

Code No: 131AB

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech I Year I Semester Examinations, May - 2018

MATHEMATICS-II

(Common to CE, ME, MCT, MMT, AE, MIE, PTM, CEE, MSNT)

Time: 3 hours

Max. Marks: 75

**Note:** This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

**PART - A****(25 Marks)**

1.a) Find  $L\{\cos^3 2t\}$ . [2]

b) Find  $L^{-1}\left\{\frac{4}{(s+1)(s+2)}\right\}$ . [3]

c) Evaluate  $\int_0^1 x^7 (1-x)^5 dx$ . [2]

d) Evaluate  $\int_0^{\infty} x^4 e^{-x^2} dx$ . [3]

e) Evaluate  $\int_0^1 \int_0^{\sqrt{x}} xy dy dx$ . [2]

f) Evaluate  $\int_{-1}^1 \int_{-2}^2 \int_{-3}^3 dx dy dz$ . [3]

g) If  $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$  then find  $\text{div } \vec{r}$ . [2]

h) State Green's theorem on a plane. [3]

i) Evaluate  $\nabla(x^2 - yz + z^2)$ . [2]

j) If  $\vec{a}$  is a constant vector then find  $\text{curl}(\vec{r} \times \vec{a})$ . [3]

**PART - B****(50 Marks)**

2.a) Find  $L\{te^{2t} \sin 3t\}$ .

b) Find  $L^{-1}\left\{\frac{s^2}{(s^2+4)(s^2+25)}\right\}$ . [5+5]

**OR**

3. Solve the differential equation  $\frac{d^2x}{dt^2} + 9x = \sin t$  using Laplace transform, given that

$x(0) = 1, x\left(\frac{\pi}{2}\right) = 1.$

[10]

4. Prove that  $\beta(m, n) = \frac{\Gamma(m) \cdot \Gamma(n)}{\Gamma(m+n)}$ . [10]

OR

5. Show that  $\beta\left(m, \frac{1}{2}\right) = 2^{2m-1} \beta(m, m)$ . [10]

6. Change the order of integration and solve  $\int_0^a \int_{x^2/a}^{2a-x} xy^2 dy dx$ . [10]

OR

7. Find the area of the loop of the curve  $r = a(1 + \cos \theta)$ . [10]

8.a) Prove that  $\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B})$ .

b) Find the directional derivative of  $2x^2 + z^2$  at  $(1, -1, 3)$  in the directional of  $\vec{i} + 2\vec{j} + 3\vec{k}$ . [5+5]

OR

9. Show that  $\nabla^2 [f(r)] = f''(r) + \frac{2}{r} f'(r)$  where  $r = |\vec{r}|$ . [10]

10. Verify Green's theorem for  $\int_C (xy + y^2) dx + x^2 dy$  where 'C' is bounded by  $y = x$  and  $y = x^2$ . [10]

OR

11. Verify the Stoke's theorem for  $\vec{F} = y\vec{i} + z\vec{j} + x\vec{k}$  and surface is the part of the plane  $x^2 + y^2 + z^2 = 1$  above the  $xy$  - plane. [10]