Code No: 111AB

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JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech I Year Examinations, June - 2015

MATHEMATICS-I

(Common to all Branches)

Time: 3hours **Max.Marks:75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART-A (25 Marks)

1.a) Define skew-symmetric matrix and give a suitable example. [2M]

b) If λ is an Eigen value of an orthogonal matrix, then prove that $\frac{1}{\lambda}$ is also its Eigen value. [3M]

c) State Lagrange's mean value theorem. [2M]

d) If x = uv, y= u/v, then find $\frac{\partial(x, y)}{\partial(u, v)}$. [3M]

e) Find the value of $\Gamma\left(\frac{1}{2}\right)$. [2M]

f) Sketch the region of integration of the integral $\int_{0}^{2a}\int_{\sqrt{(2ax-x^2)}}^{\sqrt{(2ax)}} f(x, y)\,dxdy$. [3M]

g) Write the working procedure to find out the orthogonal trajectories of the family of curves f(x, y, c) = 0. [2M]

h) Solve $\frac{d^3x}{dt^3} - x = 0$. [3M]

i) Find the inverse Laplace transform of $\log\left(\frac{s+1}{s-1}\right)$ [2M]

j) Find $L\left\{e^{t-3}u(t-3)\right\}$. [3M]

PART-B (50 Marks)

2.a) Reduce the following matrix A to normal form and hence find its rank

$$A=\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}.$$

b) Diagonalise the matrix $A=\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. [5+5]

OR

3.a) Using Cayley-Hamilton theorem, find the inverse of $A=\begin{bmatrix} 7 & -1 & 3 \\ 6 & 1 & 4 \\ 2 & 4 & 8 \end{bmatrix}$.

b) Show that the equations $3x + 4y + 5z = a$, $4x + 5y + 6z = b$ and $5x + 6y + 7z = c$ do not have a solution unless $a + c = 2b$. [5+5]

4.a) If x is positive, show that $x > \log(1+x) > x - \frac{1}{2}x^2$.

b) Given $u + v + w = a$, find the maximum value of $u^m v^n w^p$. [5+5]

OR

5.a) Verify Rolle's theorem for $f(x) = \log\left\{\frac{x^2+ab}{x(a+b)}\right\}$ in the interval (a, b).

b) If $x = u\sqrt{(1-v^2)} + v\sqrt{(1-u^2)}$ and $y = \sin^{-1}u + \sin^{-1}v$, then show that x and y are functionally related. Also, find the relationship. [5+5]

6.a) Express $\int_0^1 x^m(1-x^n)^p\,dx$ in terms of Gamma function and evaluate $\int_0^1 x^7(1-x^5)^{14}\,dx$.

b) Find the volume enclosed by the cylinders: $x^2 + y^2 = 2ax$ and $z^2 = 2ax$. [5+5]

OR

7.a) Evaluate $\int_0^{4a}\int_{y^2/4a}^{y} \frac{(x^2-y^2)}{(x^2+y^2)}\,dx\,dy$ by changing to polar coordinates.

b) Find by double integration, the area lying between the parabola $y = 4x - x^2$ and the line $y = x$. [5+5]

8.a) Solve $\frac{d^2y}{dx^2} + a^2y = \tan ax$.

b) If the air is maintained at 30^0C and the temperature of the body cools from 80^0C to 60^0C in 12 minutes, find the temperature of the body after 36 minutes. [5+5]

OR

9.a) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = e^x \tan x$ by method of variation of parameters.

b) An electrical circuit consists of an inductance of 0.1 henry, a resistance of 20 ohms and a condenser of capacitance 25 micro-farads. Find the charge q and the current i at any time t, given that at t = 0, q = 0.05 coulomb, $i = \frac{dq}{dt} = 0$, when t=0. [5+5]

10.a) Find the Laplace transform of square wave function (with period T = a) given by

$$f(t) = \begin{cases} 1, & \text{if } 0 < t < \frac{a}{2} \\ -1, & \text{if } \frac{a}{2} < t < a \end{cases}$$

b) Solve the integral equation $y(t) = a\sin t - 2\int_0^t y(u)\cos(t-u)du$ by Laplace Transform method. [5+5]

OR

11.a) State and prove the Convolution theorem for Laplace transform.

b) Solve $\left(D^2+9\right)y = \cos 2t,\ y(0)=1,\ y(\frac{\pi}{2}) = -1$ by using transform method.

[5+5]

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