selected at random is found to be defective, what is the probability that it was manufactured by plant 3? [5+5]
- OR**
- 4.a) Obtain the characteristic function of a uniformly distributed random variable.
 - b) Obtain the variance of Poisson random variable. [5+5]
- OR**
- 5.a) A random variable X uniformly distributed in the interval $(0, \pi/2)$. Consider the transformation $Y = \cos X$, obtain the pdf of Y .
 - b) Obtain the characteristic function of Poisson random variable. [5+5]

- 6.a) Consider the linear transformations given by $Y_1 = X_1 - 3X_2$, $Y_2 = -2X_1 + 3X_2$ where X_1 and X_2 are independent random variables. Obtain the joint density of Y_1, Y_2 in terms of joint density of X_1, X_2 .
- b) Prove the joint characteristic function of two independent random variables is equal to the product of marginal characteristic functions. [5+5]

OR

- 7.a) Write the properties of joint density function.
- b) Define covariance of random variables X, Y and derive the relationship between covariance and correlation of X and Y . [5+5]

8. Define cross-correlation function of a random processes. Write properties processes and prove any three of them. [10]

OR

- 9.a) Explain the properties of covariance.
- b) Check whether the random process $X(t) = A \cos(\omega_0 t + \theta)$ is WSS or not? Given θ is uniformly distributed in the interval $(0, 3\pi/2)$. [5+5]

10. State and prove Wiener- Khinchine relations. [10]

OR

- 11.a) The power spectral density of a WSS random process is $\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$. Find its autocorrelation function.
- b) Obtain the average power in the random process $X(t) = A \cos(\omega_0 t + \theta)$ where A, ω_0 are real constants and θ is a random variable uniformly distributed in the range $(0, \pi/2)$. [5+5]

