CHAPTER – 25

SPUR – GEAR DRIVES

25.1. GEAR DRIVES - AN INTRODUCTION :

A gear drive is a mechanical drive which transmits power through toothed wheels called as gears. In this drive, the driving wheel is in direct contact with driven wheel in contrast with other mechanical drives such as belt drives and chain drives where an intermediate link, like belt or chain, is needed to connect the driven wheel with the driving wheel. Gears are transmitting power from one shaft to another by means of the positive contact of successively engaging teeth. When compared to belt and chain drives, gear drives are more compact, can operate at higher speeds and can be used where precise timing is desired. In the gear drive arrangement, the smaller wheel is named as ***"pinion"*** and the bigger wheel is known as ***"wheel or gear"*** regardless of the nature of working such as the smaller wheel may be operated as driving member or driven member and so on. Usually gears require more attention to lubrication, cleanliness, shaft alignment etc. and they may be operated in a closed case with provision for proper lubrication.

25.2. APPLICATIONS OF GEAR DRIVES :

The gear transmission system is most widely used because of its high load carrying capacity, high efficiency and compact layout. Gears are used in many fields and under a wide range of conditions such as from smaller watches and instruments to the heaviest and most powerful machineries like lifting cranes. They can be operated to transmit the power from negligibly small values to thousands of kilowatts, using gears of diameter from a few millimeters to many metres. Some of the common applications of gears are in all *automobiles, hoisting machineries, rolling mill, machine tools such as lathes, milling machine, shaping machine etc. and even in toys and so on*. Shortly saying that the use of gears is enormous compared to other power transmitting drives.

25.3. ADVANTAGES AND DISADVANTAGES OF GEAR DRIVES :

(a) Advantages :

1. Gear drives are more compact than belt drives and chain drives and hence less space is sufficient for installation.
2. Gear drives are having high efficiency.
3. They have long service life and high reliability.
4. Gear drive can transmit more power than other drives like belt or chain drives.
5. They have a greater range of speed ratios and power than other drives.
6. They have constant speed ratio owing to the absence of slipping which may be happened in belt drives.
7. Metal gears do not deteriorate with age, heat, oil and grease.

(b) Disadvantages :

1. The design and manufacturing of gears are more complicated compared to other drives.
2. It produces noise at high velocities due to inaccurate manufacturing or because of excessive wear.
3. For "long centre distance - power transmission" such as in rice mill, flour mill etc. gears can not be employed.
4. They require careful maintenance and proper lubrication than belt or chain drives.

25.4. CLASSIFICATION OF GEARS :

One factor influencing the choice of the right gear is the geometric arrangements of the apparatus which needs a gear drive. Based on various operating conditions, gears can be classified in the following ways.

1. ***Based on the position of the axes of gear shafts such as***
   (a) Parallel - Example : Spur, helical, herring-bone gears.
   (b) Intersecting - Example : Bevel gears - (straight and spiral)
   (c) Non intersecting and non-parallel - Example :- Crossed helical, worm gear, hypoid, spiroid, planoid, beveloid, helicon, face gear etc.

2. ***Based on the type of engagement :***
   a) External gearing.
   b) Internal gearing.
   c) Rack and pinion type
   d) Sector gearing.

3. ***Based on the position of teeth on the wheel rim :***
   a) Teeth parallel to axis of gear-Eg. spur gears
   b) Teeth inclined to axis of gears-Eg. Helical, Herringbone gears.
   c) Curved teeth on wheel rim - Eg. Spiral gears.

4. ***Based on the pheriperal speed of gears :***
   a) Low speed gears : $v < 3$ m/s
   b) Medium speed gears : $3 < v < 15$ m/s
   c) High speed gears : $v > 15$ m/s.

5. ***Based on profile :***
   a) Involute profile gears.
   b) Cycloidal profile gears

6. ***Based on pressure angle :***
   a) Gears with 20° pressure angle
   b) Gears with 14 ½° pressure angle.

7. ***Based on tooth height (or) working depth :***
   a) Full depth gears or standard gears.
   b) Stub gears.

The common types of gears are

1. Spur gears
2. Helical gears
3. Bevel gears
4. Worm gears.

The schematic diagrams of various gears are shown in figure 25.1.

25.5. OUTLINE OF VARIOUS TYPES OF GEARS :

(a) General types :

1. Spur gear :

The most prevalent and best-known type of gear is the spur gear. It is used to provide a constant speed and torque relation between parallel shafts. It is simple in design, easy in manufacturing and maintenance. They can be operated as external or internal gears. Usually they are slow speed gears and produce noise at high speeds. It is having teeth on the wheel rim which are parallel to the axis of gear.

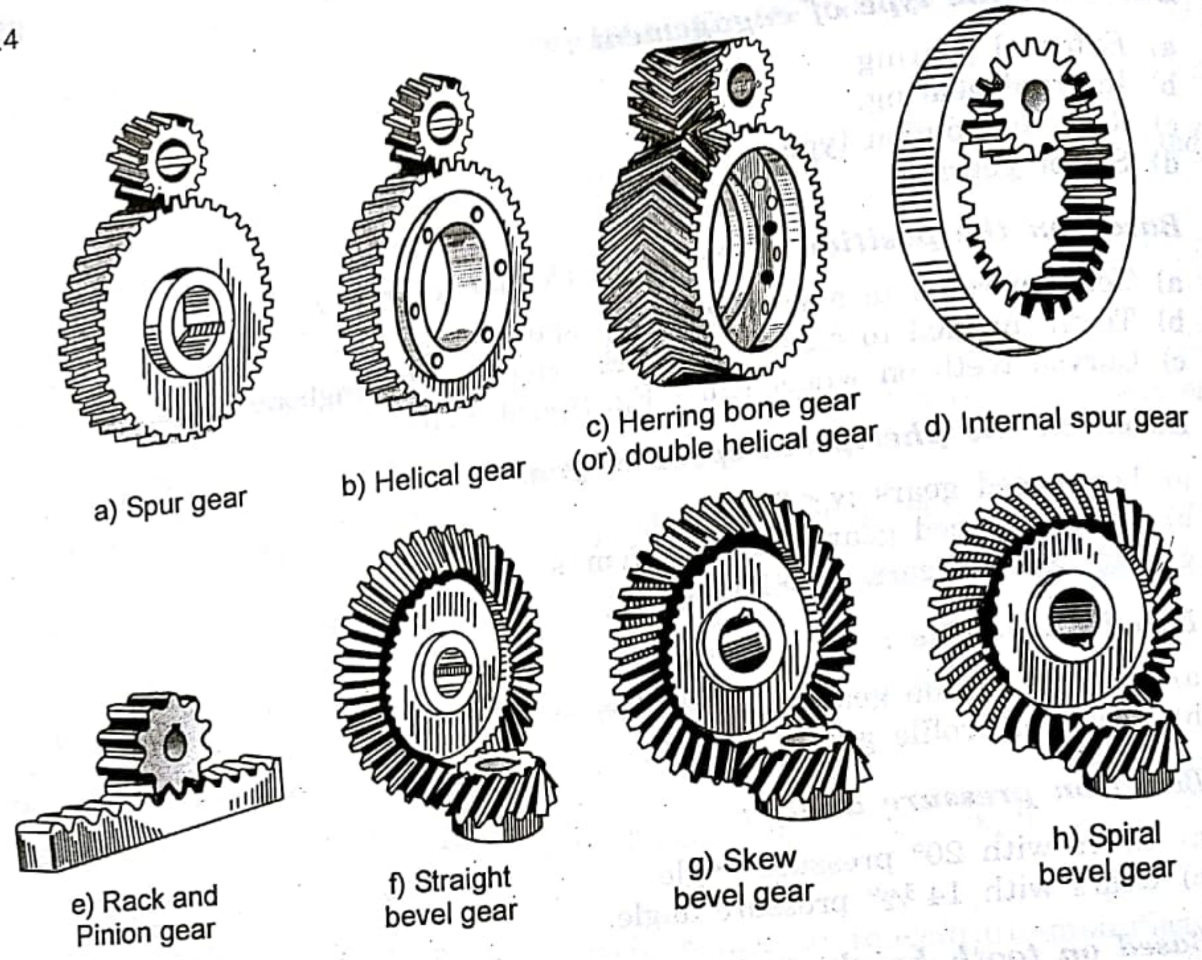


Fig. 25.1: Principal types of toothed gears

2. Helical gear :

The structure of this gear is similar to spur gear except the arrangement of teeth in the wheel rim.That is, the teeth are cut at an angle, known as ***helix angle***, to the axis of the shaft. Since they are having inclined teeth, engagement will be gradual in contrast with spur gear in which the entire width of tooth of one gear will engage with the tooth of mating gear at a time. Also because of gradual engagement, the helical gears may operate without producing much noise like spur gears. Helical gears can be operated for higher speed and higher power transmission systems compared to spur gears. The main types of helical gears are *(i) Simple helical (i.e., single helical) (ii) Double helical (or) Herringbone and (iii) Crossed helical gears etc.*

3. Bevel gears :

Bevel gears are operated for transmitting power at a constant velocity ratio between two shafts whose axes intersect at a certain angle. The blanks required for making bevel gears are similar to frustum of cones. That is, bevel gears are cut on conical pitch surface in contrast with spur and helical gears for which the teeth are cut on the cylindrical pitch surfaces. There are two main types of bevel gears namely ***straight bevel gears*** in which the teeth are straight and ***spiral bevel gears*** which are having curved teeth.

3. Worm gears :

Worm gears are commonly employed for transmitting power at high velocity ratio between non-intersecting shafts. It can give velocity ratio of about 300 : 1 or more in a single step in a minimum space but it has lower efficiency due to sliding contact. In worm gear drive, the driving shaft is known as ***worm*** which is similar to a "Lead screw" used in a lathe and the driven gear is similar to a "Curved teeth helical gear having a small concavity at the pitch surface". This is the available maximum speed reducer in a single step which may sometimes replace the multi-stage spur gear or helical gear speed reducer if necessary.

(b) Special Types :

1. Zerol Bevel gears :

Zerol bevel gears are a special form of spiral bevel gears with curved teeth and having a ***zero degree mean spiral angle***. They can be used in the same type of drives as straight bevel gears.

2. Face gears :

Face gears, as the name implies, have teeth cut on the end face of gear. The mating pinion is either a spur or a helical gear. The pinion and face gear are usually mounted with a 90° shaft angle. Functionally, this type of gear is akin to bevel gears.

3. Beveloid gears :

The beveloid gear is a completely generalized form of involute gear. It is an involute gear with tapered tooth thickness, root, and outside diameter. Beveloid gears are useful primarily for precision-instrument drives, where the combination of high precision and limited load-carrying ability fits the application.

4. Crossed-axis Helical gears :

Crossed-axis helical gears are the simplest form of gearing for non-intersecting shafts with conjugate action. They can be thought of as non-enveloping work gears. The action of crossed-helical gears consists primarily of screwing or wedging with a resultant high degree of sliding on the tooth flanks.

5. Hypoid gears :

They are similar to spiral bevel gears except that the pinion axis is offset above or below the gear axis. The pitch surfaces of hypoid gears are hyperboloids of revolution. In operation, hypoid gears smoother and quiter than spiral bevel gears. However, as in all cases of non-intersecting gear sets, sliding takes place across the face of the teeth, with a resultant loss of efficiency.

25.17. DESIGN PROCEDURE :

For the design of spur gear drive, the following steps may be observed.

1. From the statement of problem, note down the power to be transmitted, pinion speed, gear ratio, life of gear drive and other working conditions.

2. Based on the transmitting power and gear ratio, select a suitable material. Usually the pinion is subjected to more loading cycles than gear and hence the material selected for pinion should be stronger than gear material (From table 25.6 or 25.7) (PSG 8.4 or 8.5) (JDB 25.9)

3. Note the design surface compressive stress and bending stress for the selected material from the table 25.7 (or) find them by using the formula.

$$[\sigma_c] = C_B \cdot HB\, k_{cl} \text{ (or) } [\sigma_c] = C_R \cdot HRC \cdot k_{cl}$$

and $$[\sigma_b] = \frac{1.4\, k_{bl}\, \sigma_e}{n\, k_\sigma} \text{ for rotation in one direction only}$$

$$= \frac{k_{bl}\, \sigma_e}{n\, k_\sigma} \text{ for rotations in both directions}$$

The values of C_B, C_R, HB, HRC, k_{cl}, k_{bl}, σ_e, n, k_σ are noted from the tales 25.20 to 25.28 (PSG 8.16 to 8.20) (JDB 25.17 to 25.20)

4. Based on surface compressive stress, determine the minimum centre distance required for the gear drive as

$$a \geq (i \pm 1) \sqrt[3]{\left\{\frac{0.74}{[\sigma_c]}\right\}^2 \frac{E\,[M_t]}{i\,\psi}} \text{ for } 20° \text{ pressure angle.}$$

$$\text{(or) } a \geq (i \pm 1) \sqrt[3]{\left\{\frac{0.85}{[\sigma_c]}\right\}^2 \frac{E\,[M_t]}{i\,\psi}} \text{ for } 14\frac{1}{2}° \text{ pressure angle.}$$

In the above expressions.

a = Centre distance

i = Gear ratio $= \frac{Z_2}{Z_1}$

(i + 1) for external gearing and (i – 1) for internal gearing.

$[M_t]$ = Design torque

= $M_t \cdot k \cdot k_d$ where M_t is the nominal twisting moment (torque) and is obtained from the power as,

$$P = \frac{2\pi n_1 M_t}{60} \text{ in which,}$$

P = Power in watts.

n_1 = Speed in rpm of pinion

M_t = Nominal twisting moment in N – m

Initially $k \cdot k_d$ may be assumed as 1.3,

and $\psi = \frac{b}{a} = 0.3$ (Table 25.12 & 25.13) (PSG8.14) (JDB 25.3)

E = Equivalent young's modules

$= \frac{2 E_1 E_2}{E_1 + E_2}$ (Table 25.11) (PSG 8.14) (JDB 25.12)

The design stress $[\sigma_c]$, to be substituted in the above expression should be the minimum value, and usually formula based on 20° pressure angle is preferred.

5. Based on beam strength or bending stress, determine the minimum module as

$$m \geq 1.26 \sqrt[3]{\frac{[M_t]}{y\,[\sigma_b]\,\psi_m\,Z_1}}$$

where $[\sigma_b]$ = Design bending stress which should be the minimum value.

ψ_m = $\frac{b}{m} = 10$ (Initially assumed)

Z_1 = Number of teeth on pinion

(Usually selected from 14 to 20 initially)

y = Form factor corresponding to Z_1 (Table 25.15) (PSG 8.18)

6. After calculating the minimum module, select the next standard module from table 25.16. (PSG 8.2)

7. Then correct the number of teeth on pinion using the standard module and minimum centre distance as $Z_1 = \frac{2a}{m(i+1)}$

8. Similarly finalise the centre distance using standard module and corrected number of pinion teeth as $a = \frac{m Z_1 (i+1)}{2}$

9. Find out the pitch circle diameters for pinion and gear as $d_1 = m Z_1$ and $d_2 = m Z_2$ where $Z_2 = i Z_1$. Also the centre distance "a" is equal to $\frac{d_1 + d_2}{2}$

10. Find the face width "b" as $b = \psi a$ (or) $b = \psi_m \cdot m$ and adopt the higher value.

11. Calculate the pitch line velocity (i.e., peripheral velocity) using $v = \frac{\pi d_1 n_1}{60 \times 1000}$ m/s and also note the values of load concentration factor (k) and the dynamic load factor (k_d) from table 25.18, 25.19 based on (b/d_1) ratio and pitch line velocity and evaluate the actual transmitted torque as $[M_t] = M_t \cdot k \cdot k_d$

12. Then determine the induced surface compressive stress and bending stress as

$$\sigma_c = 0.74 \left(\frac{i \pm 1}{a}\right) \sqrt{\frac{(i \pm 1)}{i\,b} E\,[M_t]} \text{ for } = 20° \text{ pressure angle}$$

and $\sigma_b = \frac{(i \pm 1)\,[M_t]}{a\,m\,b\,y}$ (Here y = Form factor for corrected Z_1)

And check these induced σ_c and σ_b with minimum design stress $[\sigma_c]$ and $[\sigma_b]$. For optimum design, $\sigma_c \le [\sigma_c]$ and $\sigma_b \le [\sigma_b]$. If σ_c and σ_b are greater than $[\sigma_c]$ and $[\sigma_b]$, then the face width can be increased.

13. Then check the strength of teeth for plastic deformations using the formula given in the table 25.29 (PSG 8.21).

14. Evaluate the other parameters of gear such as addendum, dedendum, tip circle diameter, root circle diameter, circular pitch etc. as follows.

Addendum = $f_0 \cdot m$

Dedundum = $(f_o + c)\,m$ where f_0 = Height factor

= 1 for full depth teeth

= 0.8 for stub teeth

c = Clearance factor

= 0.25 for full depth

= 0.3 for stub teeth.

Tip circle diameter	=	Pitch circle diameter + (2 × addendum)
Root circle diameter	=	Pitch circle diameter − (2 × Dedendum)
Tooth height	=	Addendum + Dedendum
Working depth	=	2 × Addendum
Clearance	=	Dedendum − Addendum
Circular pitch	=	$\frac{\pi d}{Z}$

15. Draw a neat sketch of spur gear drive.

Table 25.2: Peripheral Speed of Gears

IS Quality		Preferred Quality	Speed of gears in metres per second		
			Cylindrical Gears	Straight Bevel	Spiral Bevel
High Precision	3 & 4	4	Above 15	upto 9	upto 18
Precision	5 & 6	6	Above 8 & upto 15	upto 6	upto 12
Medium	7, 8 & 9	8	Above 1 & upto 8	upto 3	upto 7
Coarse	10 & 12	10, 12	upto 1	upto 2	upto 4

Table 25.3: Backlash for Gears, mm

Upto 8 m/s pitch line velocity			Above 8 m/s pitch line velocity	
Module	Backlash Min.	Backlash Max.	Module	Backlash
20	0.75	1.25	8	0.40
16	0.50	0.85	7	0.38
12	0.35	0.60	6	0.36
10	0.30	0.51	5	0.28
8	0.22	0.40	4	0.23
6	0.20	0.33	3.5	0.22
5	0.15	0.25	3	0.21
4	0.13	0.20	2.75	0.20
3	0.10	0.15	2.5	0.19
2.5	0.08	0.13	2	0.18
2	0.08	0.13		
1.5 & Finer	0.00	0.10		

Specifications :

For Spur gear drive

Sl.No.	Description	Pinion	Gear
1.	Material	15 Ni2 Cr1 Mo 15	C 45
2.	No.of teeth	27	135
3.	Module	5 mm	5 mm
4.	Face width	125 mm	125 mm
5.	P.C.D.	135 mm	675 mm
6.	T.C.D.	145 mm	685 mm
7.	R.C.D.	122.5 mm	662.5 mm
8.	Centre distance	= 405 mm between two axes	

Problem 25.3 :

Design a spur gear drive to transmit 22 kW at 900 rpm, speed reduction is 2.5. Materials for pinion and wheel are C15 steel and Cast Iron grade 30 respectively. Take pressure angle of 20° and working life of the gears as 10,000 hours.

Solution :

Power to transmitted = 22 kW

Speed = 900 rpm

Speed ration (i) = 2.5

Pinion material = C15 steel

Wheel material = Cast Iron grade 30

$\alpha = 20°$

Life = 10,000 hours

Minimum centre distance based on surface compressive stress is given by

$$a \geq (i+1) \times \sqrt[3]{\left(\frac{0.74}{[\sigma_c]}\right)^2_{min} \frac{E\,[M_t]}{i\,\psi}} \quad \text{(PSG 8.13) (JDB 25.3)}$$

(Table 25.17) (PSG 8.15)

$[M_t] = M_t \cdot k \cdot k_d$

$= 97420 \times \frac{kW}{n} \times k.\ k_d$

$= 97420 \times \frac{22}{900} \times 1.3$ (Assuming $k.\ k_d = 1.3$ initially)

$= 2387 \times 1.3 = 3104$ kgf – cm

$\psi = \frac{b}{a} = 0.3$ (initially)

E = Equivalent young's modulus $= \frac{2\,E_1\,E_2}{E_1 + E_2}$

E_1 for C15 steel $= 2.08 \times 10^6$ kgf/cm^2 (From table 25.10) (PSG 1.1)

E_2 for Cast Iron grade 30 $= 1.00 \times 10^6$ kgf/cm^2

$\therefore E_{eq} = \frac{2 \times 2.08 \times 10^6 \times 1.0 \times 10^6}{(2.08 \times 10^6) + (1.0 \times 10^6)}$

$= \frac{4.16 \times 10^6 \times 10^6}{3.08 \times 10^6} = 1.35 \times 10^6$ kgf/cm^2

Design compressive stress $[\sigma_c] = C_B\, H_B\, k_{cl}$ (or) $= C_R$ HRC k_{cl}

For pinion C15 steel; $[\sigma_c] = C_R$ HRC k_{cl}

Now $C_R = 220$; (Table 25.20) (PSG 8.16)

HRC = 60

No. of cycles on pinion teeth for 10,000 hours $= 60\, n_1\, T$.

$= 60 \times 900 \times 10{,}000 = 54 \times 10^7$ cycles

$\therefore k_{cl} = 0.585$ (for steel $N > 25 \times 10^7$ cycles) (Table 25.22) (PSG 8.17)

$\therefore [\sigma_c]$ for pinion $= 220 \times 60 \times 0.585 = 7722$ kgf/cm^2

Similarly for wheel $[\sigma_c] = C_B$ HB k_{cl}

$C_B = 23$ and HB = 250 (Table 25.20)

$N = 60\, n.T. = 60 \times \frac{900}{2.5} \times 10000 = 21.6 \times 10^7$

$$\therefore k_{cl} = \sqrt[6]{\frac{10^7}{N}} = \sqrt[6]{\frac{10^7}{21.6 \times 10^7}} = 0.599 = 0.6 \quad \text{(Table 25.22)}$$

$\therefore [\sigma_c]$ for wheel $= 23 \times 250 \times 0.6 = 3500 \text{ kgf/cm}^2$

$\therefore$ Minimum centre distance required is

$$a \geq (2.5 + 1) \left\{ \left(\frac{0.74}{3500} \right)^2 \times \frac{1.35 \times 10^6 \times 3104}{2.5 \times 0.3} \right\}^{1/3} \geq 22.04 \text{ cm}$$

Minimum module based on beam strength is given by

$$m \geq 1.26 \left\{ \frac{[M_t]}{y\,[\sigma_b]\,\psi_m\,Z_1} \right\}^{1/3} \text{cm.}$$

where $\quad [\sigma_b] = \dfrac{1.4\, k_{bl}}{n\, k_\sigma}\, \sigma_e$

For pinion :

$\sigma_e = 0.25\,(\sigma_u + \sigma_y) + 500$ (Table 25.24 & 25.8)

$= 0.25\,(4000 + 2400) + 500$ (PSG 8.19 & 1.9) (JDB 25.19)

$= 2100 \text{ kgf/cm}^2$

$k_{bl} = 0.7$ (Table 25.27)

$k_\sigma = 1.2$ (Table 25.26)

$n = 2.0$ (Table 25.25)

$$\therefore [\sigma_b] \text{ for pinion} = \frac{1.4 \times 0.7}{2 \times 1.2} \times 2100 = 857.5 \text{ kgf/cm}^2$$

Similarly for wheel :

$\sigma_e = 0.45\, \sigma_u$ (Cast Iron)

$= 0.45 \times 3000 = 1350 \text{ kgf/cm}^2$ (Table 25.9) (PSG 1.5)

$n = 2\,;\ k_\sigma = 1.2$

$$k_{bl} = \sqrt[9]{\frac{10^7}{N}} = \sqrt[9]{\frac{10^7}{21.6 \times 10^7}} = 0.71$$

$$\therefore [\sigma_b] = \frac{1.4 \times 0.71}{2 \times 1.2} \times 1350 = 558 \text{ kgf/cm}^2$$

Take $\psi_m = \frac{b}{m} = 10$ (initially)

$Z_1 = 20$

$y = 0.389$ (for $Z_1 = 20$)

Minimum module required

$$m \geq 1.26 \left\{ \frac{3104}{0.389 \times 558 \times 10 \times 20} \right\}^{1/3} \geq 0.5 \text{ cm}$$

Take the nearest module $m = 5$ mm

No. of pinion teeth is corrected using "m" and "a"

i.e., $Z_1 = \frac{2\,a}{m\,(i+1)} = \frac{2 \times 22}{0.5 \times 3.5} = 25.14$

Take $Z_1 = 28$ (nearest to 25.14 from R20 series)

$\therefore Z_2 = i \,.\, Z_1 = 2.5 \times 28 = 70$

Pitch circle diameter of pinion $d = m\,Z_1 = 0.5 \times 28 = 14$ cm

Pitch circle dia of wheel $D = m\,Z_2 = 0.5 \times 70 = 35$ cm

Corrected centre distance

$$a = \left\{ \frac{d + D}{2} \right\} = \frac{14 + 35}{2} = 24.5 \text{ cm}$$

Since $a > a_{min}$, design is safe.

Face width $b = \psi_m \cdot m = 10 \times 0.5 = 5$ cm

(or) $b = \psi \cdot a = 0.3 \times 24.5 = 7.35$ cm

Take $b = 7.5$ cm

Load correction factor

k for, $\psi_p = \frac{b}{d} = \frac{7.5}{14} = 0.53$, is given by

$k = 1.0$ and $k_d = 1.2$

$\therefore [M_t] = M_t \cdot k \cdot k_d = 2387 \times 1.2 = 2865$ kgf – cm

Check for Compressive and Bending stress :

Surface compressive stress

$$\sigma_c = 0.74 \left(\frac{i+1}{a}\right)\left\{\frac{(i+1)}{i\,b} \times E\,[M_t]\right\}^{1/2}$$

$$= 0.74 \left(\frac{2.5+1}{24.5}\right)\left\{\frac{(2.5+1)}{2.5 \times 7.5} \times 1.35 \times 10^6 \times 2865\right\}^{1/2}$$

$$= 2860 \text{ kgf/cm}^2 < [\sigma_c] = 3500 \text{ kgf/cm}^2$$

Our design is safe.

Bending stress

$$\sigma_b = \frac{(i+1)}{a.m.b.y.}[M_t] = \frac{3.5 \times 2865}{24.5 \times 0.5 \times 7.5 \times 0.434} \quad (y \text{ for } Z_1 = 28 \text{ is } 0.434)$$

$$= 255 \text{ kgf/cm}^2 < [\sigma_b]_{min} = 558 \text{ kgf/cm}^2$$

Our design is safe.

Check for Plastic Deformation : (Refer Table 25.29) (PSG 8.21)

Assuming the starting torque or maximum instantaneous torque is twice the mean torque

$$\sigma_{c_{max}} = \sigma_c \left\{\frac{[M_t]_{max}}{[M_t]}\right\}^{1/2} \leq [\sigma_c]_{max}$$

Now $\sigma_{c_{max}} = \sigma_c \left\{\frac{2.\,M_t}{M_t}\right\}^{1/2} = \sigma_c\,[2]^{1/2}$

$$= 2860 \times \sqrt{2} = 4050 \text{ kgf/cm}^2$$

$$[\sigma_c]_{max} = 420 \times HRC = 420 \times 60 = 25200 \text{ kgf/cm}^2$$

Since $\sigma_{c_{max}} < [\sigma_c]_{max}$, our design is safe.

Under Bending :

$$\sigma_{b_{max}} = \sigma_b \frac{M_{t\,(max)}}{M_t} = 255 \times \frac{2 \cdot M_t}{M_t} = 255 \times 2 = 510 \text{ kgf/cm}^2$$

$$[\sigma_b]_{max} = 0.36 \times \frac{\sigma_u}{k_\sigma} = 0.36 \times \frac{4000}{1.2} = 1200 \text{ kgf/cm}^2$$

Since $\sigma_{b\,.\,max} < [\sigma_b]_{max}$, our design is safe.

Specifications :

1. Type of drive	–	Spur gear drive
2. Material for pinion	–	C15 steel
3. Material for gear	–	Cast Iron grade 30
4. Centre distance	–	245 mm
5. No. of teeth of pinion	–	28
6. No. of teeth of gear	–	70
7. Module	–	5 mm
8. Face width	–	75 mm
9. Pitch circle dia of pinion	–	140 mm
10. Pitch circle dia of gear	–	350 mm
11. Tip circle dia of pinion	–	$(d_1 + 2\text{ m}) = 150\text{ mm}$
12. Tip circle dia of gear	–	$(d_2 + 2\text{ m}) = 360\text{ mm}$
13. Root circle dia of pinion	–	$(d_1 - 2\text{ m} - 2\text{ c}) = 127.5\text{ mm}$
14. Root circle dia of gear	–	$(d_2 - 2\text{ m} - 2\text{ c}) = 337.5\text{ mm}$

Problem 25.4 :

The pitch circles of a train of spur gears are shown in figure 25.11. The spur gear A receives 3 kW power at 600 rpm through its shaft and rotates clockwise. Gear B is an idler and gear C is the driven gear. The teeth are 20° full depth. Determine

(i) the torque transmitted by each shaft.

(ii) the tooth load for which each gear must be designed

(iii) the force applied to the idler shaft as a result of the gear tooth loads.

(Anna University, Dec. 2004)

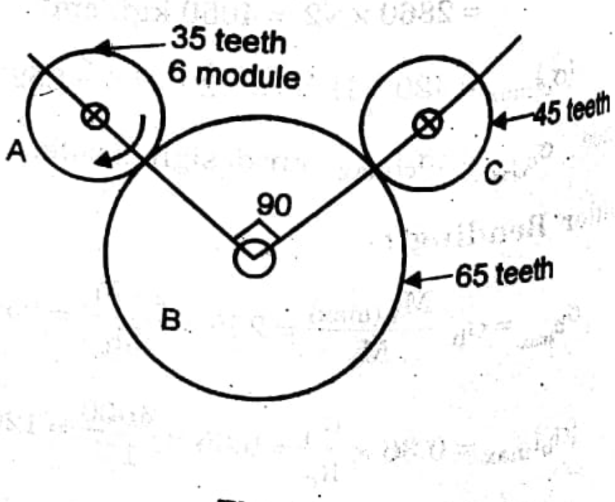


Fig. 25.11

HELICAL – GEAR DRIVES

26.1. INTRODUCTION :

Helical gears are the modified form of spur gears, in which all the teeth are cut at a constant angle, known as ***helix angle***, to the axis of the gear, where as in spur gear, teeth are cut parallel to the axis. Helical gears are also employed to transmit power between parallel shafts. Because of the inclined structure of teeth, more than one pair of teeth will be in engagement and hence the operation may be smooth due to their gradual contact and more power can be transmitted at higher speeds than spur-gear drive.

In a helical gear drive, one of the gears has a right hand helix and the mating gear has a left-hand helix.

26.2. TYPES OF HELICAL GEARS :

The helical gears may be of ***single-helical type or double-helical type***. Double helical gears are having both sets of teeth (i.e., left hand and right hand helix) cut on a single blank. In single helical gears, some axial thrust will be produced between the teeth which is a disadvantage. This deficiency may be rectified in double-helical gears, also known as ***herring-bone gears***. It is equivalent to two single helical gears in which equal and opposite thrusts are applied on each gear and the resulting axial thrust is zero.

Sometimes a little-bit of confusion may arise between the terms such as a pair of helical gears and a double helical gear. A pair of gears (or a gear-set) comprises one pinon and one gear whereas in a double-helical gear drive, both pinion and gear are having two-two gears of similar size meshed side by side which seem to be mirror imaged. That is, pinion is having one right-hand helical gear and one left-hand helical gear of same size and meshed together. Similarly the same set-up will be formed in the large gear also. A small clearance will be provided between the two gears of pinion or gears of herring-bone gears for easy manufacturing.

There is one more type of helical gear-drive called as ***crossed helical gear-drive*** .(or) ***skew gear-drive*** which can be operated to transmit power between nonparallel and non intersecting shafts. The schematic diagram for helical gear and its types are given in figure 26.1

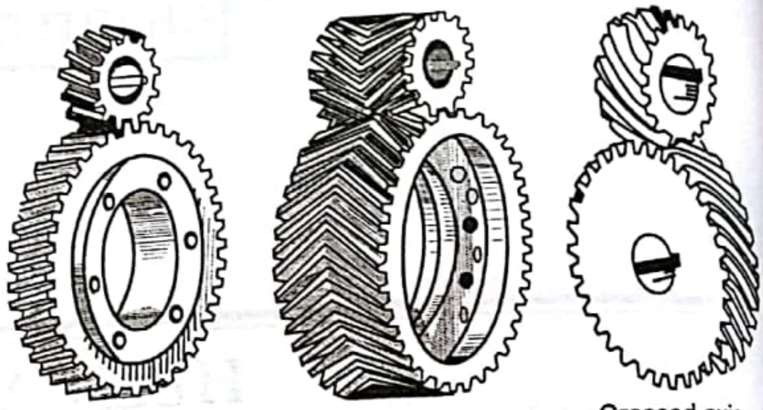


Fig. 26.1

26.3 DESIGN FORMULAS :

The designing work of all the commonly used gears such as spur gears, helical gears, bevel gears ,and worm gears is carried out usually through two main parameters which are (1) required minimum centre distance based on surface compressive stress and (2) required minimum module based on bending stress.

Since the structure of helical gear is almost similar to spur-gear, except the teeth arrangement, its designing method is also almost same with slight modification. Consider a helical gear, whose teeth are cut at the helix angle (β). Since the teeth are cut at an angle (i.e., β); three pitches have been taken into account. They are

1. p_c, ***the circular pitch*** which is the distance between the corresponding points of successive teeth, measured along the plane perpendicular to the axis of gear.

2. p_n, ***the normal pitch*** which is the distance between two successive teeth, measured along the plane perpendicular (i.e., normal) to teeth. This plane is at the angle of $(90 + \beta)$ degrees to the axis of gear.

3. p_a, ***the axial pitch*** which is distance between two successive teeth, measured along the plane parallel to the axis of the gear.

The relation between them is shown in figure 26.2.

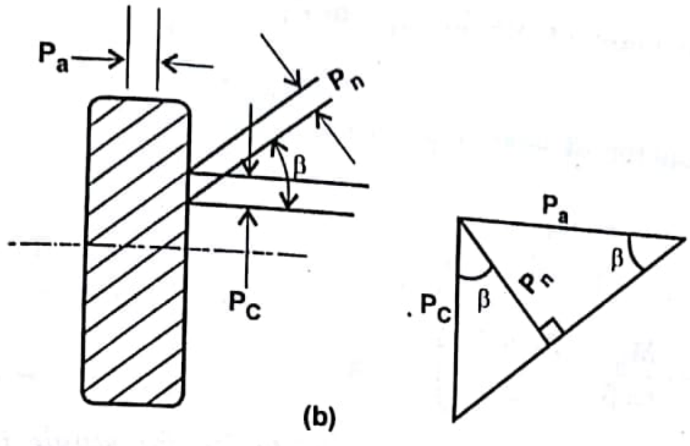


Fig. 26.2

From the figure, we know that, $\cos\beta = \frac{p_n}{p_c}$

(or) $p_n = p_c \cos\beta$

Since $p_c = \pi\, m$, we have

$$p_n = \pi\, m \cos\beta = \pi\, m_n \qquad \left(\text{where } m_n = m\cos\beta \text{ (or) } m = \frac{m_n}{\cos\beta}\right)$$

Also $p_a = \frac{p_n}{\sin\beta} = \frac{p_c\cos\beta}{\sin\beta} = \frac{p_c}{\tan\beta}$

For cutting the spur gears, the gear blank is held such that its axis is parallel to the direction of cutter movement, whereas for cutting helical gears, the gear-blank is held such that its axis is at the helix angle (β) to the cutter movement as shown in figure 26.3 and hence the normal module (m_n) may be considered for helical gear design.

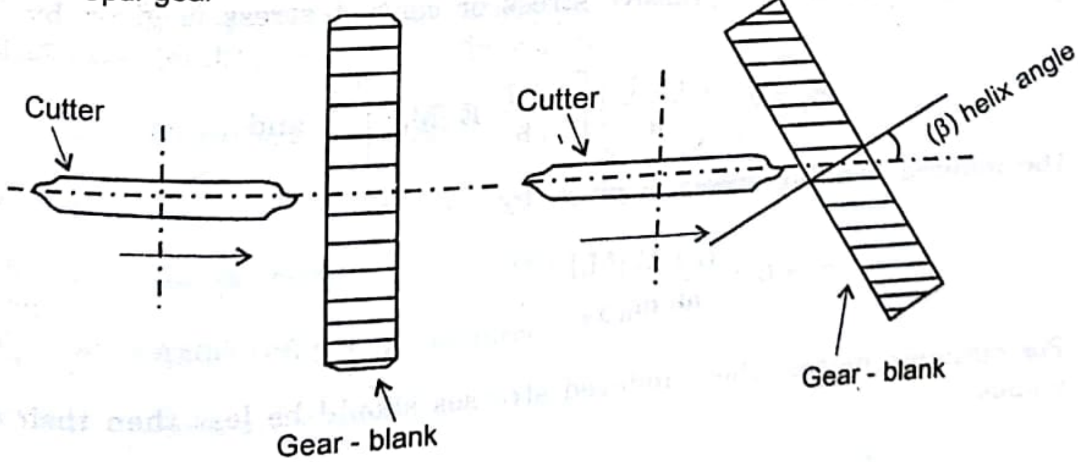


Fig. 26.3

Now, pitch circle diameter of pinion, $d_1 = m\, Z_1 = \frac{m_n Z_1}{\cos\beta}$

The pitch circle diameter of gear, $d_2 = m\, Z_2 = \frac{m_n Z_2}{\cos\beta}$

Centre distance, $a = \left(\frac{d_1 + d_2}{2}\right)$

$$= \frac{M_n}{\cos\beta}\left(\frac{Z_1 + Z_2}{2}\right)$$

The usual value of helix angle ranges from 8° to 25° for single helical gears and 25° to 40° for double-helical gears (i.e., herring-bone gears). For the design of herringbone gears, we can consider that the power transmitted by each portion is half of the total power.

Similar to spur gear drive, the basic formulas used for the design of helical gear drive are as follows.

1. Minimum centre distance based on surface compressive stress.

$$a \geq (i \pm 1)\left[\left\{\frac{0.7}{[\sigma_c]}\right\}^2 \frac{E\,[M_t]}{i\,\psi}\right]^{1/3} \quad \text{.... (26.1)}$$

2. Minimum normal module based on beam strength,

$$m_n \geq 1.15 \cos\beta \left[\frac{[M_t]}{[\sigma_b]\,\psi_m\, Z_1\, y_v}\right]^{1/3} \quad \text{.... (26.2)}$$

3. The induced surface compressive stress or contact stress is given by

$$\sigma_c = 0.7\left(\frac{i \pm 1}{a}\right)\left[\frac{i \pm 1}{i\, b} E\,[M_t]\right]^{1/2} \quad \text{and} \quad \text{.... (26.3)}$$

4. The induced bending stress is given by

$$\sigma_b = 0.7\,\frac{(i \pm 1)\,[M_t]}{a b\, m_n\, y_v} \quad \text{.... (26.4)}$$

For optimum design, these induced stresses should be less than their design values.

i.e.; $\sigma_c \leq [\sigma_c]$ and $\sigma_b \leq [\sigma_b]$ for safe design.

26.6 DESIGN PROCEDURE :

The steps given below may be observed for the design of helical and herring-bone gear drives.

1. From the given problem note down the amount of power to be transmitted, pinion speed, gear ratio, life of gear drive and other working conditions. Then select proper materials and determine their design stresses and Youngs modulus similar to spur gear drive.

2. Calculate the minimum centre distance required for the gear drive as

$$a \geq (i \pm 1) \sqrt[3]{\left\{\frac{0.7}{[\sigma_c]}\right\}^2 \frac{E\,[M_t]}{i\,\psi}}$$

Here $[M_t] = M_t.\,k.\,k_d$

where $M_t = \dfrac{60\,P}{2\,\pi\,n_1}$ in the case of single helical gear

$= \dfrac{60\,(P/2)}{2\,\pi\,n_1}$ in the case of herring-bone gears.

and assume ψ from 0.5 to 0.8 initially.

3. Based on bending stress, evaluate the minimum normal module using

$$m_n \geq 1.15 \cos\beta \sqrt[3]{\frac{[M_t]}{[\sigma_b]\,\psi_m\,Z_1\,y_v}}$$

As usual assume ψ_m as 10 initially and Z_1 from 14 to 20.

y_v is the form factor corresponding to Z_{v1}, the virtual number of teeth of pinion which is equal to $\dfrac{Z_1}{\cos^3\beta}$

4. Then select the nearest standard normal module from table 25.16.

5. Correct the number of teeth of pinion with the help of corrected module and minimum centre distance as

$$Z_1 = \frac{2\,a\cos\beta}{m_n\,(i+1)}$$

6. Determine the pitch circle diameters of pinion and gear using the relations.

$$d_1 = \frac{m_n}{\cos\beta} Z_1 \quad \text{and} \quad d_2 = \frac{m_n}{\cos\beta} Z_2$$

where $Z_2 = i\, Z_1$

7. Decide the corrected centre distance as

$$a = \frac{d_1 + d_2}{2}$$

8. The face – width of gear teeth may be determined as $b = \psi \cdot a$ (or) $b = \psi_m \cdot m$ (or) $b = 1.15\, p_a$.

where $p_a =$ Axial pitch $= \frac{p_n}{\sin\beta} = \frac{\pi\, m_n}{\sin\beta}$

In order to make atleast two pairs of teeth in mesh, the face width should be equal to or greater than 1.15 p_a.

(i.e., $b \geq 1.15\, p_a$). Otherwise, similar to spur gear, one set (or pair) of teeth will mesh resulting lower power transmission.

By calculating the face width using above three relations, the higher value may be adopted.

9. The values of load concentration factor and dynamic factor may be corrected as usual.

10. Check the induced compressive stress and bending stress with their design values as given in table 25.7. (for herring-bone gear, the face-width may be doubled).

11. Find out the essential parameters of gear-drive.

12. Draw the schematic sketch of helical gear-drive neatly.

26.7. DESIGN OF HELICAL GEAR BY "AGMA" METHOD :

The design of helical gear is almost similar to spur gear design with slight modifications in Lewis and Buckingham equations due to helix angle.

According to Lewis equation, the beam strength of helical gear tooth is given by

$$F_b = [\sigma_b] \cdot b \cdot \pi\, m_n \cdot y_v$$

where,

$[\sigma_b]$ = Allowable contact stress in N/mm^2

b = Face width of gear blank $= 10\ m_n$

m_n = Normal module which must be standardized.

y_v = Lewis form factor which depends on the virtual number of teeth $Z_v = \left(\dfrac{Z}{\cos^3 \beta}\right)$

For safe working, the beam strength should be greater than the design tooth load F_D which is given by

$$F_D = F_t \times K_s \times C_v = \frac{P \times K_s \times C_v}{v}$$

the values of K_s, C_v, v etc. are calculated similar to spur gears.

The dynamic load acting on helical gear tooth may be found out using Buckingham equation as

$$F_d = F_t + \frac{21\,v\,(Cb \cos^2 \beta + F_t) \cos \beta}{21\,v + \sqrt{Cb \cos^2 \beta + F_t}}$$

and the wear tooth load is given by

$$F_w = \frac{d_1 \cdot b \cdot Q \cdot K_w}{\cos^2 \beta}$$

The values of Q and K_w etc. are all common with spur gears. The design procedure is also very similar to spur gears.

26.8. SOLVED PROBLEMS :

Problem 26.1 :

Design a pair of helical gears to transmit 10 k.W at 1000 rpm of the pinion. Reduction ratio of 5 is required. Give details of the drive in a tabular form.

Solution :

Power	=	10 k.W
Pinion speed	=	1000 rpm
Speed ratio	=	5
∴ Gear speed	=	1000/5 = 200 rpm
Helical angle	=	15° (Assume)

Minimum centre distance based on surface compressive strength is given by,

$$a \geq (i+1)\sqrt[3]{\left(\frac{0.7}{[\sigma_c]}\right)^2 \frac{E\,[M_t]}{i\,\psi}}$$ [Table 25.5, (PSG 8.13)]

Material selection

Let the material for pinion and gear as 40 Ni2 Cr1 Mo28 Steel.

Its $[\sigma_c] = 11000\ \text{kgf/cm}^2$; $[\sigma_b] = 4000\ \text{kgf/cm}^2$ [Table 25.7] (PSG 8.5)

$E = 2.15 \times 10^6\ \text{kgf/cm}^2$; $i = 5$; $\psi = \frac{b}{a} = 0.5$ (Assumed)

$$[M_t] = M_t \cdot k \cdot k_d = 97420 \times \frac{10 \times 1.3}{1000} = 1266\ \text{kgf}-\text{cm}$$ (Table 25.17) (PSG 8.15)

$$\text{Now } a \geq (5+1)\sqrt[3]{\left(\frac{0.7}{11000}\right)^2 \frac{2.15 \times 10^6 \times 1266}{5 \times 0.5}} \geq 9.84\ \text{cm}$$

Minimum module based on beam strength

$$m_n \geq 1.15 \cos\beta \times \sqrt[3]{\frac{[M_t]}{y_v\,[\sigma_b]\,\psi_m\,Z_1}}$$ (Table 25.5) (PSG 8.13)

Let $Z_1 = 20$ and $\psi_m = \frac{b}{m} = 10$ (Assumed)

$$Z_v = \frac{Z_1}{\cos^3\beta} = \frac{20}{\cos^3 15} = 22.2$$

Hence $y_v = 0.402$

$$m_n \geq 1.15 \cos 15 \times \sqrt[3]{\frac{1266}{0.402 \times 4000 \times 10 \times 20}}$$

$$\geq 0.175\ \text{cm} = 1.75\ \text{mm}$$

Let $m_n = 2\ \text{mm} = 0.2\ \text{cm}$

No. of teeth of pinion, $Z_1 = \frac{2\,a\cos\beta}{m_n\,(i+1)}$

$$Z_1 = \frac{2 \times 9.84 \times \cos 15}{0.2\,(5+1)} = 15.84 = 16 \text{ (say)}$$

$$Z_2 = i\,Z_1 = 5 \times 16 = 80$$

Now

$$a = \frac{d_1 + d_2}{2}$$

$$d_1 = \frac{m_n}{\cos \beta} \times Z_1 = \frac{0.2 \times 16}{\cos 15} = 3.3 \text{ cm}$$

$$d_2 = i\,d_1 = 16.5 \text{ cm}$$

$$a = \frac{3.3 + 16.5}{2} = \frac{19.8}{2} = 9.9 \text{ cm (O.K)}$$

Check calculations :

[Table 25.5] (PSG 8.13)

$$\sigma_c = 0.7\,\frac{(i+1)}{a}\sqrt{\frac{(i+1)}{i\,b} \times E\,[M_t]}$$

Now $b = 0.5 \times a = 0.5 \times 9.9 = 4.95$ cm

(or) $b = 10 \times m_n = 10 \times 0.2 = 2$ cm

Let $b = 4.95$ cm $= 5$ cm (say)

$$\sigma_c = \frac{0.7\,(6)}{9.9}\sqrt{\frac{6}{5 \times 5} \times 2.15 \times 10^6 \times 1266} = 10843 < [\sigma_c] = 11000 \text{ kgf/cm}^2$$

Our design is safe.

(Table 25.5)

$$\sigma_b = 0.7\,\frac{(i+1)}{a.b.\,m_n\,y_v}\,[M_t] \le [\sigma_b]$$

$$= \frac{0.7 \times 6 \times 1266}{9.9 \times 5 \times 0.2 \times 0.402} = 1336 \text{ kgf/cm}^2 < [\sigma_b] = 4000 \text{ kgf/cm}^2$$

Our design is safe.

Addendum $= m_n = 2$ mm

Dedundum $= 1.25 \times m_n = 1.25 \times 2 = 2.5$ mm

Tip circle diameter of pinion $= d_1 + (2 \times \text{addendum})$

$= 33 + (2 \times 2) = 37$ mm

Tip circle diameter of gear	$= d_2 + (2 \times \text{addendum})$
	$= 165 + 4 = 169$ mm
Root circle diameter of pinion	$= d_1 - (2 \times \text{dedundum})$
	$= 33 - (2 \times 2.5) = 28$ mm
Root circle diameter of gear	$= 160 - 5 = 160$ mm

Specifications :

	Description	Pinion	Gear
1.	Material	40 Ni 2 Cr1 Mo28	40Ni 2Cr1 Mo28
2.	No. of teeth	16	80
3.	Pitch circle diameter	33 mm	165 mm
4.	Tip circle diameter	37 mm	169 mm
5.	Root circle diameter	28 mm	160 mm
6.	Face width	50 mm	50 mm
7.	Module	2 mm	2 mm
8.	Centre distance	= 9.9 cm (or) 99 mm.	

Problem 26.2 :

Design a helical gear drive to transmit the power of 20 h.p. Speed ratio 6, pinion speed 1200 rpm, helix angle is 25°. Select suitable materials and design the gear.

Solution :

Given : Power = 20 h.p.

$i = 6$

$n = 1200$ rpm

$\beta = 25°$

BEVEL – GEAR DRIVES

27.1. INTRODUCTION :

Some times, power is to be transmitted in an angular direction. For such purpose, another type of gear-drive, called as bevel gear drive can be employed. They are used to transmit power between the shafts whose axes are intersecting at an angle. In bevel gears, teeth are cut on conical surface in contrast with spur and helical gears for which the teeth are cut on cylindrical surfaces. The structure of bevel gear is similar to an uniformly serrated frustum of a cone. The teeth are tapered in both parameters such as in tooth thickness and tooth height. At one end, the tooth height is large while at other end its height is small. The tooth dimensions are specified by the large end of teeth. However in calculating bearing loads, the central section dimensions and forces can be adopted. The bevel gear provides positive drive and a constant velocity ratio between the shafts. Also bevel gears are made in pairs only and not in interchangeable sets.

27.2. CLASSIFICATIONS OF BEVEL GEARS :

Bevel gears can be classified in two ways (1) Based on the shape of teeth such as ***straight teeth bevel gears*** and curved teeth bevel gears, specified as ***spiral bevel gears***.

2) Based on the angle between the shaft axes. The angle is known as ***shaft angle***, denoted as ϕ. The bevel gear drive having shaft angle, less than 90° is known as ***external gear drive*** whereas if the shaft angle is more than 90°, then it may be called as ***internal gear-set***. But most of the times, the shaft angle, ϕ, may be taken equal to 90°. In this arrangement, it may be called as ***crown-gears***. At the same time, if the shaft angle is equal to 90° and the angles of mating gears (pitch angles of gears) are equal to 45° then those gears are mentioned as ***miter gears***.

For clear understanding, let δ_1 and δ_2 be the pitch angles (i.e., half of cone angles) subtended by the pinion and gear at the apex.

Now $\phi = \delta_1 + \delta_2$

$\phi < 90°$ for external gears

$\phi > 90°$ for internal gears

$\phi = 90°$ and $\delta_1 \neq \delta_2$ for crown gears

$\phi = 90°$ and $\delta_1 = \delta_2 = 45°$ for miter gears.

Since the teeth are tapered in height and thickness, the face of teeth converges at the point apex. This is a common point for the two mating bevel gears and also this is the point of intersection of axes of gears. The schematic diagram of a bevel gear-drive and its types are shown in figure 27.1, 27.2 and 27.3.

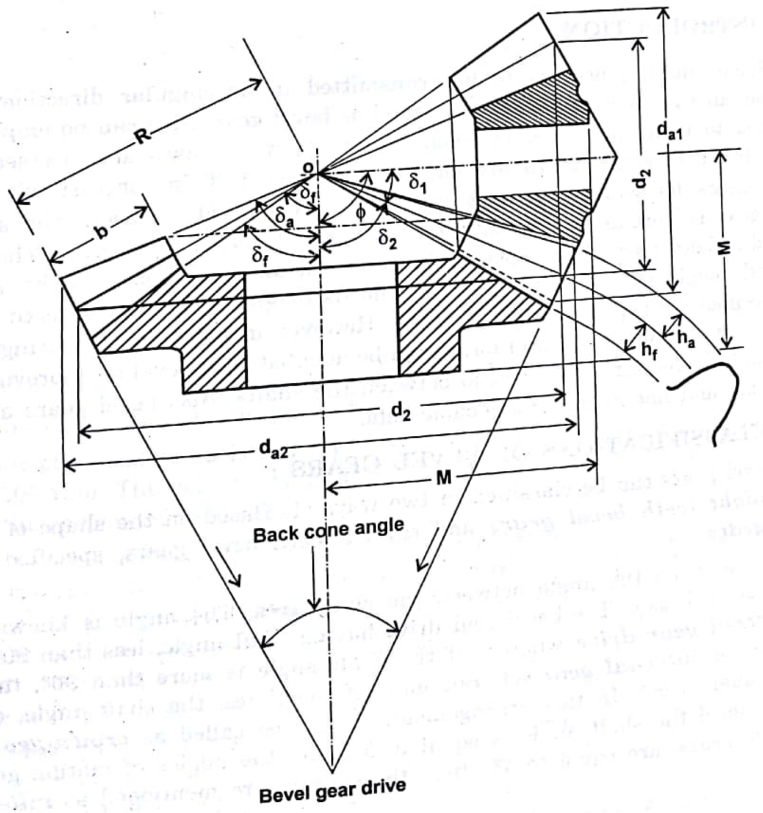


Fig. 27.1

i.e., $\sigma_c = \dfrac{0.72}{(R - 0.5\,b)} \sqrt{\dfrac{\sqrt{(i^2+1)^3}}{i\,b} E\,[M_t]} \le [\sigma_c]$ .... (27.3)

and $\sigma_b = \dfrac{R\sqrt{i^2+1}\,[M_t]}{(R - 0.5\,b)^2\, b\, m_t\, y_v} \dfrac{1}{\cos\alpha} \le [\sigma_b]$ .... (27.4)

27.4. DESIGN PROCEDURE :

The following steps may be observed for the design of straight bevel gears.

1. From the given problem, as usual note down the power to be transmitted, gear ratio, pinion speed, life of drive etc. Then select proper materials and find out their design stresses and Young's modulus.

2. Calculate the required minimum cone distance, R, based on surface compressive stress as

$$R \ge \psi_y \sqrt{i^2+1} \sqrt[3]{\left\{\frac{0.72}{(\psi_y - 0.5)\,[\sigma_c]}\right\}^2 \frac{E\,[M_t]}{i}}$$ (PSG 8.13) (JDB 26.3)

Determine $[M_t]$, E, $[\sigma_c]$ as described in spur and helical gears. Select the value of ψ_y based on gear ratio using the table 27.1 (PSG 8.15) (JDB 26.6)

3. Decide the minimum average module, m_{av}, based on bending stress as

$$m_{av} \ge 1.26 \sqrt[3]{\frac{[M_t]}{[\sigma_b]\,\psi_m\, Z_1\, y_v}}$$ (PSG 8.13) (JDB 26.3)

Z_1 may be assumed from 18 to 22 and $\psi_m = \dfrac{b}{m_{av}} = 10$ (As assumed)

y_v is the form factor corresponding to the virtual number of teeth of pinion, $Z_{v1} = \dfrac{Z_1}{\cos\delta_1}$ where δ_1 is half of the cone angle subtended by pinion, i.e., pitch angle of pinion.

Using the relations, $\tan\delta_2 = i$ and $\phi = \delta_1 + \delta_2$ where ϕ is the shaft angle, δ_1 and δ_2 can be evaluated.

4. Find out the transverse module, m_t, using the relation as

$$m_t = m_{av} \times \frac{\psi_y}{(\psi_y - 0.5)}$$

(or) $m_t = m_{av} + \frac{b \sin \delta}{Z}$ (From table 25.5) (PSG 8.38)

5. Standardize the transverse module, m_t, using the table 25.16. (PSG 8.2)

6. Correct the number of teeth of pinon using the relation as

$Z_1 = \frac{R}{0.5\, m_t \sqrt{i^2 + 1}}$ and then finalise the cone distance using the corrected module, m_t, and corrected number of teeth, Z_1 as

$$R = 0.5\, m_t\, Z_1 \sqrt{i^2 + 1}$$

(OR) $$R = 0.5\, m_t \sqrt{Z_1^2 + Z_2^2}$$

7. Findout the reference diameters (or) diameters at the outer pitch circles for pinion and gear as

$$d_1 = m_t\, Z_1 \text{ and } d_2 = m_t\, Z_2$$

and the face width is found out as $b = \frac{R}{\psi_y}$

8. Evaluate the induced surface compressive stress, σ_c, and bending stress, σ_b, and compare them with their allowable values.

i.e., $$\sigma_c = \frac{0.72}{(R - 0.5\, b)} \sqrt{\frac{(i^2 + 1)^{3/2}}{i\, b} E\, [M_t]} \le [\sigma_c]$$

and $$\sigma_b = \frac{R\sqrt{i^2 + 1}\; [M_t]}{(R - 0.5\, b)^2\, b\, m_t\, y_v} \frac{1}{\cos \alpha} \le [\sigma_b]$$

9. If σ_c and σ_b are not within the allowable limits, then modify face width suitably.

10. Calculate the other essential parameters for the bevel gear drive such as tip-diameter, addendum angle, dedendum angle, tip angle, root angle, addendum, dedendum, tooth height, working depth etc. as follows.

i) Tip diameter for pinion, $d_{a1} = m_t (Z_1 + 2\cos\delta_1)$

ii) Tip diameter for gear, $d_{a2} = m_t (Z_2 + 2\cos\delta_2)$

iii) Addendum angle for pinion and gear,

$$\theta_{a1} = \theta_{a2} = \tan^{-1}\frac{m_t f_0}{R}$$

iv) Dedendum angle for pinion and gear,

$$\theta_{f1} = \theta_{f2} = \tan^{-1}\left[\frac{m_t (f_0 + c)}{R}\right]$$

where $f_0 = 1$ and $c = 0.2$ (Standard)

v) Tip angle for pinion, $\delta_{a1} = \delta_1 + \theta_{a1}$

vi) Tip angle for gear, $\delta_{a2} = \delta_2 + \theta_{a2}$

vii) Root angle for pinion, $\delta_{f1} = \delta_1 - \theta_{f1}$

viii) Root angle for gear, $\delta_{f2} = \delta_2 - \theta_{f2}$

ix) Addendum at outer edge, $h_a = m_t$

x) Dedendum at outer edge, $h_f = 1.1236\, m_t$ (Reinecker)

xi) Tooth height $h = h_a + h_f$

xii) Working depth $h_w = 2\, m_t$

11. Draw a neat sketch of bevel-gear drive.

Table 27.1

Type of gear transmission	$\psi_y = \frac{R}{b}$
(a) Housed in roller bearings	
$i = 1$ to 4	3
$i = 4$ to 6	4
(b) Housed in Journal & Thrust bearings	
$i = 6$	5

The wear strength of gear tooth is given by

$$F_w = \frac{0.75\, d_1 \cdot b \cdot Q_v \times K_w}{\cos \delta_1}$$

where d_1 = Pitch circle diameter of the big end of pinion

b = Face width

Q_v = Ratio factor based on virtual number of teeth

$$= \frac{2 \times Z_{v2}}{Z_{v1} + Z_{v2}}$$

(Table 25.37)

$$K_w = \text{Load stress factor} = \frac{\sigma_e^2 \times \sin \alpha}{1.4}\left[\frac{1}{E_1} + \frac{1}{E_2}\right]$$

For safe operation $F_w > F_d$

The design procedure is similar to spur gears.

27.6. SOLVED PROBLEMS :

Problems 27.1

Design a bevel gear drive to transmit 7 kW at 1600 rpm for the following data.

Gear ratio	**= 3**
Material for pinion and gear	**= C 45 steel**
Life	**= 10,000 hours**

(Madras University, April 2000)

Solution :

Since pinion and gear are made of same material, pinion is weaker than gear and its teeth are subjected to more number of cycles.

Let the tooth profile is having 20° pressure angle. Minimum cone distance based on surface compressive strength is given by

$$R \geq \psi_y \sqrt{i^2 + 1}\ \sqrt[3]{\left\{\frac{0.72}{(\psi_y - 0.5)\,[\sigma_c]}\right\}^2 \frac{E\,[M_t]}{i}}$$

(Table 25.5) (PSG 8.13)
(JDB 26.3)

$$\text{Design torque } [M_t] = M_t \cdot k \cdot k_d = \frac{60 \times P}{2\,\pi\, n_1} \times k \cdot k_d$$

$$= \frac{60 \times 7 \times 10^3}{2\,\pi \times 1600} \times 1.5 = 62.7 \text{ N–m}$$ (Assuming $k \cdot k_d = 1.5$ initially)

$$= 62.7 \times 10^3 \text{ N–mm.}$$

Equivalent Youngs modulus $= 2.15 \times 10^5$ N/mm^2 [Table 25.11](PSG 8.14)

$[\sigma_c] = 500$ N/mm^2

$\psi_y = \frac{R}{b} = 3$ (for $i = 3$) (Table 27.1) (PSG 8.15)

$$\text{Now } R \geq 3 \times \sqrt{3^2+1}\;\sqrt[3]{\left\{\frac{0.72}{(3-0.5)\,500}\right\}^2 \times \frac{2.15 \times 10^5 \times 62.7 \times 10^3}{3}}$$

≥ 108 mm

Average module based on beam strength is given by

(Table 25.5) (PSG 8.13)

$$m_{av} \geq 1.26 \times \sqrt[3]{\frac{[M_t]}{y_v\,[\sigma_b]\,\psi_m\,Z_1}}$$

(Table 25.7)

$[\sigma_b] = 140$ N/mm^2

$\psi_m = \frac{b}{m_{av}} = 10$ initially

Assume $Z_1 = 20$ initially

$y_v =$ Form factor based on equivalent no. of teeth on the virtual cylinder (i.e., Z_v)

$Z_v = \frac{Z_1}{\cos \delta_1}$ (For pinion)

Now $\tan \delta_2 = i = 3$

$\delta_2 = \tan^{-1} i = \tan^{-1} 3 = 71.56°$

$\delta_1 = 90 - 71.56° = 18.43°$ (Since $\delta_1 + \delta_2 = 90°$)

Now $Z_v = \frac{Z_1}{\cos \delta_1} = \frac{20}{\cos 18.43°} = \frac{20}{0.95} = 21$ (Table 25.15) (PSG 8.39)

$y_v = 0.396$

$$\text{Now } m_{av} = 1.26 \sqrt[3]{\frac{62.7 \times 10^3}{0.396 \times 140 \times 10 \times 20}}$$

$$= 2.3 \text{ mm}$$

(PSG 8.38)

$$\text{Transverse module } m_t = m_{av} \times \frac{\psi_y}{(\psi_y - 0.5)}$$

$$m_t = 2.3 \times \frac{3}{(3 - 0.5)} = 2.3 \times \frac{3}{2.5} = 2.76 \text{ mm}$$

Nearest higher standard module $= 3$ mm

$$\text{Now } R = 0.5\, m_t\, Z_1 \sqrt{i^2 + 1}$$

$$\therefore Z_1 = \frac{R}{0.5\, m_t \sqrt{i^2+1}} = \frac{108}{0.5 \times 3 \sqrt{3^2 + 1}} = 22.8$$

Let $Z_1 = 24$ and $Z_2 = i\, Z_1 = 3 \times 24 = 72$

Now final cone distance

$$R = 0.5\, m_t\, Z_1 \sqrt{i^2 + 1}$$

$$= 0.5 \times 3 \times 24 \sqrt{3^2 + 1} = 114 \text{ mm}$$

Since the final cone distance is greater than initial cone distance, our design is safe.

$$\text{Face width } b = \frac{R}{\psi_y} = \frac{114}{3} = 38 \text{ mm}$$

Take $b = 40$ mm

Checking :

(Table 25.5)
(PSG 8.13)

$$\sigma_c = \frac{0.72}{(R - 0.5\, b)} \sqrt{\frac{\sqrt{(i^2 + 1)^3}\, E\, [M_t]}{i\, b}}$$

$$= \frac{0.72}{(114 - 0.5 \times 40)} \sqrt{\frac{(3^2 + 1)^{3/2} \times 2.15 \times 10^5 \times 62.7 \times 10^3}{3 \times 40}}$$

$$= 457 \text{ N/mm}^2$$

i.e., $\sigma_c = 457 < [\sigma_c] = 500 \text{ N/mm}^2$

Our design is safe.

$$\sigma_b = \frac{R\sqrt{i^2+1}\,[M_t]}{(R-0.5\,b)^2\,b\,m_t\,y_v} \times \frac{1}{\cos\alpha} \le [\sigma_b]$$

$$= \frac{114\sqrt{3^2+1} \times 62.7 \times 10^3}{(114-0.5\times 40)^2 \times 40 \times 3 \times 0.396} \times \frac{1}{\cos 20}$$

$$= 57.2\ N/mm^2 < [\sigma_b] = 140\ N/mm^2$$

Our design is safe.

Pitch circle diameter

For pinion, $d_1 = m_t\,Z_1 = 3 \times 24 = 72$ mm

For gear, $d_2 = m_t\,Z_2 = 3 \times 72 = 216$ mm

Tip Circle diameters :

$da_1 = m_t\,(Z_1 + 2\cos\delta_1)$

$= 3\,(24 + 2\cos 18.43) = 77.7$ mm $= 78$ mm

$da_2 = m_t\,(Z_2 + 2\cos\delta_2)$

$= 3\,(72 + 2\cos 71.56) = 218$ mm

Addendum angle θ_a :

$$\theta\,a_1 = \theta\,a_2 = \tan^{-1}\left(\frac{m_t\,f_0}{R}\right) = \tan^{-1}\left(\frac{0.3\times 1}{11.4}\right) = 1.5^\circ$$

Dedundum angle

$$\theta\,f_1 = \theta\,f_2 = \tan^{-1}\left[\frac{m_t\,(f_0+c)}{R}\right] = \tan^{-1}\left[\frac{0.3\,(1+0.2)}{11.4}\right] = 1.8^\circ$$

Tip angle δ a :

For pinion $\delta\,a_1 = \delta_1 + \theta\,a_1 = 18.43 + 1.5 = 19.93^\circ$

For gear $\delta\,a_2 = \delta_2 + \theta\,a_2 = 71.56 + 1.5 = 73.06^\circ$

Root angle δf :

For pinion $\delta f_1 = \delta_1 - \theta f_1 = 18.43 - 1.8° = 16.63°$

For gear $\delta f_2 = \delta_2 - \theta f_2 = 71.56 - 1.8 = 69.76°$

Addendum $= h_a = m_t = 3$ mm

Dedundum $= h_f = 1.1236 \times m_t = 1.1236 \times 3 = 3.4$ mm

Tooth height $= h = h_a + h_f = 3 + 3.4 = 6.4$ mm

Specifications :

Sl.No.	Description	Pinion	Gear
1.	Material	C45 steel	C45 Steel
2.	Cone distance	114 mm	114 mm
3.	Module	3 mm	3 mm
4.	No. of teeth	24	72
5.	Face width	40 mm	40 mm
6.	Semi cone angle	18.43°	71.56°
7.	Addendum	3 mm	3 mm
8.	Dedundum	3.4 mm	3.4 mm
9.	Pitch circle diameter	72 mm	216 mm
10.	Tip circle diameter	78 mm	218 mm
11.	Tip angle	19.93°	73.06°
12.	Root angle	16.63°	69.76°
13.	Addendum angle	1.5°	1.5°
14.	Dedundum angle	1.8°	1.8°

CHAPTER – 28

WORM – GEAR DRIVE

28.1. INTRODUCTION :

When a speed reducer is required to have a very high velocity ratio of about 100 or even more, sometimes upto 500, then, by two ways, this requirement may be fulfilled. One is by using spur, helical and bevel gears individually or in combined form in multi-stage arrangement. For example, if the required gear ratio is 100, then, this gear ratio may be obtained as $i = 5 \times 5 \times 4 = 100$ i.e., in three steps, because the maximum speed reduction for the above gear-drive in a single-step is about 6. Sometimes by using another type of gear-drive known as worm-gear drive, this much amount of speed ratio may be obtained in a single-step itself.

In the worm gear drive, the driving member is similar to an Archimedian screw and the driven member is similar to a helical gear with curved teeth and also the pitch surface of this helical gear is slightly concaved. In this drive, the driving member is known as worm in stead of calling as pinion and the driven member is known as worm-gear or worm wheel and the power is transmitted from worm to worm-wheel by sliding contact in contrast with spur, helical or bevel gears, where the power is transmitted by rolling contact.

The worm gear drive is employed to transmit power between non-parallel and non-interesting shafts whose axes are usually at 90°. Since they are having sliding contact, their operation is smooth and noiseless, but their efficiency is lower because of power loss due to friction caused by sliding. This drive is very compact for very high velocity ratio. Some of the drawbacks of this drive are such that the maximum power that can be transmitted by the drive is around 100 kW and usually this drive's elements may be kept inside the box, having full of oil in order to reduce the heat produced by the sliding friction. The worm may be cut usually with a single thread, (or) multiple threads, sometimes represented as single start or multiple starts.

$$\text{Velocity ratio, } i = \frac{\text{Number of teeth on gear}}{\text{Number of threads (or starts) on worm}}.$$

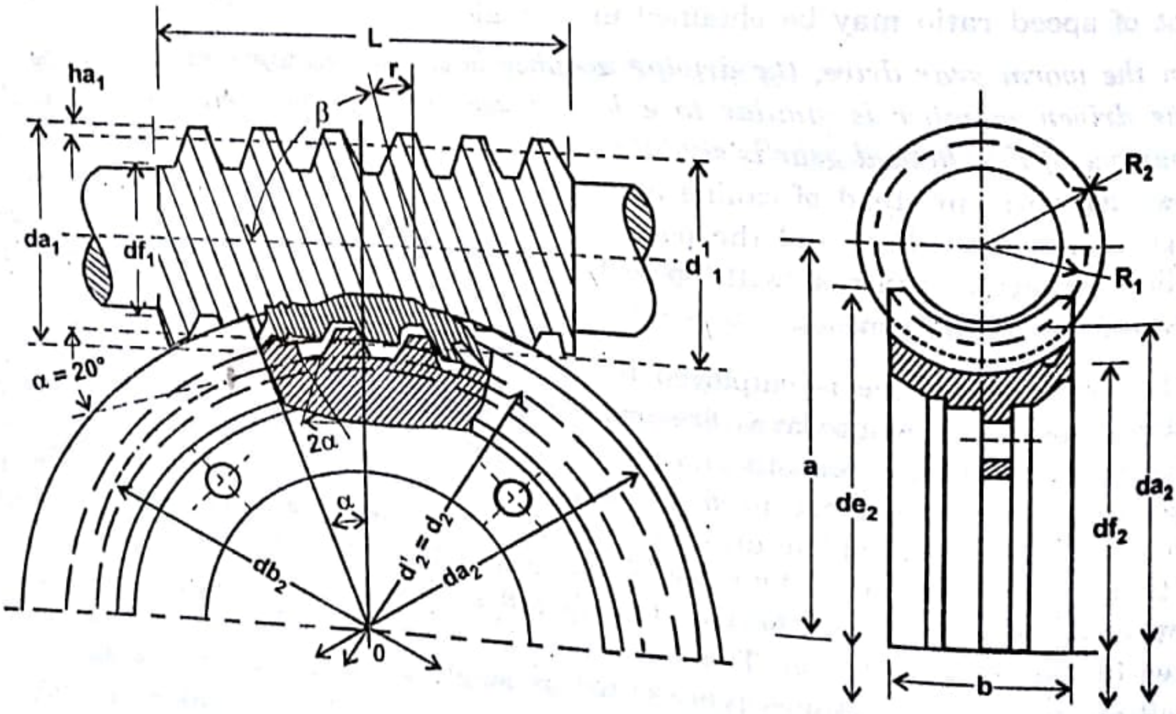
28.2. MATERIALS USED :

Since the worm is operated with worm-wheel by the sliding contact, the materials should have low coefficient of friction in order to reduce loss due to friction. This is ensured by combining heterogeneous materials with high quality finish of the contact surfaces. Usually *carbon or alloy steels may be used to make worm* and *Gray cast-iron or bronze may be selected for making worm wheels. Among all the materials, "hardened steel and phospher bronze combination"* has the lowest coefficient of friction and hence this material - combination may be preferred.

The helix angle, here known as lead angle, of worm-wheel should be kept within 30° so as to make the drive irreversible. If the lead angle is very high of about 70° to 80°, then, the worm gear drive may be operated as reversible drive. For example: Hand blower. For transmitting very high power of more than 500 kW at a very high velocity ratio, spur gear drive with multi-stage speed reduction is preferred to worm gear-drive in order to avoid more heat produced during operation in worm gear drive and also its inefficiency to transmit such a high power.

28.3. DESIGN PARAMETERS :

Since the worm gear drive transmits power through sliding contact, we should consider the friction effect when designing. Let us consider a worm gear drive as shown in figure 28.1. for which the essential parameters regarding the design point of view are given below.



Worm-gear drive

Fig. 28.1

system is changed from MKS to S.I. At the same time, there is no change in equations 28.3 and 28.4 when the unit systems are changed.

28.5. DESIGN PROCEDURE :

1 From the given problem, note down the amount of power to be transmitted, speed ratio, worm speed, materials required etc. Usually the steel for worm and bronze for wheel are preferred.

2. Estimate the minimum centre distance, 'a', based on surface compressive strength as

$$a \geq \left\{ \frac{Z_2}{q} + 1 \right\} \sqrt[3]{\left[\frac{540}{\frac{Z_2}{q}\,[\sigma_c]} \right]^2 [M_t]} \text{ cm}$$

for steel worm and bronze wheel combination.

where $[\sigma_c]$ = Design surface compressive stress in kgf/cm^2 which depends on sliding velocity, V_s (from Table 28.1) (PSG 8.45) (JDB 27.6)

(Initially V_s is assumed as 3 m/s)

q = Diameter factor $= \frac{d_1}{m_x} = 11$ (Assume initially)

Z_2 = Number of teeth on worm wheel

= $i\,Z_1$ where Z_1 = Number of starts on worm and i = Velocity ratio

$[M_t]$ = Design torque $= M_t \cdot k \cdot k_d$, in kgf-cm

Initially k.kd may be assumed as 1.

When $[M_t]$ is in N-mm and $[\sigma_c]$ is in N/mm^2, then replace $[M_t]$ in the above formula into $\frac{[M_t]}{10}$ and get the value of 'a' in mm.

3. Determine the minimum axial module, m_x, based on bending stress as

$$m_x \geq 1.24 \sqrt[3]{\frac{[M_t]}{[\sigma_b]\, q\, Z_2\, y_v}}$$

where $[\sigma_b]$ = Design bending stress (From Table 28.2) (PSG 8.45) (JDB 27.6)

y_v = Form factor corresponding $Z_{v2} = \frac{Z_2}{\cos^3 \gamma}$

and γ is the helix angle or lead angle of worm wheel which can be obtained from the relation as

$$\gamma = \tan^{-1}\frac{Z_1}{q} \text{ or from table 28.4 (PSG 8.45)}$$

4. Standardise this module using the table 25.16 (PSG 8.2)

5. Find out the diameter factor, q, and then, the centre distance using the relation as

$$a = 0.5\, m_x\,(q + Z_2).$$

6. Estimate the actual sliding velocity using

$$V_s = \frac{\pi\, d_1\, n_1}{60 \times 1000 \times \cos\gamma} \text{ m/s}$$

and note the corresponding design surface from the table 28.1

7. Calculate the induced surface compressive stress and bending stress as mentioned below and check them with their permissible (i.e., design) values.

$$\text{i.e., } \sigma_c = \left[\frac{540}{\left(\frac{Z_2}{q}\right)}\right]\sqrt{\left[\frac{\frac{Z_2}{q}+1}{a}\right]^3 [M_t]} \leq [\sigma_c] \text{ kgf/cm}^2$$

where 'a' is in cm; $[M_t]$ is in kgf-cm. To get σ_c in N/mm^2, replace $[M_t]$ in the above formula into $\frac{[M_t]}{10}$ and also $[M_t]$ must be in N-mm and 'a' must be in mm.

and $$\sigma_b = \frac{1.9\,[M_t]}{m_x^3\, q\, Z_2\, y_v}$$

8. Determine the length of worm as

$L \geq (11 + 0.06\, Z_2)\, m_x$ for $Z_1 = 1$ to 2 (Table 28.8) (PSG 8.48) (JDB 27.7)

and $L \geq (12.5 + 0.09\, Z_2)\, m_x$ for $Z_1 = 3$ to 4

For ground worm, the length L is increased by some amount, to become a new length, L_1

i.e., $L_1 = L + 25$ mm for $m_x < 10$ mm

$= L + 35$ to 40 mm for $m_x = 10$ to 16 mm

$= L + 50$ mm for $m_x > 16$ mm

9. Decide the number of teeth on worm λ as

$\lambda = \frac{L_1}{\pi\, m_x}$ and correct it to full number and then findout the actual length (L_2) of worm for that corrected number of teeth as

$L_2 = \lambda\, \pi\, m_x$

10. Determine the face width of worm-wheel by an empirical relation given in the table 28.6 (PSG 8.48) (JDB 27.7)

11. Compute the parameters of worm and worm wheel such as their reference (or pitch) diameters, tip diameters, root diameters etc. using the relations given in table 28.3 (PSG 8.43)

12. Calculate the efficiency of the drive as

$\eta = \frac{\tan \gamma}{\tan(\gamma + \rho)}$ where $\rho = \tan^{-1} \mu$ (Refer figure 28.2)

where $\rho =$ Friction angle in degrees and $\mu =$ coefficient of friction.

13. Draw a neat sketch of worm-gear drive.

Efficiency of worm gear drive (when worm is driving)

$$\eta = \frac{\tan \gamma}{\tan(\gamma + \rho)}$$

γ, lead angle, deg.

ρ, friction angle deg.

μ, $\tan \rho$, from graph below

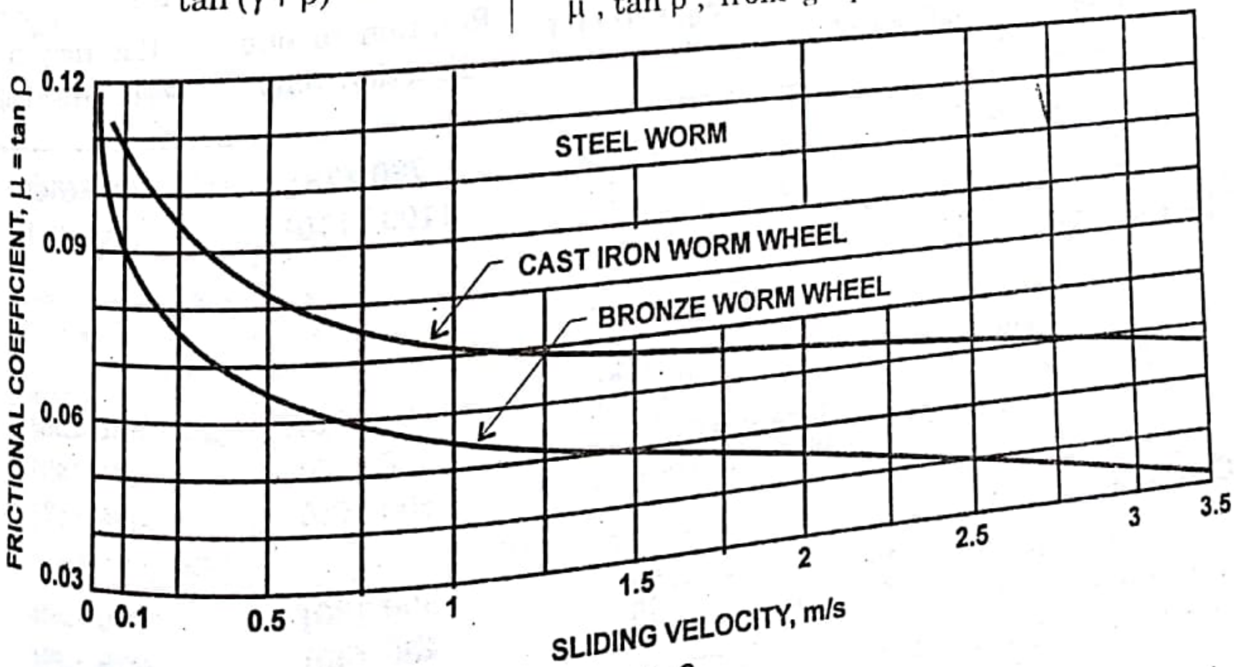


Fig. 28.2

28.6. DESIGN OF WORM GEARS BY "AGMA" METHOD :

Similar to previous gears, using Lewis and Buckingham equations, the worm gear drive can be designed.

The beam strength of worm wheel tooth, according to Lewis equation, is given by

$$F_b = [\sigma_b] \cdot b \cdot \pi \cdot m_x \cdot y_v = [\sigma_b] \cdot b \cdot m_x \cdot Y_v$$

where

$[\sigma_b]$ = Allowable static stress (i.e., design bending stress) (Table 28.2)

b = Face width of worm wheel (Table 28.6)

m_x = Axial module

y_v = Lewis form factor for worm wheel

Y_v = Modified form factor $= \pi \cdot y_v$ (Table 28.9)

The design tooth load is given by

$$F_D = F_t \times K_s \times C_v = \frac{P}{v} \times K_s \times C_v$$

where,

K_s = Service factor (Table 25.35)

C_v = Velocity factor $= \frac{6+v}{6}$ and v is the pitch line velocity of worm wheel in m/s

For satisfactory operation, $F_b > F_D$

The dynamic load as per Buckingham equation is

$F_d = F_t \left(\frac{6+v}{6}\right)$ where v is the pitch line velocity of worm wheel in m/s.

The limiting or maximum load for wear is given by,

$F_w = d_2 \cdot b \cdot K_w$

where d_2 = Pitch circle diameter of worm wheel

b = Facewidth of worm wheel.

K_w = Wear factor which depends on the materials of worm and wheel. (Table 28.10)

For safe operation $F_w > F_d$

The proportions of worm and wheel dimensions are given in table 28.3

Table 28.9: Form factor $Y_v (= \pi y_v)$ for $Z_1 + Z_2 > 40$.

Pressure angle (α)	14 ½°	20°	25°	30°
Form factor Y_v	0.314	0.392	0.47	0.55

Table 28.10: Wear factor (K_w) for worm gears

S.No.	Material		Wear factor (N/mm^2)	
	Worm	Wheel	14 ½°	20°
1.	Hardened steel	Chilled bronze	0.63	0.88
2.	Hardened steel	Bronze	0.42	0.56
3.	Steel, 250 HB	Bronze	0.252	0.35
4.	High test CI	Bronze	0.56	0.805
5.	Grey iron	Aluminium	0.07	0.084

28.7. SOLVED PROBLEMS :

Problem 28.1 : (In MKS Units)

The input to worm gear shaft is 18 kW and 600 rpm. Speed ratio is 20. The worm is to be of hardened steel and the wheel is made of chilled phospher bronze. Considering wear and strength, design worm and worm wheel.

Solution :

Power to be transmitted	$P =$	18 kW
Speed of worm	$n_1 =$	600 rpm
Speed ratio	$i =$	20
Material for worm	=	Hardened steel
Material for wheel	=	Phospher Bronze (Chilled)

Minimum centre distance between worm and worm wheel based on surface stress is given by,

$$a \geq \left(\frac{Z_2}{q}+1\right)\sqrt[3]{\left\{\frac{540}{\frac{Z_2}{q}[\sigma_c]}\right\}^2 [M_t]} \text{ cm}$$

Now, torque transmitted by the worm wheel,

$$M_t = 97420 \times \frac{kW}{n_1} i\,\eta \qquad \text{(where } n_1 = \text{speed of worm} = 600 \text{ rpm)}$$

$$i = \frac{Z_2}{Z_1} = 20, \text{ Take } q = 11 \text{ initially}$$

$Z_1 =$ No. of starts on the worm $= 3$

$\therefore \eta = 0.86$ (Assume) (Table28.5)

$\therefore Z_2 = 3 \times 20 = 60$

$$M_t = 97420 \times \frac{18}{600} \times 20 \times 0.86 = 50270 \text{ kgf} - \text{cm}$$

$[M_t] = M_t \cdot k.\, k_d$ $\qquad k = 1 \because$ load is almost constant

$= 50270 \times 1 \times 1$ $\qquad k_d = 1$ for assumed sliding velocity $V_s = 3$ m/sec.

$= 50270$ kgf – cm

Material Selection :

For worm (steel) and wheel (phospher Bronze)

$[\sigma_c] = 1590 \text{ kgf/cm}^2$ Assuming $V_s = 3$ m/sec

$[\sigma_b] = 550 \text{ kgf/cm}^2$ (Table 28.2) (PSG 8.45)

$$\text{Now } a \geq \left(\frac{60}{11}+1\right)\sqrt[3]{\left[\frac{540}{\left(\frac{60}{11}\right)(1590)}\right]^2 \times 50270} \geq 37.4 \text{ cm}$$

Axial Module based on beam strength

$$m_x \geq 1.24\sqrt[3]{\frac{[M_t]}{Z_2\, q \cdot y_v \cdot [\sigma_b]}} \text{ cm}$$

$y_v =$ Form factor for virtual no. of teeth Z_{v2}

$Z_{v2} = \frac{Z_2}{\cos^3 \gamma}$ where γ = lead angle or helix angle

$$\gamma = \tan^{-1}\left(\frac{Z_1}{q}\right) = \tan^{-1}\left(\frac{3}{11}\right) = 15° \, 15' \, 18'' = 15.255°$$

$$Z_{v2} = \frac{60}{\cos^3 15.255} = 66.8 = 67$$

$\therefore y_v = 0.493$ (Table 25.15) (PSG 8.18)

$[\sigma_b] = 550 \text{ kgf/cm}^2$

$$m_x \geq 1.24 \sqrt[3]{\frac{50270}{60 \times 11 \times 0.493 \times 550}} \geq 0.82 \text{ cm} \geq 8.2 \text{ mm}$$

Take $m_x = 10$ mm

Now the centre distance

$$a = 0.5 \, m_x (q + Z_2 + 2x)$$

$$= 0.5 \times 10 (11 + 60) \quad \text{(Assuming } x = 0)$$

$$= 355 \text{ mm}$$

Since this is less than the minimum centre distance (= 374 mm)

Let $m_x = 12$ mm

Now $a = 0.5 \times 12 (11 + 60) = 426 \text{ mm} = 42.6 \text{ cm}$ (O.K)

Sliding velocity $V_s = \frac{\pi d_1 n_1}{60 \times 1000 \times \cos v}$

$d_1 = q \, m_x = 11 \times 12 = 132$ mm

$\gamma = 15.255°$

$$V_s = \frac{\pi \times 132 \times 600}{60 \times 1000 \times \cos 15.255°} = 4.29 \text{ m/sec}$$

Using another formula : (PSG 8.44)

$$V_s = \frac{m_x \cdot n}{19100} \sqrt{Z_1^2 + q^2}$$

$$= \frac{12 \times 600}{19100} \sqrt{3^2 + 11^2} = 4.29 \text{ m/sec}$$

Since adequate data are not available for surface strength for sliding velocity beyond 4 m/sec it is assumed that $[\sigma_c] = 1490 \text{ kgf/cm}^2$. (Table 28.1) (PSG 8.45)

Checking :

$$\sigma_c = \frac{540}{\left(\frac{Z_2}{q}\right)} \sqrt{\left\{\frac{\left(\frac{Z_2}{q}+1\right)}{a}\right\}^3 [M_t]} \quad \text{kgf/cm}^2$$

$$= \frac{540}{\left(\frac{60}{11}\right)} \sqrt{\left\{\frac{\left(\frac{60}{11}+1\right)}{42.6}\right\}^3 50270}$$

$$= 1300 \text{ kgf/cm}^2 < [\sigma_c] = 1490 \text{ kgf/cm}^2$$

$$\sigma_b = \frac{1.9\,[M_t]}{m_x^3\, q\, Z_2\, y_v}$$

$$= \frac{1.9 \times 50270}{1.2^3 \times 11 \times 60 \times 0.493} = 170 \text{ kgf/cm}^2 < [\sigma_b] = 550 \text{ kgf/cm}^2$$

Our design is safe.

Length of worm $L \geq (12.5 + 0.09\, Z_2)\, m_x$

$$L \geq (12.5 + 0.09 \times 60)\, 12 \geq 214.8 \text{ mm} \geq 215 \text{ mm}$$

Number of teeth on worm $= \lambda = \frac{L}{\pi\, m_x} = \frac{215}{\pi \times 12} = 5.7$

Let $\lambda = 6$

Now the length of worm $= 6 \times \pi \times m_x = 6 \times \pi \times 12 = 226$ mm

Breadth of worm wheel (Face width) $= 0.75\, d_1$

$= 0.75 \times 132 = 99$ mm $= 100$ mm (say)

Parameters of worm : (Refer table 28.3) (PSG 8.43)

Reference diameter $d_1 = q\, m_x = 132$ mm

Tip diameter $d\,a_1 = d_1 + 2\, f_0\, m_x$

$= 132 + (2 \times 1 \times 12) = 156$ mm

Root diameter $df_1 = d_1 - 2 f_0 m_x - 2c$

$= 132 - 2 \times 1 \times 12 - 2 \times 0.2 \times 12$

$= 132 - 2 \times 12 (1 + 0.2) = 103$ mm

Pitch diameter $d_1' = m_x (q + 2x)$

$= 12 (11 + 2 \times 0)$ (Assuming $x = 0$)

$= 132$ mm

Wheel :

Reference diameter $d_2 = Z_2 m_x$

$= 60 \times 12 = 720$ mm

Tip diameter $da_2 = (Z_2 + 2 f_0 + 2x) m_x$

$= (60 + 2) 12 = 744$ mm

($\because f_0 = 1; x = 0$)

Root diameter $d f_2 = (Z_2 - 2 f_0) m_x - 2c$

$= (60 - 2) 12 - (2 \times 0.2 \times 12) = 691$ mm

Pitch diameter $= d_2' = d_2 = 720$ mm

Efficiency of worm gear drive :

$$\eta = \frac{\tan \nu}{\tan (\nu + \rho)}$$

$\tan \rho = \mu =$ Friction coefficient

$= 0.03$ for sliding velocity $V_s = 4.29$ m/sec (Refer figure 28.2)

$\rho = \tan^{-1} 0.03 = 1.72°$

$$\therefore \eta = \frac{\tan (15.255)}{\tan (15.255 + 1.72)} = 0.893 = 89.3\%$$

Specifications :

Sl. No.	Description	Worm	Wheel
1.	Material	Steel	Phosphor Bronze
2.	No. of teeth	6	60
3.	Module	12 mm	12 mm
4.	Reference Diameter	132 mm	720 mm
5.	Tip diameter	156 mm	744 mm
6.	Root diameter	103 mm	691 mm
7.	Length of worm	226 mm	–
8.	Face width of worm wheel	–	100 mm
9.	Centre distance	426 mm	
10.	Efficiency of drive	89.3 %	

Problem 28.2 : (In SI Units)

The input to worm gear shaft is 18 kW and 600 rpm. Speed of worm wheel is 30 rpm. The worm is to be of hardened steel and the wheel is made of chilled phospher bronze. Considering wear and strength, design worm and worm wheel.

Solution :

Power to be transmitted	$P =$	18 kW
Speed of worm	$n_1 =$	600 rpm
Speed ratio	$i =$	20
Material for worm	=	Hardened steel
Material for wheel	=	Phospher Bronze (Chilled)

Minimum centre distance between worm and worm wheel based on surface stress is given by,