ME 45 – STRENGTH OF MATERIALS

1. Strength of materials

Strength of materials is the science which deals with the relations between externally applied loads and their internal effects on bodies. The bodies are no longer assumed to be rigid and deformations are of major interest. During deformation the external forces acting upon the body do work. This work is transferred completely or partially into potential energy of strain. If the forces which produce the deformation of the body are gradually removed, the body returns or try to return to its original shape. During this return the stored potential energy can be recovered in form of external work. The main concern of the subject is regarding three S's, namely strength, stiffness and stability of various load carrying members

2. Simple stresses

When an external force is applied on a body, internal resistance is developed within the body to balance the effect of externally applied forces. The resistive force per unit area is called as stress. These internal forces are vectorial in nature and maintain an equilibrium with externally applied forces. It is particularly significant to determine the intensity of these forces, called stress on any arbitrarily selected section (oriented in a particular direction to fit the special requirement), as resistance to deformation and capacity of material to resist forces depend on these intensities.

In general these stresses acting on infinitesimal areas of any section vary from point to point. In general, they are inclined with respect to the sectional plane. But, in engineering practice it is customary to resolve the force perpendicular and parallel to the section investigated.

3. Types Of Stress

The stress can be classified in two categories.

a. Normal stress
b. Shear stress

The intensity of force perpendicular or normal to the section is called normal stress, denoted by a. As a particular stress holds true only at a point, we can define it mathematically as;

$$Ó = \lim_{\Delta A \to o} (\Delta F_n/\Delta A)$$

Where,

F_n is force acting normal to the section and A is its corresponding area.

These normal stresses generally causes tension or compression on the surface of section. The stresses which case tension are termed 'tensile stresses' and those which create compression are termed 'compressive stresses'.

On the other hand, component of intensity of force acting parallel to the plane of the sectional area is called shear stress. It is denoted by (tau). Mathematically we can define as;

$$r = \lim_{\Delta A \to o} (\Delta F_p/\Delta A) \text{ or } \lim_{\Delta A \to o} (\Delta F_t/\Delta A)$$

Where

F_p represents the force parallel to section cut and A represents the area of sectional plane. The shear stress causes the shearing at the sectional plane.

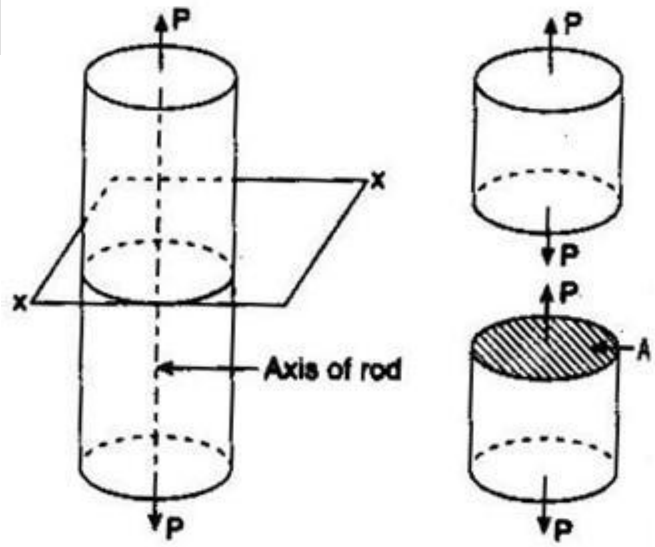
Note:

When stresses are multiplied by respective areas on which they act, it gives forces. It is the sum of these forces at any imaginary section that keeps the body in equilibrium. The SI unit of stress is N/m^2 or Pa.

4. Normal Stress

Such stress normally occurs in a straight axially loaded rod in tension or compression, at any plane perpendicular to the axis of the rod. The tensile or compressive stress acting on such a plane is maximum stress, as any other section which is not perpendicular to the axis of the rod provides a larger surface area for resisting the applied load or force.

The maximum stress is of most significance as it tends to cause the failure of the material.



If the rod is assumed weightless, two equal and opposite forces P are necessary. One at each end to maintain equilibrium. Since, the body as a whole is in equilibrium any part of it will also be in equilibrium.

Thus, from the definition of stress, the normal stress is

This normal stress is uniformly distributed over the cross-sectional area A. In general, force P is a resultant of a number of forces to one side of cut or another. Such a state of stress on an element or section is referred to as uniaxial stress.

Note:

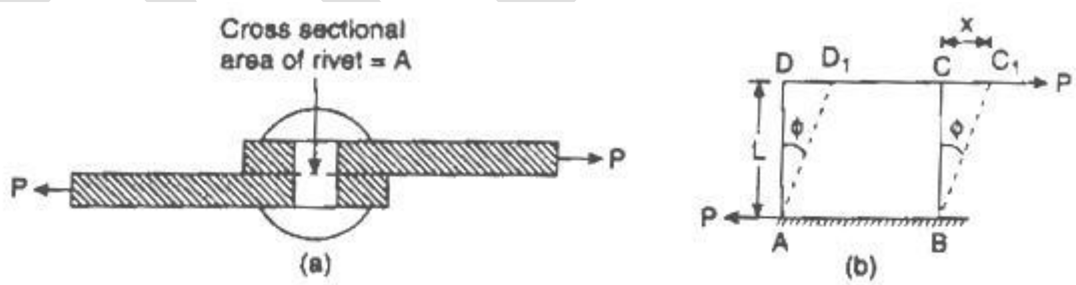
If the normal stresses try to create an extension in length of rod it is called tensile stress and in case it try to reduce the length of rod it is termed compressive stress.

5. **Shear Stress**

Such stresses are created when the forces are transmitted from one part of the body to the other by causing stresses in the plane parallel to the applied force. Assuming that the stresses that act in the sectional plane are uniformly distributed;

$$r = P/A \ 9 \ N / m^2)$$

Where x is shearing stress, P is the total force acting across and parallel to the cut and A is cross-sectional area of the sheared member. Since shear stresses actually are distributed in a non-uniform fashion across the area of the section the quantity given by above equation represents an average shearing stress.



When a body is subjected to load P consisting of two equal and opposite parallel forces not in same line, it tends to shear off across the resisting section. The stress induced in the body is called shear stress and the corresponding strain is called shear strain.

The most common occurrences of pure shear are in riveted and cotter joints.

In the resisting cross-section area parallel to load P is A, then the average shear stress is

$$r = P/A$$

In other case,

Consider of length L fixed at the bottom face AB, with unit width. Let, the force P be applied at face DC tangentially to the face AB. As a result of force P, the cuboid distorts from ABCD to ABC1D1 through an angle.

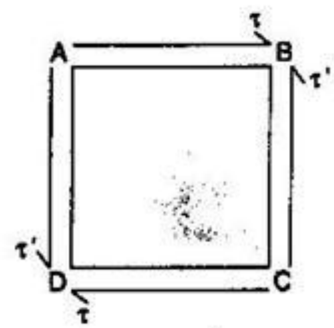
Then, we have

Shear stress, r =P/AB,, (Area = AB × 1)

6. Complementary Shear Stress

A shear stress is always accompanied by a balancing shear stress across the planes at right angles; the balancing stress is called complementary shearing stress.

Consider a rectangular block ABCD. of unit width; subjected to shear stress of intensity x on the faces or planes AB and CD.



Therefore, force acting on the faces AB and CD P = r.AB = r . CD

These forces will form a couple whose moment is x. AB. AD (i.e.. Force x Distance). To maintain the block in equilibrium there must be a restoring couple with equal moment. Let the shear stress x' be set up on faces AD and CB.

Therefore, force acting on faces AD and CB is

p = r ' .AD = z'CB

These forces will also form a couple whose moment is

r ' * AD * AB

Equating the two moments, r . AB . AD = r '.AD . AB

=> r = r '

Thus, every shear stress has an equal complementary shear stress.

7. Thermal stress

When a body is subjected to change in temperature its dimensions get changed. For metals when the temperature of a body is increased there is a corresponding increase in its dimensions. When the body is free to expand no stress develops.

But, in case the body is constrained to prevent the change in dimensions, the stresses develop in the material. These stresses are called as thermal stress. It may be tensile or compressive depending upon whether the contraction is prevented or extension is prevented.

To mathematically define the concept, let us consider a bar of length L placed between two supports to prevent the extension in its length.

If the temperature of bar is increased through Δr°C, the bar will be increased in length by an amount $\Delta L = L.\alpha.\Delta t$

Where α is the coefficient of thermal expansion.

Due to external conditions the bar can't expand by amount AL. Therefore, from Hooke's law,

Thermal stress ∞ Thermal strain

$$\acute{O} \propto (\Delta L/L)$$
$$\acute{O}_1 = E.\ \Delta L/L \qquad ...(1)$$

$$\Delta L = L.\ \alpha\Delta t \qquad ...(2)$$

Now

Where α = coefficient of thermal expansion

Δt = change in temperature

$\acute{O}_1$ = thermal stress

From equations (1) and (2), we have

$$\acute{O}_1 = E.\ L\ \alpha.\ \Delta t/L = E.\ \alpha.\ \Delta T$$
$$\acute{O}_1 = E.\ .\ \Delta T$$

The nature of the stress developed will be compressive. For the fall in temperature the stress developed will be E a Δt but its nature will be tensile.

Note: If the supports used to prevent the expansion yield by an amount 8, the total amount prevented will become (ΔL –δ).

Then,

Ó E. (ΔL - δ) / L = E (LαΔt - δ)/L

8. Concept and type of strains

It is a dimensionless quantity. This is a measure of deformation in the shape of a body on application of force. In engineering practice it is customary resolve the force perpendicular and parallel to the sections investigated. The strains can also be classified into two categories.
Normal or longitudinal strain
Shear strain

9. Longitudinal Strain

It could be defined as elongation per unit of length. Mathematically:

$$\in = \delta L/L \text{ or } l/L$$

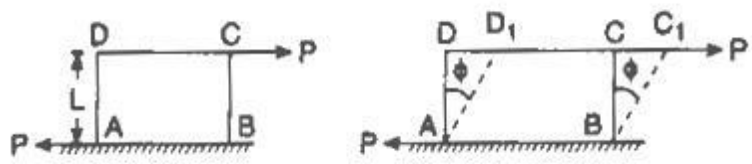
Where E is linear or normal strain, 8L is change in length L due to application of some axial force. It is a dimensionless quantity, but sometimes expressed as meter per meter.

10. Tensile strain:

When a member is subjected to equal and opposite axial tensile forces at its ends, it creates elongation in its length. The ratio of elongation in the length to the original length of the member is termed as tensile strain.

11. Compressive strain

When a member is subjected to equal and opposite axial compressive forces at its length. The ratio of this reduce d length to the original length of the member is termed as compressive strain.



Shear strain= Deformation/(Original length)
$=(CC_1)/L= ø$ ($ø$ is small, so $\tan ø = ø$)

12. Shear strain

When a body is subjected to two equal and opposite parallel forces not in same line it tends to shear off across the resisting section. As a result the cuboid (member) distorts from ABCD to $A_1BC_1D_1$ through an angle ø.

13. Volumetric Strain

It is defined as the ratio of change in volume of the speacment to the original volume of specimen The specimen is generally is cuboid shape.

Mathematically,

Volumetric strain ($\in_v$) = Change in volume of cuboid / Original volume of cuboid

$\in_v = \Delta V/V$

$\Delta V = V - V'$

Where V is original volume of cuboid and V' is final volume of cuboid.

14. **Thermal strain**

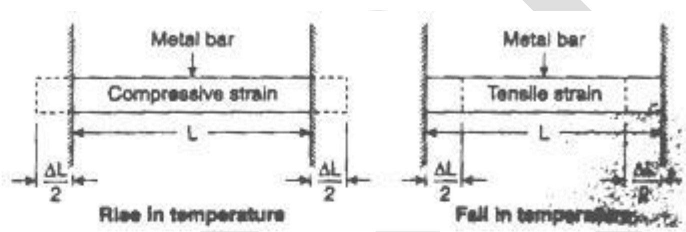
It is the ratio of extension prevented to original length.

$\in_r$ Extension prevented / original length = L.α.Δt / L= α Δt .
$\in_r = \alpha.\Delta T$

Where α = coefficient of thermal expansion

Δt = temperature rise or fall

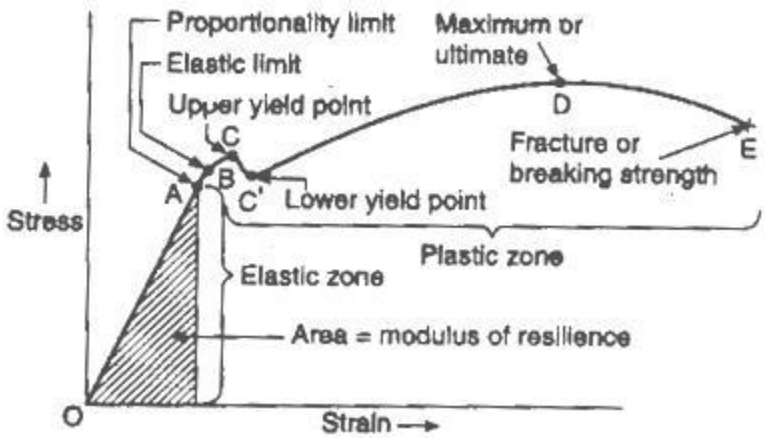
$\in_r$ = Thermal strain.



15. STRESS-STRAIN DIAGRAM FOR DUCTILE AND BRITTE MATERIALS

It is a curve between unit load or stress against unit elongation, known as strain. Stresses are usually computed on the basis of original area of a specimen before the test, although some transverse contraction of material always takes place. If the stress is computed by dividing the applied load by the corresponding actual area of the specimen, it is termed as true stress. A plot of true applied load by the corresponding actual area of the specimen, it is termed as true stress. A plot of true stress Vs strain is called true stress –strain diagram. Such a diagram is rarely used in practice.

The stress computed by dividing the applied load by the original area are termed an Engg. stress or stress. This is generally used for plotting the stress-stain curve.



16. Features of Stress-Strain Curve (for Ductile materials)

Upto a certain distance from the origin 0, the relationship between the stress and strain may be said to be linear for all materials. This generalization is stated by Hooke's law. It states that upto a point called the proportional limit the stress is directly proportional to strain i.e..

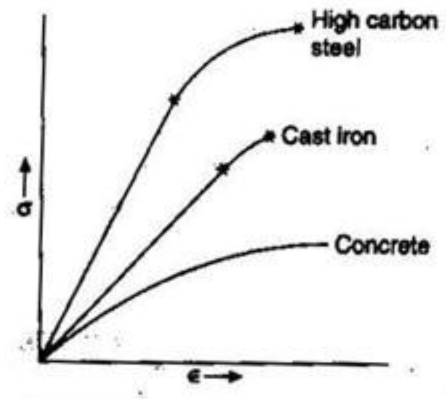
Stress ∝ Strain

Stress = E. Strain
E = Stress/Strain = P/A / δL / L

= P.L. / A.δL or P.L. / A.l

E is called as Young's modulus and is property of the material. Beyond proportionality limit 'A' stress is no longer proportional to strain. The stress-strain diagram will not be straight line after this point. Theelastic limit B' is the maximum stress that may be developed such that there is no permanent deformation after the removal of load. Thus, it is stress beyond which the material will not return to its original shape when unloaded but will retain a permanent deformation. Beyond 'B' elongation is more rapid and diagram becomes curved.

At 'C sudden elongation of the bar takes place without appreciable increase in stress (or Tensile force). This phenomenon is called yielding of metal and the corresponding stress is called yield point.Beyond 'C material recovers resistance for deformation and tensile force required increases with elongation, upto point 'D\ where the force attains a maximum value. The corresponding stress is called the ultimate strength of material. Beyond D, the elongation continues even on reduction of load and fracture finally occurs at a load correspoding to point E of the diagram and corresponding stress is called rupture strength.



The capacity of being drawn out plastically (permanently) before rupture is called the ductility of the material. It is generally measured in terms of percentage elongation or percentage reduction in area.

17. Stress-Strain Curve for Brittle Materials

Materials which show very small elongation before they fracture are called brittle materials such as CI, Tool steel, concrete etc

18. A mild steel rod 2.5 m long having a cross sectional area of 50 mm^2 is subjected to a tensile force of 1.5 kN. Determine the stress, strain, and the elongation of the rod.

Take E = 2×10^5 N/mm^2

Solution: Given that

Length of the rod 'L' = 2.5 m = 250 mm

Area of cross-section 'A' = 50 mm^2

Tensile force 'P' = 1.5 kN = 1.5×10^3 N

Young's Modulus 'E' = 2×10^5 N/mm^2

Stress = P/A= 1.5×10^3 / 50 =30N /mm^2

Since, E = Stress / Strain

Strain = Stress / E = 30 / 2 × 10^5 = 0.0015

Also, Strain = Elongation / Original length

= 0.0015 × 2500 = 0.375 mm.

19. A load of 100 kN is to be lifted with the help of a steel wire of 5 m length. The permissible limit of stress for wire is 150 N/mm2. Find the minimum diameter of the steel wire and the elongation at the permissible limit.

Take E = 2 X 10^5 N/mm^2

Solution: Given that

Tensile load 'P' = 100 kN

Length of wire 'L' = 5 m = 5000 mm

Stress 's' = 150 N/mm^2

E = 2 × 105 N/mm^2

s = P/A = P / π /4 d^2

⇒ 150 = 100 × 10^3 / π /4.d^2

⇒ d = 29.13 mm

Thus, the wire should have a minimum diameter of 29.13 mm.

Now, Elongation ' δL ' = PL/AE

⇒ δL = (100 × 10^3) × 5000 / 666.67 × 2 × 105 = 3.75 mm.

20. The following results were obtained in a tensile test on a mild steel specimen of original diameter 20 mm and guage length 50 mm. At the limit of proportionality the load was 100 kN and the extension was 0.05 mm. The specimen yielded at a load of 115 kN, and the maximum load withstood was 200 kN.

Solution: Using Hooke's Law; upto the limit of proportionality

E = PL / A . δL

E = 100 × 10^3 * 50 / π/4 $(20)^2$ 0.05 = 318.310 N/mm^2

Stress at limit of proportionality = P/A = 100,000 / π/4 $(20)^2$ = 318.3 N/mm^2

Yield stress = 115 × 103 / π/4 $(20)^2$ = 366 N/mm^2

Ultimate tensile stress = 200 × 10^3 / π/4 $(20)^2$ = 637 N/mm^2

Percentage elongation = 66.7 – 50 / 50 × 100 = 33.4%

Percentage contraction = π/4 $(20)^2$ – π/4 $(17.2)^2$ / π/4 $(20)^2$ * 100 = 26%

21. A double cover riveted butt joint is made to withstand a load of 250 kN. The plates to be joined are 20.5 cm thick. Rivets of 2 cm diameter are used, having permissible shear stress 100 N/mm^2.

Find the number of rivets required to avoid the failure of joint due to shear.

Solution: Since, we known that, in double cover joint each rivet will be in double shear.

Thus, load required to shear one river = r.2.A

= r.2 (π/4d^2)

= 100 × 2 (π/4 × 20^2)

= 62832 N

∴ Number of rivets required = 250000 / 62832 = 4

Thus, at least 4 rivets are required to avoid failure due to shear.

22. A copper rod 15 mm diameter, 0.8 m long is heated through 50oC. What is its extension when free to expand ? Suppose the expansion is prevented by gripping it at both ends, find the stress, its nature, and the force applied by the grips when:

(i)The grips do not yield

(ii)One grip yields back by 0.5 mm

[Take $\alpha_c = 18.5 \times 10^{-6}$ per ^oC, $E_C = 1.25 \times 10^5$ N/mm^2]

Solution: Given that Diameter of copper rod = 15 mm

Length of copper rod = 0.8 m

$\alpha_c = 18.5 \times 10^{-6}$ per ^oC

$E_C = 1.25 \times 10^5$ N/mm^2

Case (i): If the grips do not yield

$\delta L = L.\alpha.\Delta t$

$\delta L = (0.8 \times 10^3) \times (18.5 \times 10^{-6}) \times 50 = 0.74$ mm

Stress developed due to prevention

Ó t = E.α.Δt

$= (1.25 \times 10^5) \times (18.5 \times 10^{-6}) \times 50 = 0.74$ mm
$= 115.625$ N/mm^2 (Compressive)

$P = Ó_t \times A = 115.625 \times (\pi/4 \times (15)^2)$

$\Rightarrow P = 20.432$ kN

Case (ii): One group yield by 0.5 mm

Net expansion = $\delta L - \delta = 0.74 - 0.50 = 0.24$ mm

Thermal stress = $Ó'_t = E \times (L.\alpha.\Delta t - \delta / L)$

$= 1.25 \times 10^5 \times 0.24 / 800$

$Ó'_t = 37.5$ N/mm^2 (compressive)

And the force applied by the grips.

P = Ó'$_t$ × A = 37.5 × 176.71 = 66.27 kN

23. A steel tube with 2.4 cm external diameter and 1.8 cm internal diameter encloses a copper rod 1.5 cm diameter to which it is rigidly joined at each end. If, at a temperature of 10°C, there is no longitudinal stress, calculate the stresses in the rod and the tube when the temperature is raised to 200°C.

Given: $E_S = 210{,}000\ N/mm^2$

$E_C = 100{,}000\ N/mm^2$

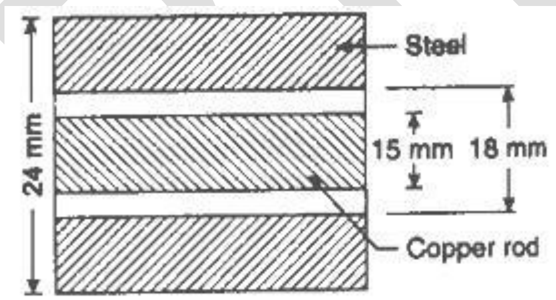
Coefficients of Linear Expansion

Solution: Applying compatibility condition

Ó$_1$ = E.α.Δt

ε_t = Ó$_t$/E = α.Δt

$\Rightarrow \varepsilon_t = \alpha_c.\Delta t$
$\varepsilon_t = \alpha_s.\Delta t$



$\varepsilon_c + e_s = (\alpha_c - \alpha_s).\Delta t$

$= (18 \times 10^{-6} - 11 \times 10^{-6}) \times (200 - 100)$

$\varepsilon_c + \varepsilon_s = 1.33 \times 10^{-3}$

From equilibrium condition

Compressive force on copper = Tensile force on steel

$P_C = P_S$

$\varepsilon_t.A_C.E_C = \varepsilon_s.A_S.E_S$

$\varepsilon_c = \varepsilon_s (A_S/A_C) \times (E_S/E_C) = \varepsilon_s [\pi/4[(24)^2 - (18)^2] / \pi/4 (15)^2] \times 210/100$

$\varepsilon_c = 2.352\ \varepsilon_s$

From equation (1) and (2)

$2.352\varepsilon_s + \varepsilon_s = 1.33 \times 10^{-3}$

$\varepsilon_s = 3.968 \times 10^{-4}$

$\varepsilon_c = 9.332 \times 10^{-4}$

Stress in steel tube; $Ó_s = \varepsilon_s.E_s$

$= (3.968 \times 10^{-4}) \times (210 \times 10^3)$

$= 83.32\ N/mm^2$ (Tensile)

Stress in copper rod; $Ó_c = \varepsilon_c.E_c$

$= (9.332 \times 10^{-4}) \times 100 \times 10^3$

= 93.32 N/mm2 (comp.)

24. ELASTIC CONSTANT- Modulus of Elasticity

The modulus of elasticity is defined by Hooke's law. This law states that up to the proportionality limit, the stress is proportional to strain or the experimental values of stress Vs strain lie on a straight line. Mathematically,

Stress ∝ Strain

$Ó \propto \varepsilon_c$

$Ó = E\varepsilon$

E is called modulus of elasticity of Young's modulus. It is property of material, s is normal stress and e is longitudinal strain. E has the same units as does the stress N/m^3.

25. ELASTIC CONSTANT- Modules of Rigidity

For many common engineering materials the modulus of elasticity in compression is very nearly equal to that found in tension.

In case of shear stress and strains the shear stress is proportional to shear strain, and the Hooke's law can be defined as:

$r \propto \gamma$

$r = G.\gamma$

where G is called the modulus of rigidity or shearing modulus of elasticity or modulus of transverse rigidity. r is shear stress and γ is shear strain measured in radians.

26. Bulk Modulus

If a cuboid is subjected to three mutually perpendicular normal stresses $Ó_x$, $Ó_y$, and $Ó_z$ of equal intensity on its faces, so that it gets volumetric strain of ε_v. Then the bulk modulus is defined as:

K= volumetric stress / volumetric strain

Volumetric strain ε_v = change in volume of cuboid / original volume of cuboid

Volumetric stress or octahedral normal stress $Ó = Ó_x + Ó_y + Ó_z / 3$

$K = Ó_x + Ó_y + Ó_z / 3.\varepsilon_v$

where, $Ó_x$, $Ó_y$, $Ó_z$ may or may not be principal stresses. \

27. Poisson's Ratio

When a bar is subjected to normal tensile stress, it produces tensile strain in the direction of normal stress, and the body elongates in the direction of stress. There is a contraction in cross-sectional area of the body. The contraction produces a lateral strain defined as:

Lateral strain = D – D' / D

Longitudinal strain = L – L' / L

Within the proportionality limit, it is found that lateral strain is proportional to longitudinal strain.

Lateral strain a longitudinal strain

Lateral strain = v (Longitudinal strain)

v = Lateral strain / Longitudinal strain

For most of the materials its value ranges from 0.25 to 0.35. For cork it is nearly zero.

28. Modules of Resilience

The work done on a unit volume of material, as a simple force gradually increased from zero to such a value that the proportional limit of the material is reached, is defined as the modulus of resilience. This may be calculated as the area under the stress-strain curve from the origin upto the proportional limit. Its unit in SI system is $N.m/m^3$.

It could also be defined as the ability to absorb energy in the elastic range.

29. Modulus of Toughness

It is the work done on a unit volume of material as a simple tensile force is gradually increased from zero to a value causing rupture. This may be calculated as the entire area under the stress-strain curve from origin to rupture.

Tangent Modulus

The rate of change of stress with respect to strain is known as the tangent modulus of the material. It is given by;

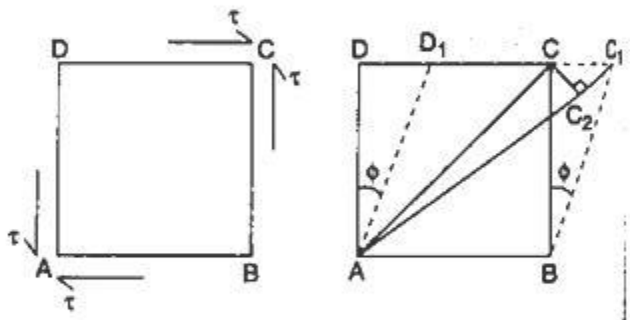
E_t = dÓ / dε

30. Relationship Between Elastic Constants

Consider a square block ABCD of side 'L' and of thickness unity perpendicular to the plane of drawing shown in figure.

Let the block be subjected to shear stresses of intensity t as shown in figure. Due to these stresses the cube is subjected to some distortion, such that the diagonal AC will get elongated and diagonal BD will be shortened.

Let the shear stress r cause shear strain Ø as shown in figure; so that AC become AC_1



Before distortion After distortion

Strain of AC = AC_1 – AC / AC = C_1C_2 / AC

= CC_1 cos 45^o / AB √2 = CC_1 / 2 AB

= 1/2 (CC_1 / AB) = 1/2 Ø

Thus, we see that the linear strain of diagonal AC is half of the shear strain and is tensile in nature. Similarly, it can be proved that the linear strain of diagonal BD is also equal to half of the shear strain, but is compressive in nature.

Now the linear strain of diagonal

AC / Ø /2 = r/2G ...(1)

where t is shear stress, and G is modulus of rigidity. The tensile strain on the diagonal AC due to tensile stress on diagonal AC is:

Strain on AC = r/E

and the tensile strain on diagonal AC due to compressive stress on diagonal BD

= v.r/E

Combined effect of the above two stresses on diagonal AC is:

r/E + v. r/E

= t/E (1 + v) ...(2)

Equating equations (1) and (2)

r/2G = r/E (1 + v)

E = 2G (1 + v) ...(3)

where G is modulus of rigidity and E is modulus of elasticity.

31. Relation between Young's Modulus and Bulk Modulus

Let
K = bulk Modulus = Fluid Pressure or Hydrostatic Pressure / Volumetric strain

K = Ó/e_v = Ó/3 Ó/E (1 – 2v) = E/3(1 – 2v)

K = 3K (1 – 2v) ...(4)

Using equations (3) and (4),

From (3),

v = E/2G = 1

Putting in (4),

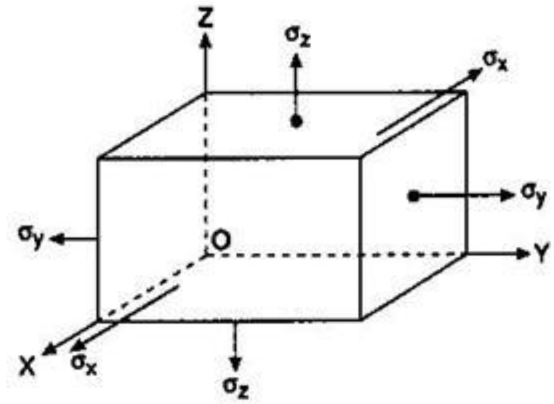
E = 3K[1 – 2 [E/2G – 1]]

E = 3K[3 – E/G]

E = 9KG/3K + G

32. GENERALIZED HOOKE'S LAW

A general state of stress acting upon an isotropic body as shown in figure can be considered. The block has dimensions a, b and c along X, Y and Z directions respectively. It is acted upon by tensile stresses uniformly distributed on all faces. The shearing stress are neglected, as it is known experimental fact that the strains caused by normal stresses are independent of small shearing deformations.



Thus, to find the change in length of the block in X-direction we can make use of the principle of superposition, which states that the resultant stress or strain in a system due to several forces is the algebraic sum of their effects when separately applied. The stress in X-direction causes a positive strain $e_x' = sx/E$ along X-direction, while each of the positive stresses in Y and Z directions causes a negative strain in X-direction as a result of Poisson's effect.

Therefore, the total strain along X-direction will be

$\varepsilon_x = + \text{Ó}_x/E - v\,\text{Ó}_y/E - v\,\text{Ó}_z/E = 1/E\,(\text{Ó}_x - v(\text{Ó}_y + \text{Ó}_z))$ ...(1)

Similarly,

$\varepsilon_y = + \text{Ó}_y/E - v\,\text{Ó}_x/E - v\,\text{Ó}_z/E = 1/E\,(\text{Ó}_y - v(\text{Ó}_x + \text{Ó}_z))$ ...(2)

$\varepsilon_z = + \text{Ó}_z/E - v\,\text{Ó}_x/E - v\,\text{Ó}_y/E = 1/E\,(\text{Ó}_z - v(\text{Ó}_x + \text{Ó}_y))$ ...(3)

The above equations 1, 2 and 3 represent the generalized Hooke's law. The application of equations 1, 2 and 3 is limited to isotropic materials in the elastic range.

Note:
In case a particular stress is compressive, the sign of corresponding term changes.

The total elongation along X-direction = $\Delta x = \varepsilon_x.a$.

33. Dilation

The change in volume per unit volume is referred to as dilation, in the infinitesimal strain theory.

$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$

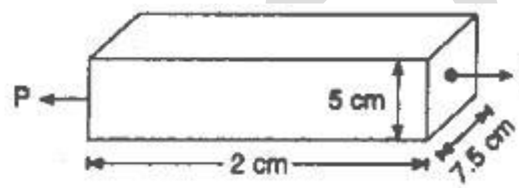
Based on generalized Hooke's law, the dilation can be found in terms of stresses and material constants. This yields

$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$

$\Rightarrow \varepsilon_v = 1 - 2v/E\ (Ó_x + Ó_y + Ó_z)$

34. A 2 m long rectangular bar of 7.5 cm × 5 cm is subjected to an axial tensile load of 1000 kN. Bar gets elongated by 2 mm in length and decreases in width by 10×10^{-6} m. determine the modulus of elasticity E and Poisson's ratio v of the material of bar.

Solution: Given that:



Length of bar 'L' = 2 m

Width 'B' = 7.5 cm

Height 'H' = 5 cm

Axial load applied P = 1000 kN
ΔL = 2 mm

$\Delta B = 10 \times 10^{-6}$ m

Normal stress s = P/A

A = B × H

$= (7.5 \times 10^{-2})\ (5 \times 10^{-2})$

$= 0.00375\ m^2$

⇒ Ó = 1000 / 0.00375 kN / m^2

= 266.67 MN/m^2

Longitudinal strain ε = ΔL / L

⇒ε = 2 × 10^{-3} / 2 = 10^{-3} or 0.001

From law of proportionality Ó = E.ε

⇒ E = Ó/ε

Where E is modulus of elasticity,

E = 266.67 × 10^6 / 0.001 = 2.67 × 10^{11} N/m^2

Or = 266.67 GPa

Lateral strain = ΔB / B

= 10 × 10^{-6} / 7.5 × 10^{-2}

= 1.333 × 10^{-4}

Poisson's ratio of the material of bar is

v = lateral strain / Longitudinal strain = 1.333 × 10^{-4} / 0.001

v = 0.1333

35. A 500 mm long bar has rectangular cross-section 20 mm × 40 mm. The bar is subjected to:

40 kN tensile force on 20 mm × 40 mm faces.

(ii) 200 kN compressive force on 20 mm × 500 mm faces.

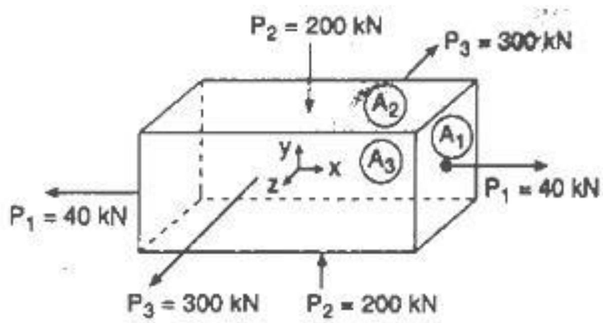
(iii) 300 kN tensile force on 40 mm × 400 mm faces.

Find the changes in dimensions and volume, if

E = 2 × 105 N/mm^2 and Poisson's ration v = 0.3

Solution: Given that $E = 2 \times 10^5\ N/mm^2$

And, $v = 0.3$



Area of faces: $A_1 = 20 \times 40 = 800\ mm^2$

$A_2 = 20 \times 500 = 10,000\ mm^2$

$A_3 = 500 \times 40 = 20,000\ mm^2$

$Ó_1 = P_1/A_1 = 40 \times 10^3 / 800$

$= 50\ N/mm^2$

$Ó_2 = P_2/A_2 = -200 \times 10^3 / 10,000 = -20\ N/mm^2$

$Ó_3 = P_3/A_3 = 300 \times 10^3 / 20,000 = 15\ N/mm^2$

$\varepsilon_x = Ó_1/E - v/E\ (Ó_2 + Ó_3)$

$\varepsilon_x = 50 / 2 \times 10^5 - 0.3/2 \times 10^5\ (-20 + 15) = 2.575 \times 10^{-4}$

Change in length $\delta y = 40 \times 1.975 \times 10^{-4} = -0.0079$ mm

$\varepsilon_z = Ó_3/E - v/E\ (s_1 + s_2)$

$= 15 / 2 \times 10^5 - 0.3 / 2 \times 10^5\ (50 - 20)$

$= 3 \times 10^{-5}$

Change in length,

$\delta z = 3 \times 10^{-5} \times 20 = 00006$ mm

Initial volume = 500 × 40 × 20 = 4 × 10^5 mm^3

Final volume = (500 + 0.12875) (40 – 0.0079) (20 + 0.0006)

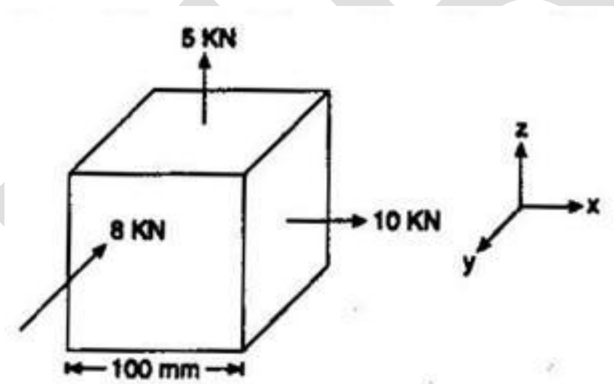
= 400035.98 – 400000

= 35.98 mm^3

36. A steel cube of 100 mm side is subjected to a force of 10 kN (Tension), 8 kN (compression) and 5 kN (Tension) along X, Y and Z direction respectively. Determine the change in volume of the block.

Take E = 2 × 10^{11} N/m^2 and v = 0.30

Solution: Given that:



Length of each side = 100 mm

Area of each plane = 10^4 mm^2

Volume of cube = 10^6 mm^3

Therefore, $Ó_x$ = 10 × 10^3 / 10^4 = 1N/mm^2

$Ó_y$ = 8 × 10^3 / 10^4 = 0.8 N/mm^2

$Ó_z$ = 5 × 10^3 / 10^4 = 0.5 N/mm^2

Thus,

$\varepsilon_x = \acute{O}_x/E + v.\acute{O}_y/E - v\ \acute{O}_z/E$

$\varepsilon_y = \acute{O}_y/E - v.\acute{O}_x/E - v\ \acute{O}_z/E$

$\varepsilon_z = \acute{O}_z/E - v.\acute{O}_x/E - v\ \acute{O}_y/E$

Putting the values we get,

$\varepsilon_x = 1/2 \times 10^5\ [1 + 0.3 \times 0.8 - 0.3 \times 0.5] = 1.09/2 \times 10^{-5}$

$\varepsilon_y = 1 / 2 \times 10^5\ [-0.8 - 0.3 \times 1 - 0.3 \times 0.5] = -1.25/2 \times 10^{-5}$

$\varepsilon_z = 1 / 2 \times 105\ [0.5 - 0.3 \times 1 + 0.3 \times 0.8] = 0.44/2 \times 10^{-5}$

37. Show that if E is assumed to be correctly determined, an error of 1% in the determination of G will involve an error of about 6% in the calculation of Poisson's ratio (v) when its correct value is 0.20.

Solution: Since we have

$E = 2G(1 + v)$ ...(1)

1% error in G will result $G' = 1.01\ G$

And the value of Poisson's ratio will become v'.

Then, $E = 2.G'(1 + v')$ ...(2)

Since E is determine correctly; from equations (1) and (2),

$2G(1 + v) = 2G'(1 + v')$

$2G(1 + v) = 2 \times 1.01\ G(1 + v')$

$1 + v = 1.01 + 1.01\ v'$

Subtracting both sides from v'.

$v' - v = -20.01 - 0.01\ v' = -0.01\ (1 + v')$ ...(3)

The percentage error in v is; using equation (3)

$v' - v / v \times 100 = -0.01\ (1 + v') / v \times 100$

$= -0.01\ (1 + 0.20) / 0.20 \times 100$

$= -6\%$

Also, we can use differential calculate to solve the problem.

Since, $E = 2G(1 + v)$

$dE = d(2G(1 + v)) = 2Gd(1 + v).d(2G)$

$0 = 2G.dv + (1 + v).2.dG$

$\Rightarrow dv = -dG(2\ (1 + v)) / 2$

$\Rightarrow dv/v \times 100 = -dG/G\ (1 + v / v) \times 100$

$= -1/100\ (1 + 0.20 / 0.20) \times 100 = -6\%$

38. Stress In Bars Of Different Lengths And Of Different Diameters

Consider a bar of different lengths and of different diameters as shown in figure. Let this bar be subjected to an axial load P.

Let L_1 = Length of section 1

L_2 = Length of section 2

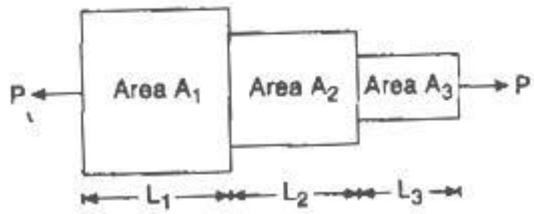
L_3 = Length of section 3

A_1 = Cross-sectional area of section 1

A_2 = Cross-sectional area of section 2

A_3 = Cross-sectional area of section 3

P = Axial sectional acting on bar



Though each section is subjected to the same load P, yet the stresses, strains and changes in length will be different for each section. The total change in length will be obtained by adding the changes in length of individual sections.

Stress on section 1 = $Ó_1 = P/A_1$

Similarly, Stress on section

2 = $Ó_2 = P/A_2$

Since on section 3 = $Ó_3 = P/A_3$

Strain of section 1 = $ε_1 = Ó_1/E = p/A_1E$

Similarly $ε_2 = P / A_2E$

$ε_3 = P / A_3E$

Change in length of section 1 = $ε_1.L_1 = P.L_1/A_1E$

Similarly, Change in length of section 2 = $ε_2.L_2 = P.L_2/A_2E$

Change in length of section 3 = $ε_3.L_3 = P.L_3/A_3E$

Total change in length of bar

$δL = ε_1.L_1 + ε_2.L_2 + ε_3.L_3$

$δL = P/E [L_1/A_1 + L_2/A_2 + L_3/A_3]$

In case the Young's modulus of different sections is different, then total change in length is given by

$δL = P [L_1/E_1A_1 + L_2/E_2A_2 + L_3/E_3A_3]$

Or $δL\ Σ^{k=n}{}_{k=1}\ P_k.\ L_k/A_k.\ E_k$ = ($p_1L_1/A_1E_1 + P_2L_2/A_2E_2 + P3L_3/A_3E3$ + ...)

Where P_k = Force acting on kth section

L_k = Length of kth section

A_k = Area of cross-section of kth section.

E_k = Modulus of Elasticity for kth section.

Note:
In case of composite bars, the ratio of their modulae of elasticity is called modular ratio of the two materials and is denoted by 'm'.

$\Rightarrow m = E_1/E_2$

In case, the equations of statics are not sufficient to solve the problem, the problem is called statically indeterminate problem. For solving such problems, the deformation characteristics of the structure are also taken into account alongwith the statical equilibrium the deformation characteristics, are called compatibility equations.

39. Stress in Bars of Uniformly Tapering Circular Cross-Section

Consider a circular bar of uniformly tapering section as shown in figure.

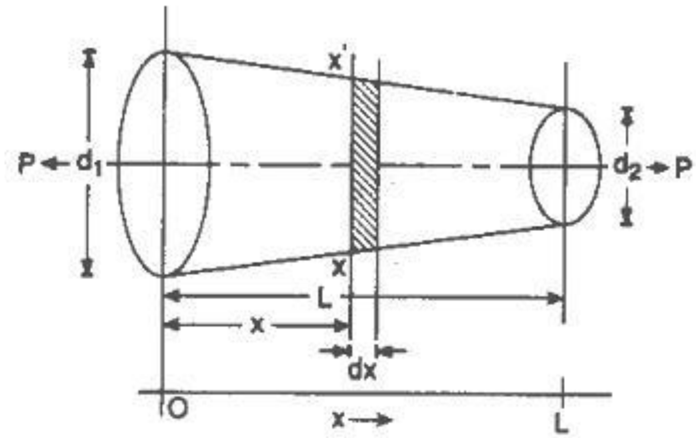
Let P = Force applied on bar (Tensile) (N)

L = Length of the bar (m)

d_1 = Diameter of the bigger end of the bar (m), and

d_2 = Diameter of the smaller end of the bar (m)

Now, consider a small element of length dx, of the bar, at a distance x from the bigger end as shown in figure.



Diameter of the bar at a distance x, from the origin,

$d = d_1 – (d_1 – d_2) / L.x$ ...(1)

Let $k = (d_1 – d_2) / L$

$\Rightarrow d = d_1 – kx$

Thus, cross-sectional area of the bar at this section.

$A = \pi/4 (d_1 – kx)^2$ ...(2)

Stress: $Ó = P/\pi/4 (d_1 – kx)^2 = 4P/\pi(d_1 – kx)^2$ ...(3)

And Strain $\varepsilon = Ó/E = 4P/\pi E(d_1 – kx)^2$ ...(4)

Elongation of the elementary length = ε = dx = $4P.dx / \pi E(d1 - kx)^2$

Therefore, total extension of the bar may be found out by integrating the equation (5) between the limits x = 0; x = L.

$$\int_0^x \varepsilon.dx = \delta L = \int_0^L \frac{4.Pdx}{\pi E\,(d_1 - kx)^2}$$

$$= \frac{4P}{\pi E}\int_0^L \frac{dx}{(d_1 - kx)^2}$$

$$= \frac{4P}{\pi E}\left[\frac{(d_1 - kx)^{-1}}{-1(-k)}\right]_0^L$$

$$= \frac{4P}{\pi EK}\left[\frac{1}{d_1 - kx}\right]_0^L$$

Substituting the value of k,

$$\delta L = \frac{4P}{\pi E\left(\frac{d_1 - d_2}{L}\right)}\left[\frac{1}{d_1 - \left(\frac{d_1 - d_2}{L}\right).L} - \frac{1}{d_1}\right] = \frac{4PL}{\pi E\,(d_1 - d_2)}.\left[\frac{1}{d_2} - \frac{1}{d_2}\right]$$

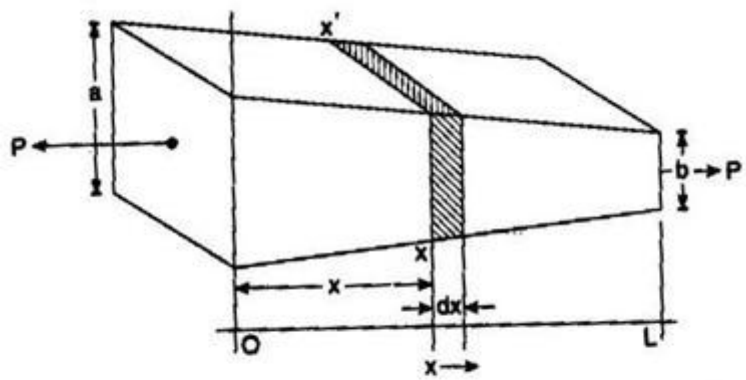
$$= \frac{4PL}{\pi E\,(d_1 - d_2)}\left[\frac{1}{d_2} - \frac{1}{d_1}\right]$$

$$= \frac{4PL}{\pi E\,(d_1 - d_2)}\left[\frac{d_1 - d_2}{d_1 d_2}\right]$$

$$\delta L = \frac{4PL}{\pi E\,d_1.d_2} = \frac{PL}{A_{gm}.E}, \qquad where\ A_{gm} = \sqrt{A_1.A_2}$$

40. Stress In Bars of Uniformly Tapering Rectangular Cross-section

Consider a bar of constant thickness and uniformly tapering in width from one end to the other end as shown in figure -



Let P = Axial load on the bar (Tensile) (N)

L = Length of bar (m)

a = Width at bigger end (m)

b = Width at smaller end (m)

E = Young's modulus

t = Thickness of bar

Now, consider a section at a distance x from origin O.

Width of bar at section x – x' = a – (a – b) / L. x

= a – kx

Where k = (a – b/L)

Thickness of bar at section x – x' = t

Area of section x – x' = (a – kx) × t

⇒ Stress on the section x – x' = P/(a – kx).t

and, strain at section x – x' = Ó/E

⇒ Extension of the small elemental length dx = strain × dx

= (P/(a - kx).t/E dx = P.dx/E(a - kx).t

Total extension of the bar is obtained by integrating the above equation between the limits x = 0 and x = L.

$$\int_0^L \varepsilon . dx = \delta L = \int_0^L \frac{P}{E(a-kx)\,t} . dx$$

$$= \frac{P}{Et}\int_0^L \frac{dx}{(a-kx)}$$

$$= \frac{P}{E.t} In\,[(a-kx)]_0^L \left(-\frac{1}{k}\right)$$

$$= -\frac{P}{Et.k}[In(a-kL) - In\,a]$$

$$= \frac{P}{Etk}\,[In\,a - In\,(a-kL)\,]$$

$$= \frac{P}{Etk}\left[In\left(\frac{a}{a-kL}\right)\right]$$

Putting the value of K;

$$\delta L = \frac{P}{Et\left(\frac{a-b}{L}\right)}\left[In\left(\frac{a}{a-\left(\frac{a-b}{L}.L\right)}\right)\right]$$

$$\delta L = \frac{P.L}{E.t(a-b)} In\left(\frac{a}{b}\right). = \frac{PL}{A_{tm}.E} \qquad where;\, A_{tm} = \frac{A_a - A_b}{In\left(\frac{A_a}{A_b}\right)}$$

A_{tm} is logrithimic mean area.

41. The Cross-section of Bar of Uniform Strength

Consider, the same stress s is developed through the bar, under an applied axial load and the self weight of bar,

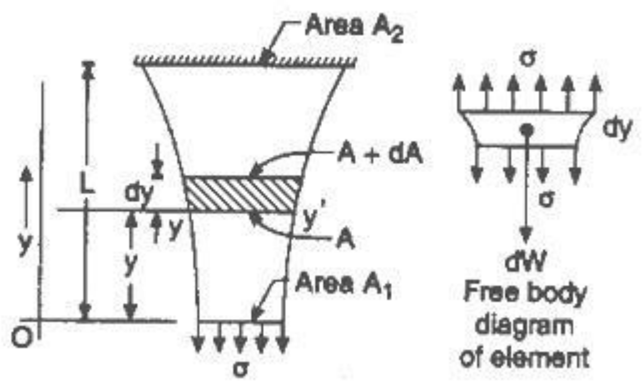
Let L = length of bar (m)

A_1 = Area of lower end (m)

A_2 = Area of upper end (m)

Ó = Stress developed throughout the bar (W/m^2)

Now, consider a small element of axial length dy at a distance y from the smaller end. Let area of cross-section be A at section Y – Y'.



For an element of length dy + (Tensile stress at section Y – Y' × area at section Y – Y') = (Tensile stress at section at (Y – Y') × Area of section at y + dy)

⇒ dW + Ó.A = Ó (A + dA)

But   dW = p.g.(A.dy)

where p = density of maternal of bar.

And, g = Acceleration due to gravity

⇒ pgA.dy + Ó.A = Ó.A + Ó.dA

$\Rightarrow dA/dy = p.g.A/Ó$

$\Rightarrow \int^{A}_{A1} dA/A = g.p./ Ó . \int^{y}_{o} dy$

$\Rightarrow In (A/A_1) = g.p./Ó.(y – 0)$

$In (A/A_1) = g.p./Ó . y$

$\Rightarrow A/A_1 = e^{(g.p/Ó.y)}$

Hence, $A = A_1.e^{pgy/Ó}$

Since p.g = w = weight density of specific weight.

$A = A_1 . e^{w.y/Ó}$ ...(1)

Also applying the limits y = 0; $A = A_1$ and y = L; $A = A_2$

$A_2 = A_1. e^{wL/Ó}$ ...(2)

The area A can also be found in terms of A_2 by dividing equation (1) by equation (2),

$A/A_2 = e^{-w(L-y)/Ó}$

$A = A2. e^{-w(L-y)/Ó}$ ...(3)

Extension in the bar can now be calculated.

Let e.dy be extension in small length dy. Then
$\int_0^L \varepsilon.dy = \delta L= \int_0^L \sigma / E.dy$

$\delta L = \sigma / E (L - o)$

$\delta L = \sigma.L / E$

42. Composite Section

A composite section is made up of two or more different materials joined in such a manner that system in elongated or compressed as a single unit. So that,

(i) the change in length is same for all materials i.e., the strains induced are also equal.

$\varepsilon_1 = \varepsilon_2 = \varepsilon_3 ...$

(ii) The sum of loads carries by individual members is equal to total load applied on the member.

$P = P_1 + P_2 + ...$

$\Rightarrow P = Ó_1 A_1 + Ó_2 A_2 + ...$

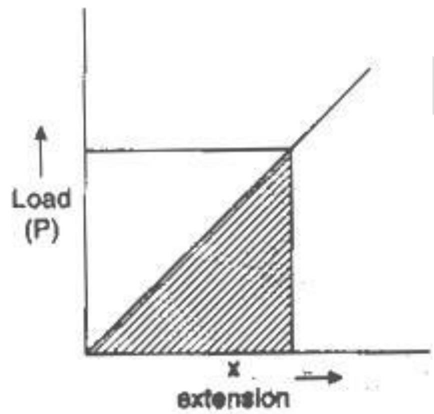
And $Ó_1/E1 = Ó_2/E_2$

43. Strain Energy due to Static Load

The strain energy is the energy stored in a body due to the work done by the applied load in staining it. For 'static' load P, the change in length x is proportional to the load.

Work done for gradually applied load = Strain energy (U)

$\Rightarrow U = 1/2\ P.x$



For a bar of uniform area of cross-section A and length L.

$Ó = P/A \Rightarrow P = Ó/A$

$\varepsilon = x/L$

$E = Ó/\varepsilon$

$\Rightarrow E = P.L./A.x \Rightarrow x = PL/AE$

$U = 1/2\ (P.L/A.E). P = 1p^2.L/AE = U = 1/2\ p^2L/AE$

Or $U = 1/2\ (P/2)2\ .\ AL/E = 1/2\ Ó^2.AL/E$

Since A.L = volume (V)

$U = 1/2\ Ó^2v/E$

Thus, strain energy per unit volume is

$U/V = 1/2 \; \acute{O}^2/E$

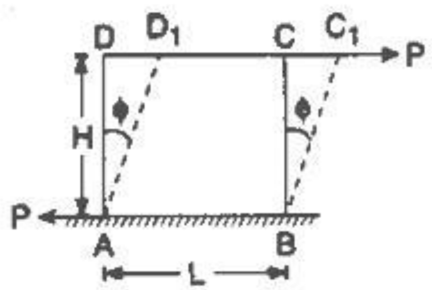
The quantity strain energy per unit volume is called modulus of resilience, and the maximum energy stored in a body at the elastic limit is known as proof resilience.

Strain energy is always a positive quantity with units of work.

44. Strain Energy Due to Simple Shear

Consider a small rectangular block of length L, height H, and unit width. The block is fixed at the bottom face AB.

Let a shear force P is applied on the top face CD and hence it moves a distance equal to CC_1.



Now shear stress r = P/A = P / L × 1   (unit width)

⇒ P = r × L × 1

And, shear strain   Ø = CC_1 / CB

∴ CC1 = CB.Ø

If the shear force P is applied gradually, then
work done by gradually applied shear force = Average load × Distance

= P/2 × CC1 = 1/2 (r × L × 1) × CB.Ø

= 1/2 r (L . 1 . H).Ø

= 1/2 r × Ø (volume of block)

= 1/2 r × r/G (volume of block)

$= 1/2\ r^2/g\ (V)$

Since, the work done is equal to the strain energy stored

∴ Strain energy stored

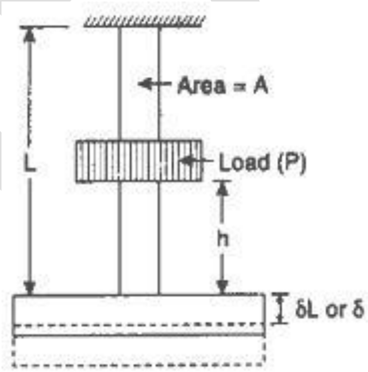
$U = r^2/2G \times V$

Strain energy per unit volume $U/V = r^2/2G$.

45. Strain energy due to impact loading

A load is said to be impact if it has kinetic energy and strikes instantaneously on a body. If the body is loaded instantaneously with the load having no kinetic energy, the load is said to be sudden. The impact or sudden load condition may occur in tension, compression, torsion or bending or combinations of these.

Whenever impact is produced, the energy of striking load is instantaneously given to the body struck. Since the body cannot be isolated form the surrounding. But, it is difficult to find the exact energy given to the body struck. In analyzing problems related to impact loading, it is assumed that whole of the energy is given to the body subjected to impact and there is no energy lost to the surroundings.



Let us assume;

A = Area of crossleft-section of rod.

L = Length of rod upto collar

P = Load falling height h on collar

δ = Extension in length of rod due to impact.

Now Energy of impact = Potential energy of the falling load

= P(h + δ) ...(1)

This energy is given to the rod of length L (without any energy dissipation to the surroundings), if Ói is the instantaneous stress produced due to impact, the strain energy 'U' produce by it is given by;

U = 1/2 × $Ó_t^2$/E × (A × L) ...(2)

Equating the loss of potential energy to the strain energy stored by the rod.

P(h + δ) = 1/2 × $Ó_i^2$/E (A × L) ...(3)

The extension 'd' in length due to stress $Ó_i$ is, δ = $Ó_i$.L/E ...(4)

⇒ From (3) and (4),

$$P\left(h + \frac{\sigma_i . L}{E}\right) = \frac{1}{2} . \frac{\sigma_i^2}{E} (A \times L)$$

$$Ph = \frac{P.\sigma_i . L}{E} = \frac{\sigma_i^2}{2E} (AL)$$

$$\Rightarrow \quad \frac{\sigma_i^2}{2E} - \frac{P.\sigma_i}{A} = \frac{2P.h.E}{A.L}$$

$$\Rightarrow \quad \sigma_i^2 - \frac{2P.\sigma_i}{A} = \frac{2P.h.E}{A.L}$$

Adding P^2/A^2 to both sides of equation,

$$\sigma_i^2 - \frac{2.P.\sigma_i}{A} + \frac{P^2}{A^2} = \frac{P^2}{A^2} + \frac{2.P.h.E}{A.L}$$

$$\left(\sigma_i - \frac{P}{A}\right)^2 = \frac{P^2}{A^2} + \frac{2.P.h.E}{A.L}$$

$$\Rightarrow \qquad \left(\sigma_i - \frac{P}{A}\right) = \sqrt{\frac{P^2}{A^2} + \frac{2P.h.E}{A.L}}$$

$$\sigma_i = \frac{P}{A} + \sqrt{\frac{P^2}{A^2} + \frac{2P.h.E}{A.L}} \qquad \ldots(5)$$

Note:

(1) If δ is very small in comparison to h.
Loss of potential energy = P.h

Equating the loss of energy to the strain energy.

p.h = $Ó_i^2$ /2E . AL

$Ó_i$ = √2p.h.E/A.L ...(6)

(2) If h = 0; from equation (5) $Ó_i$ = 2P/A

It is the case of suddenly applied load from eqn (4).

δ = (2P/A). L/E
Thus, stress produced by a suddenly applied load is twice the static stress.

46. Strain Energy due to Principal Stresses

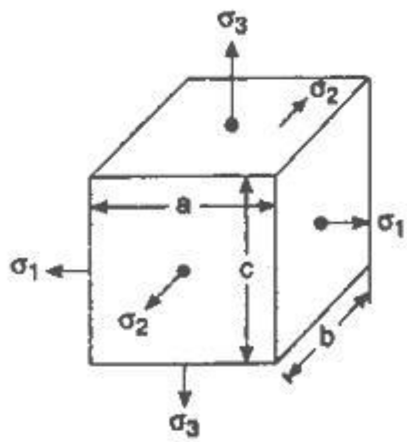
Since, any compound stress condition can be reduced to the condition of three principal stresses, we can assume it a generailsed case.

Consider; a block of length, breadth, and height of a, b and c respectively as shown in figure.

Assume v be Poisson's Ratio of the material.

Therefore, extension of the block in direction of $\sigma_1 = \varepsilon_1.a$

Or $\delta L_1 = 1/E\ [\sigma_1 - v(\sigma_2 + \sigma_3)].a$ ...(1)



⇒ Strain energy due to σ_1 = 1/2 [Load due to σ_1 along σ_1]* δL_1

$= 1/2\ [\sigma_1.(b.c)].a/E\ [\sigma_1 - v(\sigma_2 + \sigma_3)]$

$= 1/2E\ [\sigma_1^2 - v\ (\sigma_1.\sigma_2 + \sigma_1\sigma_3)]\ (a.b.c)$

$= 1/2E\ [\sigma_1^2 - v\ (\sigma_1\sigma_2 + \sigma_1\sigma_3)]V$ ...(2)

Similarly, by symmetry

Strain energy due to $\sigma_2 = 1/2E\ (\sigma_2^2 - v\ (\sigma_2\sigma_3 + \sigma_2\sigma_1)).V$ ...(3)

Strain energy due to $s_3 = 1/2E\ (\sigma_3^2 - v\ (\sigma_3.\sigma_2 + \sigma_3\sigma_1))V$ ...(4)

Total strain energy for volume 'V' = sum of strain energies due to σ_1, σ_2 and σ_3

$U = 1/2E\ [(\sigma_1^2 - v(\sigma_1\sigma_2 + \sigma_1\sigma_3) + (\sigma_2^2 - v(\sigma_2\sigma_1 + \sigma_2\sigma_3)) + \sigma_2^3 - v(\sigma_3\sigma_1 + \sigma_3\sigma_2))].1V$

$\Rightarrow U = 1/2E\ [\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2v\ (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)].V$ ...(5)

Note:
In case any of the principal stress is compressive, negative sign should be taken in account.

In case of simple shear $\sigma_1 = r$

$\sigma_2 = -r$

$\sigma_3 = 0$

Putting these values in equation (5) of article.

$U = 1/2E\,[r^2 + (-r)^2 + 0 - 2v\,(r(-r) + 0 + 0)].V$

$= 1/2E\,[r^2 + r^2 + 2v(r^2)]V$

$= 1/2E\,[2(r^2 + vr^2)]V$

$= 1/2E.2r^2\,(1 + v).V$

$U = r^2/E\,(1 + v).V$

Putting $E = 2G(1 + v)$,

$U = r^2\,(1 + v).V / 2G(1+v)$

$U = r^2.V/2G$

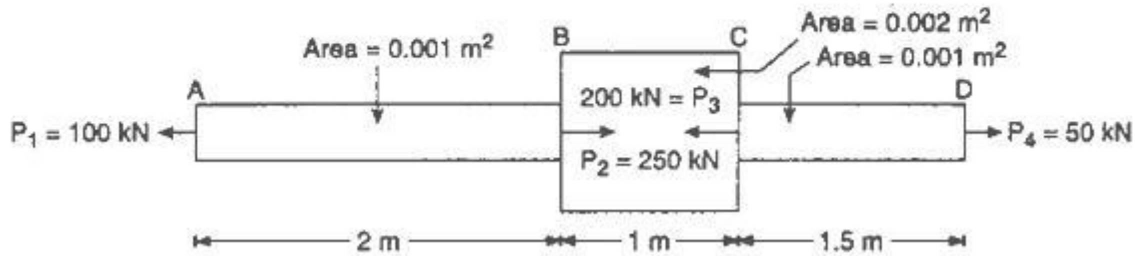
Total energy given by equation (5) of article represents the following two strain energies.
Shear strain energy or strain energy due to distortion.
Volumetric strain energy or energy of dilation or strain energy of uniform compression or tension.

$A_1 = 500\,e^{0.0001\times2500/20}$

$A_1 = 506.29\ mm^2$.

47. Calculate the relative displacement of points A and D of the steel rod as shown in figure, when it is subjected to the four concentrated loads P_1, P_2, P_3 and P_4.

Take; $E = 2\times10^{11}\ N/m^2$



Solution: The body as a whole will be in equilibrium if $\Sigma P_k = 0$

Now taking sections $x - x_1$, $y - y_1$, $z - z_1$ we see that

Thus, Between; A and B; $P_1 = +100$ kN

Between; B and C; $P_2 = -150$ kN

Between; C and D; $P_3 = 50$ kN

Thus, from Principle of Superposition:

$\delta \Sigma P_k L_k / A_k.E_k = 100(2)\times10^3/0.001(2\times10^{11}) \times 150(1)\times10^3/0.002(2\times10^{11}) + (50)(1.5)\times10^3 / 0.001(2\times10^{11})$

= 0.001 m or 1 mm.

48. A load of 300 kN is applied on a short concrete column 250 mm × 2500 mm. The column is reinforce by steel bars of total area 5600 mm2. If modulus of elasticity of steel is 15 times that of concrete. Find the stresses in concrete and steel.

If the stress is concrete should not exceed 4 N/mm^2. Find the area of steel required so that the column may support a load of 600 kN.

Solution: Given that

Load applied 'P' = 30 kN

Area of concrete column 'A' = (250 × 250) mm^2

Let σ_s = stress developed in steel

σ_c = Stress developed in concrete

For equilibrium $P_s + P_c = P$

$\sigma s \times A_s + \sigma_c \times A_c = 200 \times 10^3$

$A_s = 5600\ mm^2$

$A_c = 250 \times 250 - 5600 = 5600 = 56900\ mm^2$

$\Rightarrow \sigma_s \times 5600 + \sigma_c \times 56900 = 300 \times 10^3$ ...(1)

To avoid any slip $(\delta L)_s = (\delta L)_c$

$\sigma_s.L_s/E_s = \sigma_c.L_c/E_c$

$\sigma_s = E_s/E_c \times \sigma_c = 15\ \sigma_c\ (L_s = L_c)$ ...(2)

Putting the value of $\sigma_s = 15\sigma_c$ in equation (1)

$15\ \sigma_c \times 5600 + \sigma_c \times 56900 = 300 \times 10^3$

$\sigma_c = 2.13\ N/mm^2$

$\sigma_c = 15 \times 2.13 = 31.94\ N/mm^2$

Now, if $\sigma_c = 4\ N/mm_2$, P = 600 kN, As = ?

$\sigma_s = E_s/E_c \times \sigma_c = 15 \times 4 = 600\ N/mm^2$

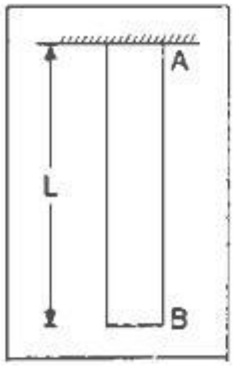
We also know that for equilibrium

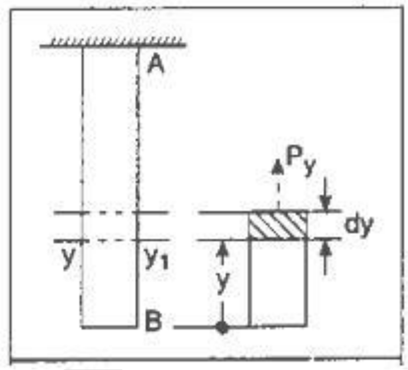
$\sigma_s \times A_s + \sigma_c \times A_c = 600 \times 10^3$

$60 \times A_s + 4 \times (62500 - A_s) = 600 \times 10^3$

$A_s = 6250\ mm^2$

49. A bar of uniform cross-sectional area A and length L hangs vertically from a rigid support of the density of a material of the bar is p kg/m3 derive the expression for maximum stress induced and elongation produced in the bar due to its own.





Solution: Taking a section $y - y_1$ as shown below:

Assume, the load develop due to weight of rod is wN/m

$P_y = w.y$; taking origin at B,

Hence, $\delta L = \int_B^A p_y.d_y/A_y.E_y = 1/AE \int_o^L (w.y)\,dy$ ...(1)

$= w/AE \mid y^2/2 \mid_o^L$ ...(2)

$= wL^2/2AE$

$= (wL)L/2AE$

$= WL / 2AE$ ...(3)

Since, W = p . (AL).g

$\delta L = pALg.L/2AE$

$\delta L = pgL^2/2E$ or $WL/2AE$ ...(4)

where, wL is the total gravitational force developed by the rod; it is designated by W. Equation (3) shown that the elongation by weight W of self is half externally applied concentrated load W.

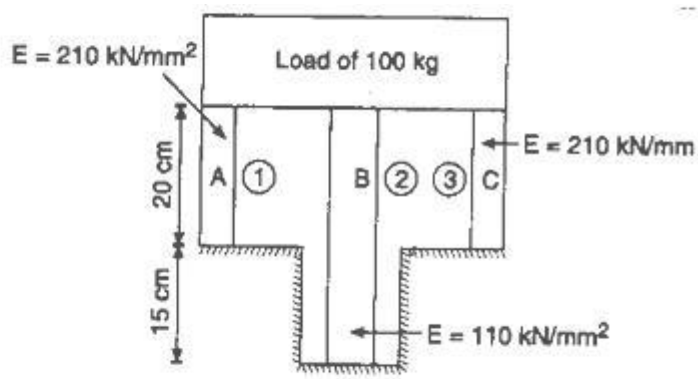
Maximum stress is induced at the bottom section of the bar. Therefore,

Maximum stress = W/A

$\sigma_{max} = pALg / A$

$\sigma max = pgL$ ...(5)

50. A load of 100 kg is supported upon the rods A and C each of 10 mm diameter and another rod B of 15 mm diameter as shown in figure. Find stresses in rods A, B and c.



Solution: Given that; Diameter of rod A = 10 mm

Diameter of rod C = 10 mm

Diameter of rod B = 15 mm

Load applied P = 100 × 9.8 = 980 N

Since, the decrease in length of all three columns will be equal

$\delta = \varepsilon_A.L_1 = \varepsilon_B L_2 = \varepsilon_c.L_3$

$\varepsilon_A = \sigma_1/E_1 = P_1/A_1.E_1$

$\varepsilon_B = \sigma_2/E_2 = P_2/A_2.E_2$

$\varepsilon_C = \sigma_3/E_3 = P_3/A_3.E_3$

And $P_1 + P_2 + P_3 = P = 980$ N

(i) $A_1 = A_3 = \pi/4\,(0.01)^2 = 7.854 \times 10^{-5}$ m^2

$A_2 = \pi/4\,(0.015)^2 = 1.757 \times 10^{-4}$ m^2

$P_1/P_2 = A_1E_1/A_2E_2 * L_2/L_1 = 7.854 \times 10^{-5} * 210 \times 35 / 1.767 \times 10^{-4} \times 110 \times 20$

= 1.485

By symmetry $P_1 = P_3$

$\Rightarrow 2P_1 + P_2 = 980$

$2 * 1.485\ P_2 + P_2 = 980$

$\Rightarrow P_2 = 246.84$ N

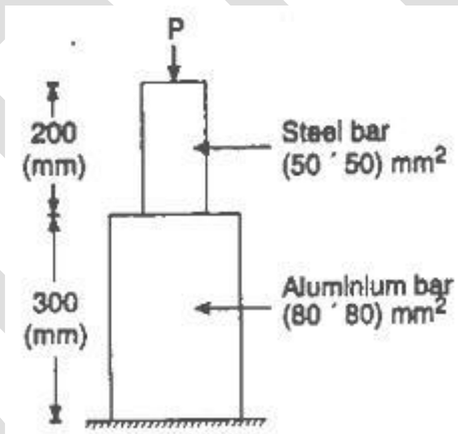
$\Rightarrow P_1 = P_3 = 366.58$ N

Stresses in the rods A, B, C can be obtained as

$\sigma_1 = \sigma_3 = P_1 / A_1 = 366.58 / 7.854 \times 10^{-5} = 4.6674\ MN/m^2$

$\sigma_2 = P_2 / A_2 = 246.84 / 1.767 \times 10^{-4} = 1.397\ MN/m^2$

51. A member formed by connecting a steel bar to an aluminum bar is shown in figure. Assuming that the bars are prevented from buckling sidewise, calculate the magnitude of force, P that will cause the total length of member to decrease by 0.50 mm. The values of elastic modulus of steel and aluminum are 2×10^{11} N/m^2 and 0.7×10^{11} N/m^2.



Solution: Given that

A_s = Area of steel bar = 50 × 50 = 2500 mm^2

A_L = Area of aluminum bar = 6400 mm^2

L_S = length of steel bar = 200 mm

L_A = length of aluminium bar = 300 mm

Decrease in length = δL = 0.50 mm

$E_S = 2 \times 10_{11}$ N/m^2 = 2×10^5 N/mm^2

$E_A = 0.7 \times 10_{11}$ N/m^2 = 0.7×10^5 N/mm^2

Let P = Magnitude of required load.

Using principle of superposition,

$\delta L = P [L_s/A_s.E_s + L_A/A_A.E_A]$

$0.50 = P[200 / 2500 \times 2 \times 10^5 + 300 / 6400 \times 0.7 \times 10^5]$

$0.50 = P [4 X 10^{-7} + 6.70 X 10^{-7}]$

P = 467.29kN

52. If the tension test bar is found to taper from (D + a) cm diameter to (D – a) cm diameter, prove that the error involved in using the mean diameter to calculate. Young's modulus is $(10a/D)^2$ percent.

Solution: Given that:

Larger diameter, d_1 = D + a (mm)

Smaller diameter, d_2 = D – a (mm)

Let, P = load applied on bar (tensile) (N)

L = length of bar (mm)

E_1 = Young's modulus, using tapered cross-section (N/mm^2)

E_2 = Young's modulus using uniform cross-section (N/mm^2)

δL = Extension in length of bar (mm)

Using the relation of extension for tapering cross-section, we have

$$\delta L = \frac{4.P.L}{\pi E.d_1.d_2} = \frac{4PL}{\pi E_1(D+a)(D-a)}$$

$$\delta L = \frac{4PL}{\pi E_1(D^2-a^2)}$$

$$E_1 = \frac{4PL}{\pi \delta L.(D^2-a^2)} \qquad ...(1)$$

Now, using the relation of extension for uniform cross-section; we have

$$\delta L = \frac{4PL}{\pi E d^2} = \frac{4PL}{\pi E_2.D^2}$$

$$E_2 = \frac{4PL}{\pi.\delta L.D^2} \qquad ...(2)$$

The percentage error in calculation of Young's modulus is:

$= (E_1 - E_2 / E_1) \times 100\%$

$$= \frac{\left[\frac{4PL}{\pi.\delta L(D^2-a^2)}\right] - \left[\frac{4PL}{\pi\delta L.D^2}\right]}{\frac{4PL}{\pi E_1(D_2-a^2)}} \times 100$$

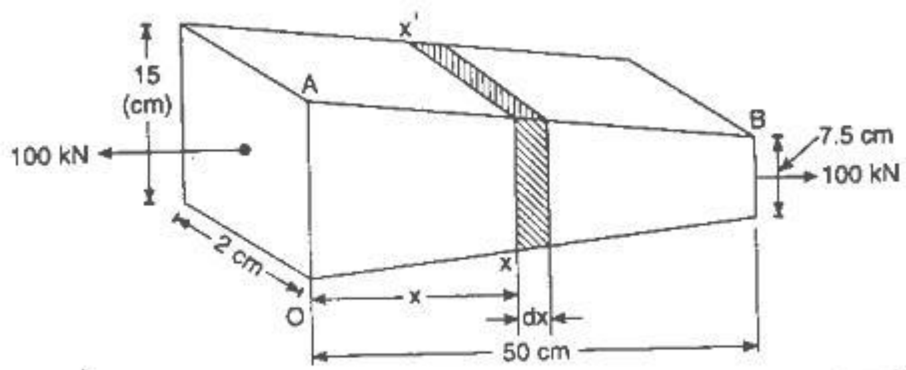
$$= \frac{\frac{1}{D^2-a^2} - \frac{1}{D^2}}{\frac{1}{D^2-a^2}} \times 100 = \frac{\frac{D^2-(D^2-a^2)}{(D^2-a^2)D^2}}{\frac{1}{D^2-a^2}} \times 100$$

$$= \frac{a^2}{D^2} \times 100 = \left(\frac{10a}{D}\right)^2$$

53. A steel bar AB of uniform thickness 2 cm, tapers uniformly from 15 cm to 7.5 cm in a length of 50 cm. From first principle determine the elongation of the plate; if an axial tensile force of 100 kN is applied on it.

Take: $E = 2 \times 10^{11}$ N/m^2.

Solution: Considering; an element of length dx, of the bar, at a distance x from A, as shown in figure.



We find that the width of the plate at a distance x form A

$= 15 - [15 - 7.5]/50 \text{ X } x \text{ cm}^2 = (15 - 0.15x)\text{cm}^2$

Area of cross-section at the section;

$A_x = 2(15 – 0.15x) = (30 – 0.30x) \text{ cm}^2$

Stress σ_x $P/A_x = 100 \times 10^3 / 30 – 0.30x \text{ N/cm}^2$

And Stress $\varepsilon_x = \sigma_x/E = 100 \times 10^3 / 2 \times 10^{11} \times 10^{-4} (30 – 0.30x) = 1/2 \times 10^2(30 – 0.30x)$

Elongation of elementary length = ε_x.dx

$$\delta L = \int_0^{50} \frac{dx}{200(30-0.30x)}$$

$$\delta L = \frac{1}{200(-0.30)}[In\ (30-0.30x)]_0^{50}$$

$$= -\frac{1}{60}[In\ 15 - In\ 30]$$

$$= -\frac{1}{60}[In\ (15/30)] = 0.0115\ cm$$

54. A metallic block having Poisson's ratio of 0.3 is subjected to direct stress s along one direction only. Compare the shear strain energy with volumetric strain energy.

Solution: Since we have,

Shear strain energy = 1/12 G $[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]$.V

Volumetric strain energy = 1/2K $[\sigma_1 + \sigma_2 + \sigma_3 / 3]^2$.V

⇒ Shear strain energy (U_d) = V/12G $[\sigma_2 + 0 + \sigma_2]$ = σ_2/6G.V

Volumetric strain energy (U_v) = 1/2K $(\pi/3)^2$ V = σ^2.V/6G

$U_d/U_v = \sigma^2$.V/6G X 18K/σ^2V = 3K/G

E = 3K (1 – 2v)

E = 2G (1 + v)

⇒ U_d/U_v = 3K/G = 2(1 + v) / (1 – 2v) = 2(1 + 0.3)/(1–0.3×2) = 6.5

∴ Energy of distortion or shear strain energy is 6.5 times of volumetric strain energy.

55. A bar of length 50 cm has varying cross-section. It carries a load of 25 kN. Find the extension if the cross-section is given by [5 + x^2/100] cm^2, where x is distance from one end in cm. Neglect weight of bar.

E = 2 × 1011 N/m^2

Solution: Given that
E = 2 × 10^{11} N/m^2 = 2 × 10^7 N/cm^2

Length of bar 'L' = 50 cm

Load applied 'P' 25 kN

Cross-sectional area at a section, x cm away from one end = (5 + x^2/100) cm^2

⇒ Stress at distance x from given end = P/A_x

⇒ σ_1 = 25 × 10^3 / (5 + x^2 / 100) N/cm^2

If de is the extension of an element of length dx at a distance x from given end.

de/dx = σ/E

$$\Rightarrow \quad \frac{de}{dx} = \frac{25\times10^3}{\left(5+\frac{x^2}{100}\right)2\times10^7} = \frac{1.25\times10^{-3}}{(5+x^2/100)}$$

⇒ de = (1.25 × 10^{-3} × 100) / 500 + x^2 dx

$$\int_0^{\delta L} de = \int_0^{50} \frac{0.125}{500 + x^2} dx$$

$$\delta L = \frac{0.125}{\sqrt{500}} \left| tan^{-1} \frac{x}{\sqrt{500}} \right|_0^{50}$$

$$= \frac{0.125}{\sqrt{500}} \left[tan^{-1} \frac{50}{\sqrt{50}} \right] = 0.368\ cm.$$

56. A tensile load of 50 kN is acting on a rod of diameter 50 mm and a length of 5 m. A bore of diameter 25 mm is made centrally in the rod at one end. To what length the rod should be bored, so that the total extension will increase by 25% under the same tensile load.

Take E = 2 × 10^{11} N/m^2

Solution: Given that:

E = 2 × 10^{11} N/m^2 = 2 × 10^5 N/mm^2

Load applied on rod 'P' = 50 kN

Diameter of rod 'D' = 50 mm

Area of cross-section of rod 'A' = (π/4) (50^2) = 1963.50 mm^2

Length of rod = 5000 mm

Diameter of bore 'd' = 25 mm

Area of bore = (π/4).d^2 = (π/4)$(25)^2$ = 490.87 mm^2

Total Extension before boring

δL = P.L/A.E = 50 X 10^3 X 5000/1963.50 X 2 X 10^2 = 0.6366mm

Total extension after boring for length 'l' is δL = 1.25 δL = 1.25 X 0.6366 = 0.7958mm

Now, using principle of superposition

$$\delta L' = \frac{P}{E}\left(\frac{l}{1472.63} + \frac{(5000 - l)}{1963.50}\right)$$

$$0.7958 = \frac{50 \times 10^3}{2 \times 10^5}\left[\frac{4l + 3(5000 - l)}{5890.5}\right]$$

18750 = l + 15000

l = 3750 mm

The rod should be bored upto the length of 3.75 metre.

57. The principal stresses at a point in an elastic material are 70 N/mm^2 tensile, 30 N/mm^2 tensile, and 50 N/mm^2 compressive. Calculate the volumetric strain and resilience.

Take E = 2 × 10^{11} N/m^2; v = 0.30

Solution: Given that:

σ_1 = 70 N/mm^2 = 70 × 10^6 N/m^2

σ_2 = 30 N/mm^2 = 30 × 10^6 N/m^2

σ_3 = –50 N/mm^2 = 50 × 10^6 N/m^2

Volumetric strain = (σ_1 + σ_2 + σ_3) [1 – 2v / E]

= [70 + 30 – 50] [1 – 2 × 0.30 / 2 × 10^{11}] × 10^6

= 50 × 0.4 / 2 × 10^5 = 1 × 10^{-4}

Resilience = 1 / 2E (σ_1^2 + σ_2^2 + σ_2^3)

= 1 / 2 × 2 × 10^{11} [70^2 + 30^2 + (–50)^2] × 10^{12} N/m^2

= 0.02075 N/mm^2 or 20750 N/m^2.

58. Calculate the strain energy of the bolt shown in figure, under a tensile load of 10 kN. Shown that the strain energy is increased for the same maximum stress by turning down the shank of the bolt to the root diameter of the thread.

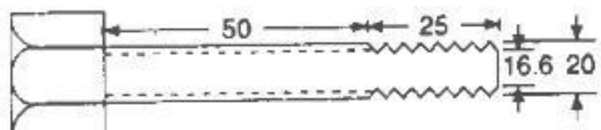
Solution: Given that:

Root diameter = 16.6 mm

⇒ Area of core = $\pi/4\ (16.6)^2 = 217\ mm^2$

Load applied 'P' = 10 kN

⇒ Stress 'σ' = $P/A = 10 \times 10^3 / 217 = 46\ N/mm^2$



(All dimension in mm)

Now, Stress in shank = Load applied / Area of cross-section of shank

$= 10 \times 10^3 / \pi/4\ (20)^2 = 10 \times 10^3 / 314 = 31.8\ N/mm^2$

Total strain energy 'U' = $1/2\ (\sigma^2/E).A.E$

$= 1(46^2 \times 217 \times 25) + (31.8^2 \times 314 \times 50) / 2 \times 205000$

= 67 N.mm

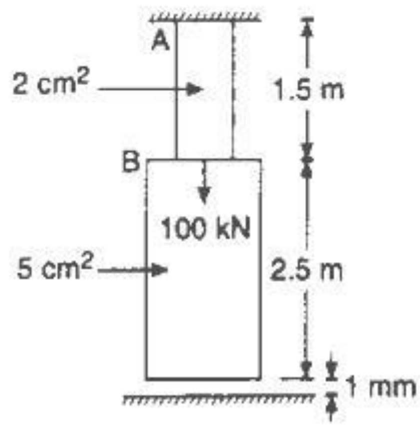
If the shank is turned down to root diameter 16.6 mm, the stress bolt will become 46 N/mm^2 throughout.

and Strain energy = $1/2\ (\sigma^2/E).A.L = 46^2/2 \times 217 \times 75 / 205000 = 84$ N.mm

Thus, strain energy is increased for the same maximum stress by turning down the shank of the bolt to the root diameter of the thread.

59. A composite bar ABC, rigidly fixed at A and 1 mm above the lower support, is loaded as shown in figure. If the cross-sectional area of section AB is 2 cm^2 and that of the section BC is 5 cm^2, determine the reactions at the ends, and the stresses in two sections.

Take E = $2 \times 10^7\ N/cm^2$.



Solution: Length: AB = L_1 = 1.5 m = 150 cm

Length: BC = L_2 = 2.5 m = 250 cm

Distance between C and lower support = 0.1 cm

Load on the bar 'P' = 100 kN

Area of AB = $A_1 = 2\ cm^2$

Area of BC = $A_2 = 5\ cm^2$

Young's modulus, $E = 2 \times 10^7\ N/cm^2$

Assume that, reaction at end A = R_A

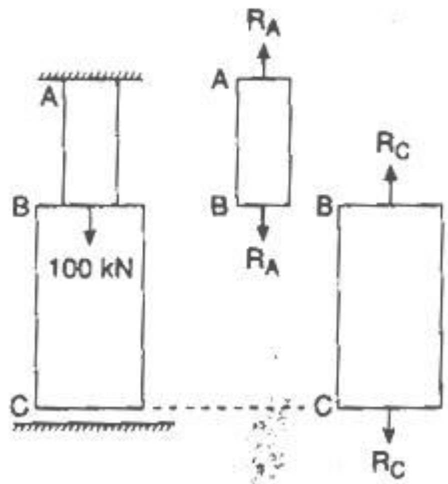
Reaction at end C = R_C

It is notable that the bar is rigidly fixed at A and loaded at B, not at C. Thus, only AB is under tension while BC feel no stress until C touches the rigid support.

Let δL = Increase in length of AB when subjected to a load to 100 kN.

We know that; δL = PL/AE with usual notations.

$\delta L = 100 \times 10^3 \times 150 / 2 \times 2 \times 10^7 = 0.375$ cm



Since, the increase in length AB is more than 0.1 cm therefore some part of load will be required to increase AB by 0.1 cm and remaining will be shared by the portions AB and BC of the bar.

Thus using;

$\delta L = PL / AE$

$0.1 = P_1 \times 150 / 2.0 \times 2 \times 10^7$

$P_1 = 26.67$ kN

$\Rightarrow P - P_1 = P_2 = 73.33$ kN

The load P_2 will be shared by AB and BC.

Let the reaction at A (beyond 0.1 cm) = R_{A1}

And the reaction at C (beyond 0.1 cm) = R_C

$R_{A1} + R_C = 73.33$ kN ...(1)

Let δL_1 = Increase in length of AB (beyond 0.1 cm)

δL_2 = Decrease in length of BC (beyond 0.1 cm)

$\delta L = PL/AE$

$\delta L_1 = R_{A1} X 150/2 X 2 X 10^7$
$\delta L_2 = R_c X 250/5 X 2 X 10^7$

$R_{A1} X 150/2 X 2 X 10^7 = R_c X 250/5 X 2 X 10^7$

$R_{A1}/ R_c = 2 X 2 X 10^7 X 250 / 150 X 5 X 2 X 10^7 2/3$

$R_{A1} = 2/3 R_c$

Substituting in equation (1),

$2/3 R_C + R_C = 73.33$

$R_C (5/3) = 73.33$

$R_C = 44$ kN

$R_{A1} = 29.33$ kN

Total reaction at A = 26.67 + 29.3 = 56 kN and total reaction at B = 44 kN

Therefore, stresses in two sections are

$\sigma_{AB} = R_A/A_1 = 56 X 10^3/2 = 28 kN/cm^2$
$\sigma_{BC} = R_c/A_2 = 44 X 10^3/5 = 8.8 kN/cm^2$

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73. Define stress.

When an external force acts on a body, it undergoes deformation. At the same time the body resists deformation. The magnitude of the resisting force is numerically equal to the applied force. This internal resisting force per unit area is called stress.

Stress = Force/Area

$\sigma = P/A$ unit is N/m^2

74. Define strain

When a body is subjected to an external force, there is some change of dimension in the body. Numerically the strain is equal to the ratio of change in length to the original length of the body.

Strain = Change in length/Original length

$e = \delta L/L$

75. Define shear stress and shear strain.

The two equal and opposite force act tangentially on any cross sectional plane of the body tending to slide one part of the body over the other part. The stress induced is called shear stress and the corresponding strain is known as shear strain.

76. **State Hooke's law.**

It states that when a material is loaded, within its elastic limit, the stress is directly proportional to the strain.

Stress α Strain

$\sigma \propto e$

$\sigma = Ee$

$E = \sigma/e$ Unit is N/m^2

Where,

E - Young's modulus

σ - Stress

e - Strain

77. Define Poisson's ratio.

When a body is stressed, within its elastic limit, the ratio of lateral strain to the longitudinal strain is constant for a given material.

Poisson' ratio (μ or 1/m) = Lateral strain /Longitudinal strain

78. State the relationship between Young's Modulus and Modulus of Rigidity.

$E = 2G\,(1+1/m)$

Where,

E - Young's Modulus

K - Bulk Modulus

1/m - Poisson's ratio

79.
80.
81.
82.
83.
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88.

12. Give the relationship between Bulk Modulus and Young's Modulus.

E = 3K (1-2/m)

Where,

E - Young's Modulus

K - Bulk Modulus

1/m - Poisson's ratio

13. What is compound bar?

A composite bar composed of two or more different materials joined together such that system is elongated or compressed in a single unit.

14. What you mean by thermal stresses?

If the body is allowed to expand or contract freely, with the rise or fall of temperature no stress is developed but if free expansion is prevented the stress developed is called temperature stress or strain.

15. Define- elastic limit

Some external force is acting on the body, the body tends to deformation. If the force is released from the body its regain to the original position. This is called elastic limit

16. Define – Young's modulus

The ratio of stress and strain is constant within the elastic limit.

E = Stress

Strain

17. Define Bulk-modulus

The ratio of direct stress to volumetric strain.

$$K = \frac{\text{Direct stress}}{\text{Volumetric strain}}$$

18. Define- lateral strain

When a body is subjected to axial load P. The length of the body is increased. The axial deformation of the length of the body is called lateral strain.

19. Define- longitudinal strain

The strain right angle to the direction of the applied load is called lateral strain.

20. What is principle of super position?

The resultant deformation of the body is equal to the algebric sum of the deformation of the individual section. Such principle is called as principle of super position

21. Define- Rigidity modulus

The shear stress is directly proportional to shear strain.

$$\tau = \frac{\text{Shear stress}}{\text{Shear strain}}$$

UNIT –II

1. **Beam**

A beam is a structural member which is primarily subjected to a system of external loads that act transverse to its axis. The forces in the longitudinal direction and twisting moments about the longitudinal axis may act in addition to transverse loading. A beam is therefore different from a bar in tension or a bar in compression because of the direction of loads acting on it. A beam has a characteristic feature that internal forces called shear forces and the internal moments called bending moments are developed in it, to resists the external loads. Many shafts of machines act as beams. The beams may be straight or curved. The actual installation of a straight beam may be vertical, inclined or horizontal. But, for convenience the beams discussed here will be shown in horizontal position. For the beams the distance (L) between the supports is called a span.

2. **Types of beam support**

The beams usually have three different types of support:

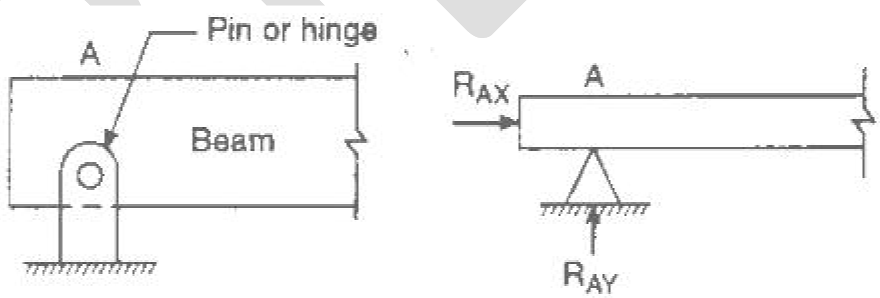
a. Hinged or pinned support

b. Roller support

c. Fixed support

3. Hinged or pinned support:

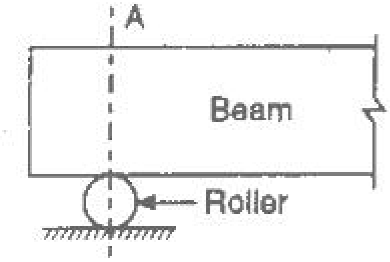
The hinged support is capable of resisting force acting in any direction of the plane. Hence, in general the reaction at such a support may have two components, one in horizontal and another in vertical direction. To determine these two components two equations of statics must be used. Usually, at hinged end the beam is free to rotate but translational displacement is not possible

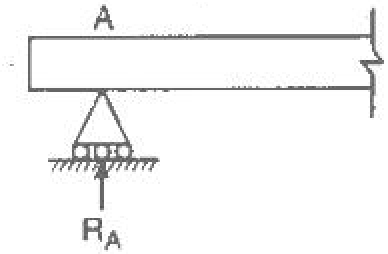


The hinged and roller supports are also termed as simple supports.

4. Roller support:

The roller support is capable of resisting a force in only one specific line or action. The roller can resist only a vertical force or a force normal to the plane on which roller moves. A reaction on this type of supports corresponds to a single unknown figure.

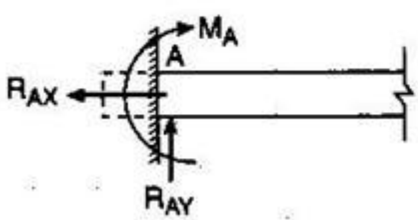




The hinged and roller supports are also termed as simple supports.

5. Fixed Support:

The fixed support is capable of resisting of force in any direction and is also capable of resisting a couple or a moment. A system of three forces can exist at such a support (i.e., two components of force and a moment).



6. **Classification of beams**

The beams may be classified in several ways, but the commonly used classification is based on support conditions. On this basis the beams can be divided into six types:

a. Cantilever beams

b. Simply supported beams

c. Overhanging beams

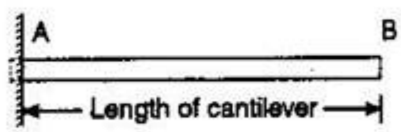
d. Propped beams

e. Fixed beams

f. Continuous beams

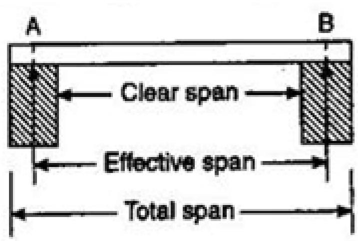
7. **Cantilever beam:**

A beam having one end fixed and the other end free is known as cantilever beam, figure shows a cantilever with end 'A' rigidly fixed into its supports, and the other end 'B' is free. The length between A and B is known as the length of cantilever.



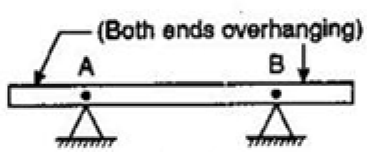
8. **Simply supported beam:**

A beam having both the ends freely resting on supports, is called a simply supported beam. The reaction act at the ends of effective span of the beam. Figure show simply supported beams. For such beams the reactions at the two ends are vertical. Such a beam is free to rotate at the ends, when it bends.



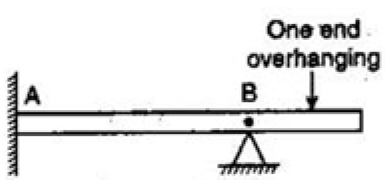
9. **Overhanging beams:**

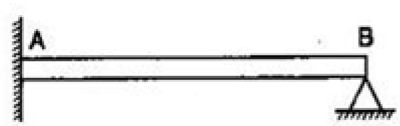
A beam for which the supports re not situated at the ends and one or both ends extend over the supports, is called an overhanging beam. Figure represents overhanging beams.



10. **Propped cantilever beams:**

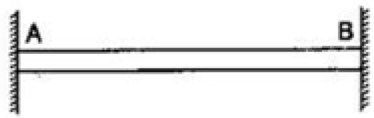
A cantilever beam for which one end is fixed and other end is provided support, in order to resist the deflection of the beam, is called a propped cantilever bema. A propped cantilever is a statically indeterminate beam. Such beams are also called as restrained beams, as an end is restrained from rotation.





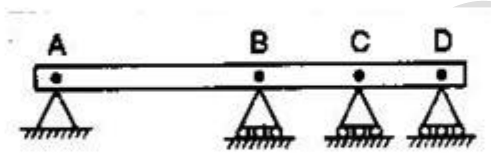
11. **Fixed beams:**

A beam having its both the ends rigidly fixed against rotation or built into the supporting walls, is called a fixed beam. Such a beam has four reaction components for vertical loading (i.e., a vertical reaction and a fixing moment at both ends) figure shows the fixed beam.



12. Continuous beam:

A beam having more than two supports is called as continuous beam. The supports at the ends are called as the end supports, while all the other supports are called as intermediate support. It may or may not have overhang. It is statically indeterminate beam. In these beams there may be several spans of same or different lengths figure shows a continuous beam.



13. **Types of loading**

A beam may be loaded in a variety of ways. For the analysis purpose it may be splitted in three categories:

a. Concentrated or point load

b. Distributed load:

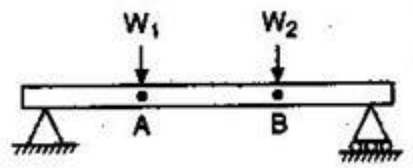
c. Uniformly distributed load

d. Uniformly varying load

e. Couple

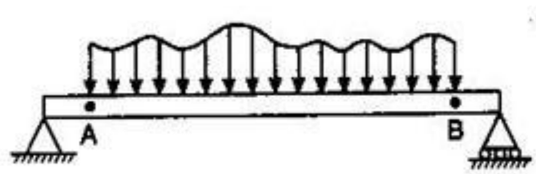
14. **Concentrated load:**

A concentrated load is the one which acts over so small length that it is assumed to act at a point. Practically, a point load cannot be places as knife edge contact but for calculation purpose we consider that load is being transmitted at a point. Figure represents point loading at points A and B.



15. Distributed load:

A distributed load acts over a finite length of the beam. A distributed load may be uniformly. Such loads are measured by their intensity which is expressed by the force per unit distance along the axis of the beam. Figure represents distributed loading between point A and B.

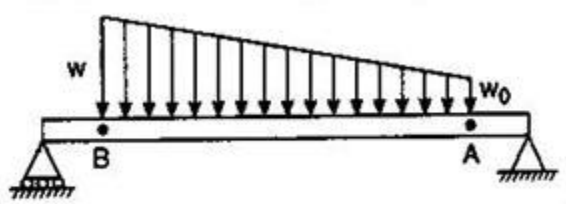


16. Uniformly distributed load:

A uniformly varying load implies that the intensity of loading increases or decreases at a constant rate along the length.

$w = w_0 = k . x$

Where k is the rate of change of the loading intensity, w0 being the loading at the reference point.



Such a loading is also known as triangularly distributed load. Figure represents such a loading between points A and B. Sometimes, the distributed loading may be parabolic, cubic or a higher order curve for non-uniformly varying load i.e.,

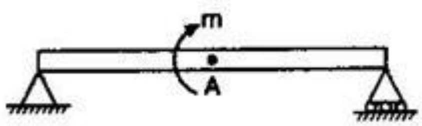
$w = w_0 + k_1x + k_2x^2$ (Parabolic)

$w = w_0 + k_1x + k_2x^2 + k_3x^3$ (Cubic) and so on.

17. Couple

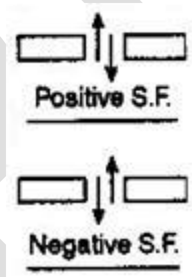
A beam may also b subjected to a couple 'µ' at any point. As shown in figure.

Note: In general, the load may be a combination of various types of loadings.



18. SHEAR FORCE

When a beam is subjected to any type of loading, at any section of the beam, an internal vertical force is developed to maintain the segment of the beam in equilibrium. This internal vertical force acting at right angles to the axis of the beam is called the shearing force. It is numerically equal to the algebraic sum of all the vertical components of the external forces acting on the isolated segment but it is opposite in direction.



The shearing force at any section may be computed by considering the forces either at right hand segment or at the left hand segment. The shearing force at the section is numerically equal and opposite in direction to the sum of all the vertical forces, including the reaction components on either side of the section. Shearing force at any other section may be computed similarly.

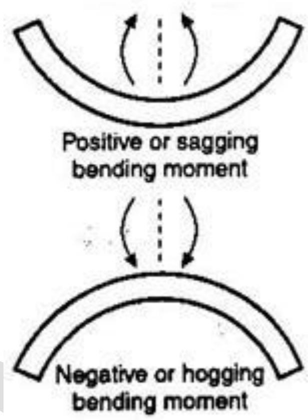
19. Sign convention for shear force

In order to draw the shear force diagram, it is necessary to follow a sign convention. The shear force should be taken to be positive, if the resultant of all the forces is in upward direction at the left hand side of a section or in downward direction on the right hand side of the section.

It is taken to be negative if it has resultant of all the forces in downward direction on the left hand side of the section and in upward direction on the right hand side of the section

20. Bending moment

An internal resisting moment or a couple developed within the cross-sectional area of the cut, to counteract the moment caused by external forces, it termed as bending moment. It acts in the direction opposite to the external moment to satisfy the governing equation; $\Sigma M_z = 0$ i.e., the magnitude of the internal resisting moment equal the external moment. These moments tend to bend a beam in the plane of the loads. Thus, BM at any section of a beam is the algebraic sum of the moments (about a horizontal axis passing through that section and at right angles to the axis of the beam) that are caused by all the vertical loads acting on either side of the section.



the internal bending moment (M) can be developed only within the cross-sectional area of the beam and is equivalent to a couple. For equilibrium the sum of all the moments caused by the forces may be made about any point in the plane. The summation of moments is conveniently made around the centroid of the section at the cut.

21. Sign convention for bending moment

When the bending moment causes concavity at the top, it is taken to be positive. It is also called as sagging bending moment.

On the other hand, the bending moment which causes convexity at the top is taken to be negative. It is also called as hogging moment.

It can also be observed that bending moment will be considered positive when the moment on the left portion is clockwise and on the right portion is anticlockwise.

22. Shear force diagram (SFD)

The shear force diagram (SFD) is drawn to represent the variation of shear force along a beam. The positive shear force is drawn as ordinate above arbitrary reference line and negative shear force below it. The straight lines or curves joining the tips of all such ordinates at salient points from the SFD.

The steps to draw a S.F.D. are as follows:

a. Draw the symbolic loading diagram of the given beam to some scale, along the length of the beam.

b. Find the reactions at the supports using equations of equilibrium.

c. Starting from the right hand end obtain the shear force at various sections, and at all salient points.

d. If there is no loading between two sections, the shear force will not change between these sections.

e. Plot the SD to a suitable scale, under the loading diagram, with the same scale along the length.

23. Special features of SFD

a. The SFD consists of rectangles for point loads.

b. It consists of an inclined line for the portion on which U.D.L. is acting.

c. It consists of a parabolic curve for the portion over which uniformly varying load acts.

d. It may be a cubic or higher order depending upon the type of distributed load.

24. Bending moment diagram

The bending moment diagram (BMD) is a graph showing the variation of bending moment along a beam. The positive BM is drawn above the arbitrary reference line while the negative bending moment is drawn below it and a line joining the extremities of the ordinate at salient points from the BMD.

The steps to draw BMD are as follows:

a. Starting from the right hand end of the beam for convenience, obtain the bending moment at various sections in magnitude and direction. The BM at a section is obtained by the summation of the moments due to the reactions and other forces acting on the right hand side only, or on the left hand side only.

b. Determine the BM at all salient points. At these points the BM changes in magnitude or sign.

c. Plot the BMD to a suitable scale preferably under the loading diagram, and SFD, using either straight lines or smooth curves.

25. **Special features of BMD:**

a. A BMD consists of inclined lines for the beam loaded with point loads.

b. The BMD consists of a parabolic curve for the portion over which U.D.L. is acting.

c. The BMD consists of cubic curve for uniformly varying load.

d. The BMD should be a higher degree curve for distributed loading.

26. Cantilever With Concentrated Or Point Load At The End

Consider a cantilever beam AB, carrying concentrated load (W) at end B, as shown in figure (a). Let the length of beam be 'L'. Then at any section X – X', at a distance x from end B.

Shear force = Total unbalanced vertical force on either side of the section

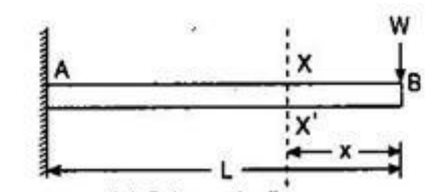
$\Rightarrow F_x = + W$

(The sign is taken to be positive because the resultant force is in downward direction on the right hand side of the section)

Now, Bending Moment = Algebraic sum of moments vertical loads acting on either caused by side of the section.

$\Rightarrow M_x = - W \cdot x$

(The sign is taken to be negative because the load creates hogging).



(a) Schematic diagram
(b) S.F.D
(c) B.M.D.

To draw SFD and BMD, x is varied from 0 to L. Since, shear force is not dependent on x the SFD is a rectangle with constant ordinate W, and Bending moment is proportional to x. Therefore, BMD is a triangle with M = 0 at x = 0 and M = WL at x = L.

27. Simply supported beam with point load at center

Consider a simply supported beam AB, with span 'L', and subjected to point load (W) at the centre, as shown in figure.

To draw SFD and BMD, we need R_A and R_B.

$R_B.L – W. L/2 = 0$

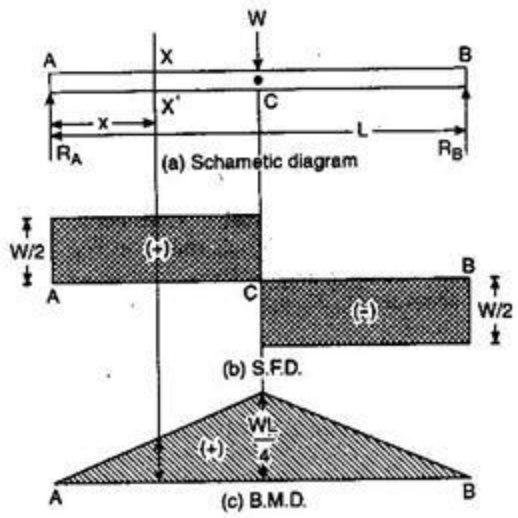
$\Rightarrow R_B = W/2$

Also, from condition of static equilibrium $\Sigma F_y = 0$ i.e.,

$R_A + R_B – W = 0$

$\Rightarrow R_A = W – R_B = W – W/2$

$R_A = W/2$



Consider a section (X – X') at a distance x from end 'A'.

Shear force = Total unbalanced vertical force on either side of the section

$F_x = + R_A = + W/2$

The F_x remains constant between A and C.

(The sign is taken to be positive because the resultant force is in upward direction on the left hand side of the section).

By taking a section between C and B, we get

$F_x = + RA – W$

$F_x = + W/2 – W = – W/2$

Now, the bending moment between A and C,

Bending moment = Algebraic sum of moments caused by vertical loads acting on either side of the section.

Mx = + RA . x

(The sign is taken to be positive because the load creates sagging).

The bending moment between C and B, can be obtained by taking section between C and B, at a distance x from A.

$M_x = + R_A . x – W (x – L/2) = + W/2\ x – W(x – L/2)$

To draw SFD and BMD, x is varied from O to L. Since, shear force is not dependent on x, the SFD is a rectangle with constant ordinate W/2 but it changes sign at point C. In between A and C, shear force is positive and between C and B it is negative. The bonding moment is a function of x and its values can be obtained from equations (5) and (6).

M_A = + W/2. 0 =0

M_C = + W/2 . L/2 = WK/4

Also from equation (6) we will get the same value of M_C.

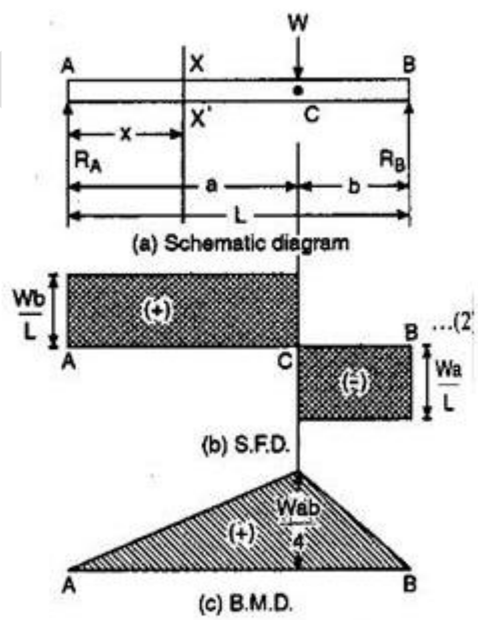
Now, M_B = + W/2 L – W (L – L /2) = 0

The BMD is therefore a triangle with maximum ordinate of + WL/4, under the point loading at centre.

28. Simply supported beam with eccentric load

Consider a simply supported beam A_B, with span 'L' and subjected to an eccentric point load (W), as shown in figure.

To draw SFD and BMD we need R_A and R_B. By taking moment of all the forces about point 'A', we get;



$R_B . L – W. a = 0$

$\Rightarrow R_B = W.a/L$

Also, from condition of static equilibrium $\Sigma F_y = 0$

i.e., $R_A + R_B – W = 0$

$R_A = W – Wa/L = W(L – a) / L$

$\Rightarrow R_A = W.b / L$

Consider a section (X – X') at a distance x from end 'A'.

Shear force = Total unbalanced vertical force on either side of the section

$F_x = + R_A = + W.b / L$

The F_x remains constant between A and C.

(The sign is taken to be positive because the resultant force is in upward direction on the left hand side of the section).

By taking a section between C and B, we get

$F_x = + R_A – W = + W.b / L – W = W/L (b – L) = – Wa / L$

Now, the bending moment between A and C,

Bending moment = Algebraic sum of moments caused by vertical loads acting on either side of the section

$\Rightarrow M_x = + R_A . x$

(The sign is taken to be positive because the load creates sagging).

The bending moment between C and B can be obtained by taking section between C and B at a distance x from A.

$M_x = + R_A.x – W (x – a) = + Wb/L. x – W(x – a)$

To draw SFD and BMD, x is carried from 0 to L.

Since, shear force is not dependent on x the SFD is a rectangle with constant ordinate Wb/L but if changes both sign and magnitude at point C. In between A and C, shear force is positive and between C and B it is negative.

The bending moment is a function of x and its values can be obtained from equations (4) and (6).

M_A = + Wb/L.0 = 0

M_C = + Wb/L.a = + Wab/L

M_B = + Wb.L/L – W(L – a) = Wb – WL + Wa

= W(b + a) – WL

= WL – WL

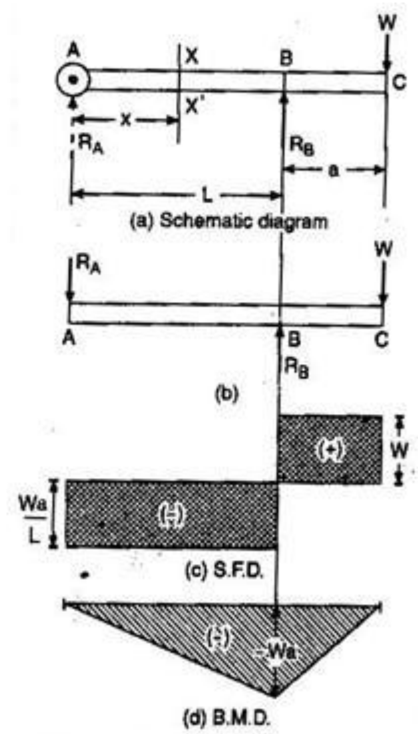
= 0

The BMD is therefore a triangle with a maximum ordinate of + Wab / L, under the point loading at 'C'.

29. Simply supported beam with one side overhang

Consider a simply supported AB with overhang on one side. The beam is subjected to point load (W) at free end, as shown in figure.

To draw SFD and BMD we need R_A and R_B.



Let RA and RB be the reactions in upward direction.

For equilibrium of the beam, Σ M = 0; at point A

Therefore, by taking moment about A, we get,

$R_B.L – W(L + a) = 0$

$R_B = W/L (L + a)$ …(1)

Also from condition of static equilibrium S Fy = 0

$\Rightarrow R_A + R_B – W = 0$

$R_A = W – R_B = W – W/L (L + a)$

$\Rightarrow R_A = W – W – Wa / L$

$\Rightarrow R_A = – Wa / L$ …(2)

The negative sign shows that the assumed direction of RA is opposite to the actual. Therefore, R_A must act vertically downward, as shown in figure and $R_A = Wa / L$.

Consider a section (X – X’) at a distance x from end ‘A’.

$F_x = -R_A = -Wa/L$ ...(3)

(The sign is taken to be negative because the resultant force is in downward direction on the LHS of the section)

The Fx remains constant between A and B.

By taking a section between B and C, we get

$F_x = -R_A + R_B$

$F_x = -Wa / L + W / L (L + a) = - Wa / L + W + Wa / L = + W$

This value of F_x remains constant between B and C.

Now the bending moment between A and B;

Bending moment = Algebraic sum of moments caused by vertical loads acting on either side of the section.

Portion AB; $M_x = -R_A.x$

⇒ At x = 0; $M_A = 0$

At x = L; $M_B = -Wa/L.L = -Wa$

Portion BC; $M_x = -R_A.x + R_B (x - L)$

$M_x = Wa/L\ x + W/L (L + a)(x - L)$

At x = L; $M_B = -Wa / L.L + W/L (L + L).0$

$= -Wa$

At x = (L + a),

$M_C = - Wa / L (L + a) + W/L (L + a) (L + a - L)$

$= -Wa/L (L + a) + Wa/L (L + a) = 0$

To draw SFD and BMD, x is varied from 0 to L.

Since SF is independent of x, SFD is formed by rectangles but BM is dependent on x and varies linearly with x. The BMD is therefore a triangle with a maximum ordinate of $M_B = -Wa$.

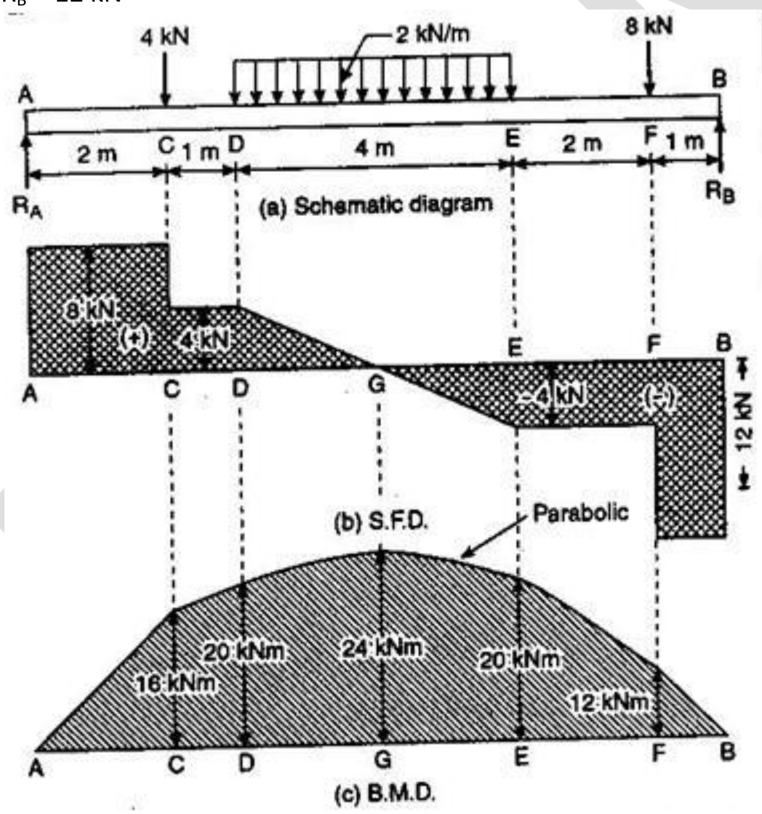
30. A simply supported beam is subjected to a combination of loads as shown in figure. Sketch the S.F. and B.M. diagrams and find the position and magnitude of maximum B.M.

Solution: To draw the SFD and BMD. We need R_A and R_B.

By taking moment of all the forces about point A.

We get $R_B \times 10 - 8 \times 9 - 2 \times 4 \times 5 - 4 \times 2 = 0$

$R_B = 12$ kN



From condition of static equilibrium $\Sigma F_y = 0$

$R_A + R_B - 4 - 8 - 8 = 0$

$R_A = 20 - 12 = 8$ kN

To draw SFD we need S.F. at all salient points:

For AC; $F_A = + R_A = 8$ kN

For CD, $F_C = + 8 - 4 = 4$ kN

$F_D = 4$ kN

For DE, $F_x = 8 - 4 - 2(x - 3) = 10 - 2x$

At x = 3 m; $F_D = 10 - 6 = 4$ kN

At x = 7 m; $F_E = 10 - 2 \times 7 = -4$ kN

The position for zero SF can be obtained by $10 - 2x = 0$

$\Rightarrow x = 5$ m

For EF; $F_x = 8 - 4 - 8 = -4$ kN

For FB; $F_x = 8 - 4 - 8 - 8 = -12$ kN

To draw BMD, we need BM at all salient points.

For region AC, $M_x = + 8x$

At x = 0; $M_A = 0$

x = 2; $M_C = 8 \times 2 = 16$ kN m

For region CD; $M_x = + 8x - 4(x - 2)$

$\Rightarrow$ at x = 2 m; $M_C = 8 \times 2 - 4(2 - 2) = 16$ kN m

At x = 3 m; MD = $8 \times 3 - 4(3 - 2) = 20$ kN m

For region DE,

$M_x = + 8x - 4(x - 2) - 2(x - 3)^2 / 2 = 10x - x^2 - 1$

At x = 3 m; $M_D = 8 \times 3 - 4(3 - 2) - 2(3^2 - 3)^2 / 2 = 20$ kN m

At x = 7 m; $M_E = 10 \times 7 - (7)^2 - 1 = 20$ kN m

At x = 5 m; $M_G = 10 \times 5 - (5)^2 - 1 = 24$ kN m

For region EF,

$M_x = 8x - 4(x - 2) - 2 \times 4(x - 5) = 48 - 4x$

At x = 9 m, $M_F = 120 - 12 \times 9 = 12$ kN m

At x = 10 m; $M_B = 120 - 12 \times 10 = 0$

The SFD and BMD can now be drawn by using the various value of SF and BM. For BMD the BM is proportional to x, so it depends, linearly on x and the lines drawn are straight lines.

The maximum bending moment exists at the point where the shear force is zero, and also dM/dx = 0 in the region of DE

$d/dx\,(10x - x^2 - 1) = 0$

$\Rightarrow 10 - 2x = 0$

$\Rightarrow X = 5$ m

$M_{max} = 10 \times 5 - (5)^2 - 1 = 24$ kN m

Thus, the maximum bending is 24 kN m at a distance of 5 m from end A.

31. A simply supported beam overhanging on one side is subjected to a U.D.L. of 1 kN/m. Sketch the shear force and bending moment diagrams and find the position of point of contraflexure

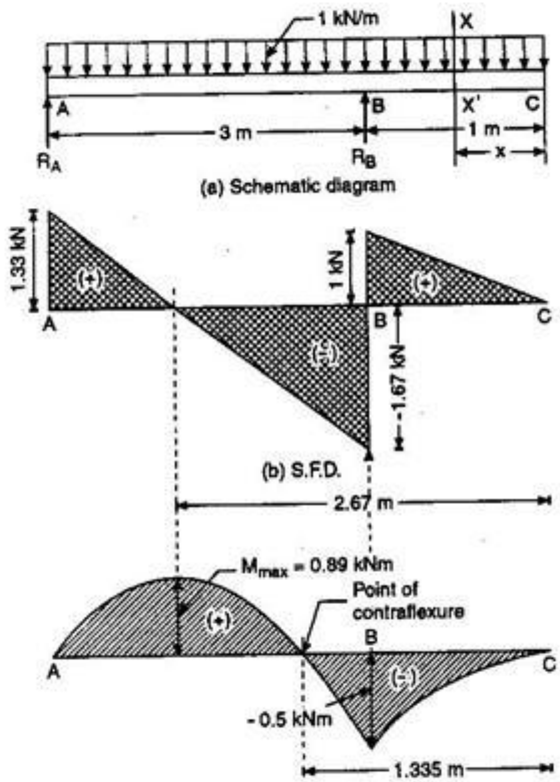
Solution: Consider a section (X – X') at a distance x from end C of the beam.

To draw the SFD and BMD we need R_A and R_B.

By taking moment of all the forces about point A.

$R_B \times 3 - w/2 \times (4)^2 = 0$

$R_B = 1 \times (4)^2 / 2 \times 3 = 8/3$ kN



(a) Schematic diagram
(b) S.F.D.
(c) B.M.D.

For static equilibrium;

$R_A + R_B - 1 \times 4 = 0$

$R_A = 4 - 8/3 = 4/3$ kN

To draw SFD we need S.F. at all salient points:

Taking a section between C and B, SF at a distance x from end C. we have,

$F_x = + w.x$ kN

At x = 0; $F_C = 0$

x = 1 m; $F_{B\ just\ right} = 1 \times 1 = +1$ kN

Now, taking section between B and A, at a distance x from end C, the SF is:

$F_x = x - 8/3$

When, x = 1 m; $F_B = 1 - 8/3 = -5/3$ kN $= -1.67$ kN

At x = 4 m; $F_A = 4 - 8/3 = +4/3$ kN $= +1.33$ kN

The S.F. becomes zero;

$F_x = x – 8/3 = 0$

$\Rightarrow x = 2.67$ m

(The sign is taken positive taken when the resultant force is in downward directonon the RHS of the section).

To draw BMD we need B.M. at all salient points. Taking section between C and B, BM at a distance x from end C, we have

$M_x = – wx^2 / 2 = –1.x2/2$ kN m

When x = 0, $M_C = 0$

At x = 1 m. $M^B = –1 \times (1)^2 / 2 = –0.5$ kN m

Taking section between B and A, at a distance x from C, the bending moment is:

$M_x = –x^2/2 + 8/3 (x – 1)$

At x = 1 m, $M_B = – 0.5$ kN m

x = 4 m; $M_A = –(4)^2 /2 – 8/3 (4 – 1) = 0$

The maximum bending moment occurs at a point where

$dMx / dx = 0$

$\Rightarrow d/dx [–x^2 / 2 + 8/3 x – 8/3] = 0$

$\Rightarrow –1/2 \times 2x + 8/3 = 0$

$\Rightarrow x = 8/3$ m from end C.

$\Rightarrow M_{max} = –1/2 (8/3)^2 + 8/3 (8/3 – 1) = 0.89$ kN m

The point of contraflexture occurs at a point, where

$M_x = 0$

$\Rightarrow –x^2/2 + 8/3 (x – 1) = 0$

$\Rightarrow x^2 = 16/3 (x – 1)$

$\Rightarrow x_2 - 16/3\, x + 16/3 = 0$

$x = 1.335$ m or 4 m

The SFD and BMD can be drawn by joining the ordinates at salient point by straight line or smooth curves, as defined by governing equations.

32. A simply supported beam overhanging on both sides is subjected to loads as shown in figure. Sketch the SF and BM diagrams and locate the position of point of contraflexture.

Solution: Consider a section (X – X') at a distance x from end C of the beam.

To draw SFD and BMD. We need R_A and R_B.

By taking moment of all forces about point A, we get

$-R_B \times 6 + (2 \times 7.2) + 1 \times 1.2 \times (6 + 1.2/2) + 2 \times 6 \times (6/2) - 1 \times 1.8 \times (1.8/2) = 0$

$\Rightarrow R_B = 9.45$ kN

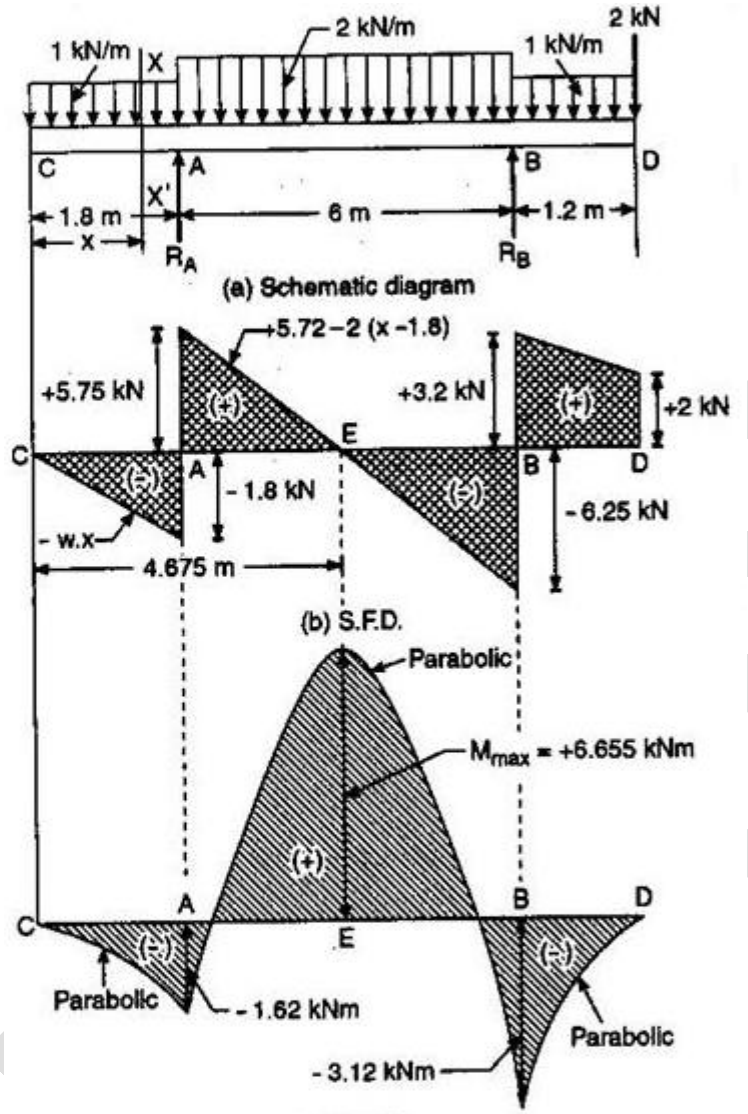
From condition of static equilibrium,

$R_A + R_B - 2 - (1 \times 1.8) - (2 \times 6) - (1 \times 1.2) = 0$

$\Rightarrow R_A = 2 + 1.8 + 12 + 1.2 - 9.45$

$R_A = 7.55$ kN

To draw SFD we need SF at all salient points.



Taking a section between C and A. SF at a distance x from end C is;

$F_x = -\, w.x = -1.x$ ...(1)

Thus, at x = 0; $F_C = 0$

x = 1.8 m, $F_{A\ just\ right} = -\,1.8$ m

Now, consider a section between A and B, at a distance x from C.

$F_x = -1.8 + 7.55 - 2\,(x - 1.8)$ ...(2)

Thus, at x = 1.8 m; $F_{A\ just\ right} = -1.8 + 7.55 - 2\,(1.8 - 1.8) = +\,5.75$ kN

At x = 9.0 m; $F_{B \text{ just left}} = -1.8 + 7.55 - 2\,(7.8 - 1.8) = -6.25$ kN

Again, considering a section between B and D, at a distance x from C,

$F_x = -1.8 + 7.55 - 12 + 9.45 - 1\,(x - 7.8) = +\,3.2 - (x - 7.8)$ ...(3)

Thus, at x = 7.8 m; $F_{B \text{ just right}} = +\,3.2 - (7.8 - 7.8) = +\,3.2$ kN

At x = 9.0 m; $F_D = +\,3.2 - (9 - 7.8) = +\,2$ kN

S.F. becomes zero between A and B, when changes sign,

$\Rightarrow F_x = +5.75 - 2\,(x - 1.8) = 0$

$\Rightarrow x = 4.675$ m from end C

(The sign is taken positive when the resultant force is in upward direction on the LHS of the section).

To draw BMD, we need B.M. at all salient points, taking a section between C and A, BM at a distance x from end C is:

$M_x = -wx^2 / 2$ ...(4)

Thus, at x = 0, $M_C = 0$

At x = 1.8 m $M_A = -1 \times (1.8)^2/2 = -1.62$ kN m

Now, considering a section between A and B, at a distance x from C;

$M_x = -1.8\,(x - 0.9) + 7.55\,(x - 1.8) - 2(x - 1.8)^2 / 2$ ...(5)

Thus, at x = 1.8 m;

$M_A = -1.8\,(1.8 - 0.9) + 7.55\,(1.8 - 1.8) - 2(1.8 - 1.8)^2 / 2$

$M_A = 1.62$ kN m

At x = 7.8 m,

$M_B = -1.8\,(7.8 - 0.9) + 7.55\,(7.8 - 1.8) - 2(7.8 - 1.8)2 / 2 = -3.12$ kN m

Again, considering a section between B and D, at a distance x from end C.

$M_x = -\,1.8\,(x - 0.9) + 7.55\,(x - 1.8) - 12\,(x - 4.8) + 9.45\,(x - 7.8) - 1 \times (x - 7.8)2 / 2$ ...(6)

Thus, at x = 7.8 m;'

M_B = –1.8 (7.8 – 0.9) + 7.55 (7.8 – 1.8) – 12 (7.8 – 4.8) + 9.45 (7.8 – 7.8) – 1 × $(7.8 – 7.2)^2$ / 2 = – 3.12 kN m

At x = 9.0 m; M_D = –1.8 (9.0 – 0.9) + 7.55 (9.0 – 1.8) – 12 (9.0 – 4.8) + 9.45 (9.0 – 7.8) – 1 × $(x – 7.8)^2$ / 2

= –14.58 + 54.36 – 50.4 + 11.34 – 0.72

= 0

The maximum bending moment occurs at a point where SF is zero, i.e., at x = 4.675 m from end C. it lies between A and B. Therefore from equation (5), we get

M_{max} = –1.8 (4.675 – 0.9) + 7.55 (4.674 – 1.8) – $(4.675 – 1.8)^2$

= + 6.655 kN m

The points of contraflexture occur at position where M_x = 0, therefore, from equation (5), we get

$-1.8 (x - 0.9) + 7.55 (x - 1.8) - (x - 1.8)^2 = 0$

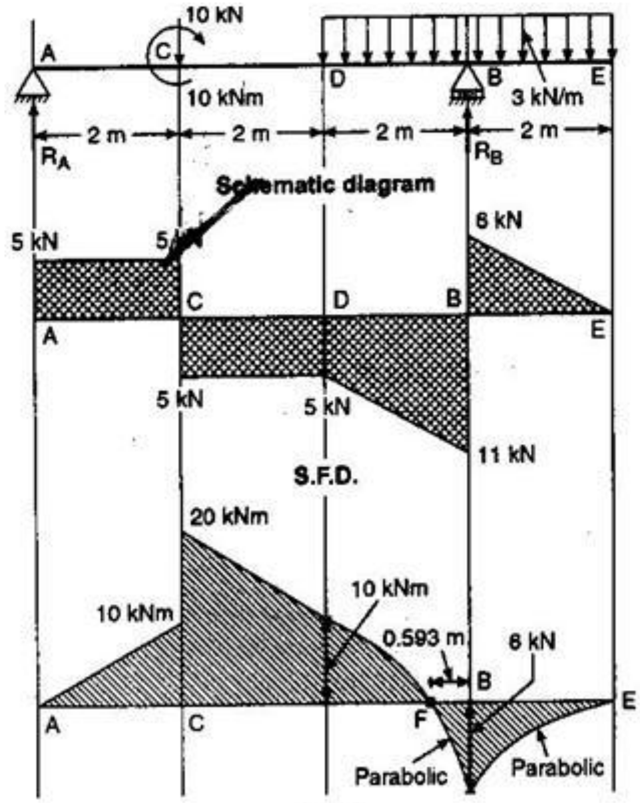
$\Rightarrow -1.8 + 1.62 + 7.55 x - 13.59 - x^2 + 3.6 x - 3.24 = 0$

$\Rightarrow -x^2 + 5.75 x + 3.6 x - 11.97 - 3.24 = 0$

$\Rightarrow -x^2 + 9.35 x - 15.21 = 0$

$\Rightarrow$ X = 7.25 m or 2.1 m

33. Draw the S.F.D. and B.M.D. for the beam shown in figure. Also find the point of contraflexure.



Solution: By taking moment of all forces about A, we get

$R_B \times 6 = (10 \times 3) + (3 \times 4 \times 6)$

$\Rightarrow R_B = 17$ kN

And $R_A = 10 + (3 + 4) - 17 = 5$ kN

To draw S.F.D. we need S.F. at all salient points.

S.F. at any section between A and C is

$F_x = +5$ kN

S.F. at any section between C and D

$F_x = +5 - 10 = -5$ kN

S.F. at any section between D and B at a distance x from D is

$F_x = -5 - 10 - 3x = -5 - 3x$

$\Rightarrow F_D = -5 - (3 \times 2) = -11$ kN

And $F_B = -5 - (3 \times 2) = -11$ kN

S.F. at any section between B and E, at a distance x from E

$F_x = 3x$

$\Rightarrow F_E = 0$

$F_B = +3 \times 2 = +6$ kN

To draw B.M.D. we need bending moment at all salient points.

B.M. at A = 0

B.M. on LHS at C = + 5 × 2 = + 10 kNm

B.M. on RHS at D = + 10 + 10 = + 20 kNm

B.M. at D = + 5 × 4 – 10 = + 10 kNm

B.M. at B = –3 × 2 × 1 = – 6 kNm

B.M. at E = 0

Point of contraflexure lies between D and B at point F. Let F be at a distance x from B. Then

$17x - 3.(x + 2)^2 / 2 = 0$ Þ $3x^2 - 22x + 12 = 0$

Or x = 0.593 m from B.

34. Draw S.F.D. and B.M.D. for the beam shown in figure.

Solution: The beam AB can be considered to consist of two parts:

(i) a cantilever ACDE; and

(ii) a simply supported beam EB.

For beam EB the reaction at B

$= R_B = 3 \times 10 / 2 = 15$ kN

The common reaction at E = 15 kN acting upwards in the beam and downwards in the cantilever.

Now, S.F.D. for cantilever and simply supported beam can easily be drawn.

To draw S.F.D. we need S.F. at all salient points.

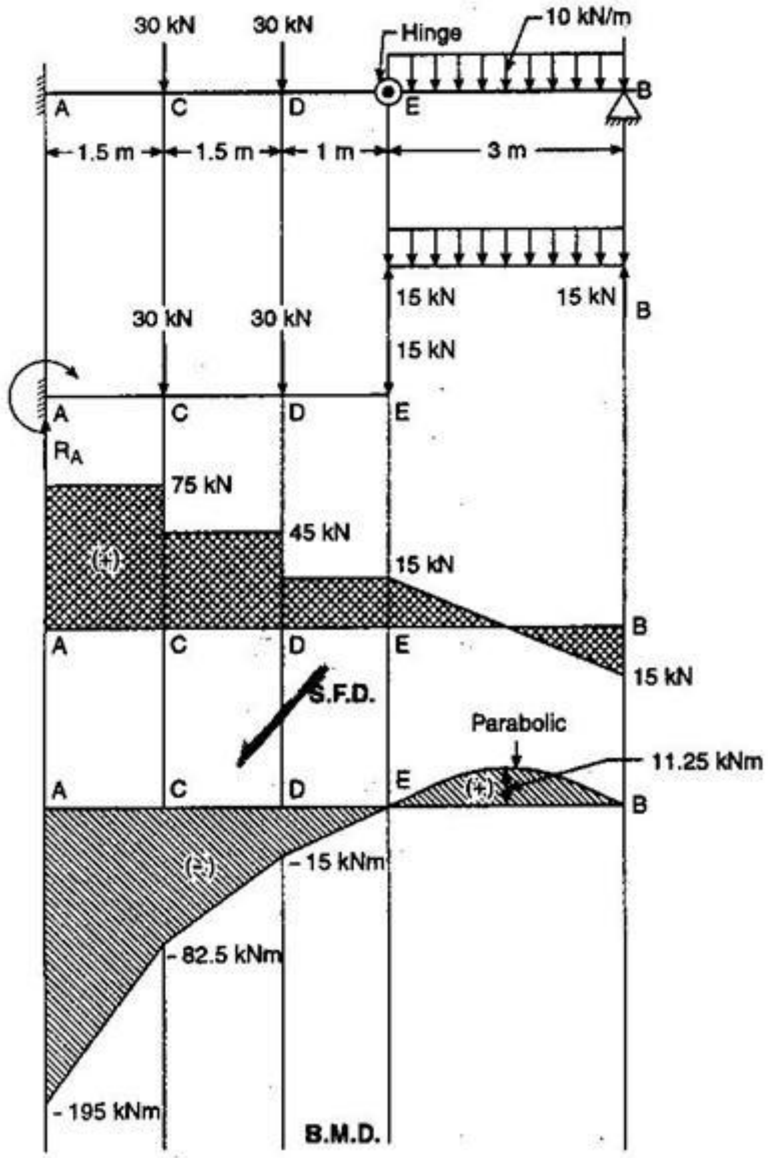
$F_B = -15$ kN

$F_E = -15 + 10 \times 3 = 15$ kN

$F_D = +15 + 30 = 45$ kN

$F_C = +15 + 30 + 30 = 75$ kN

$F_A = 75$ kN



Now, to draw B.M.D. we need bending moment at all salient points.

B.M. at E = 0

B.M. at D = –15 × 1 = –1.5 kNm

B.M. at C = –15 × 2.5 – 30 × 1.5 = –82.5 kNm

B.M. at A = –15 × A – 30 × 3 – 30 × 1.5 = –195 kNm

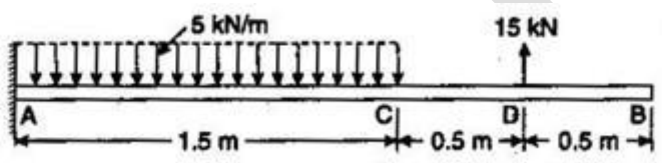
35. What force and moment is transmitted to the supporting wall at A in the given cantilever beam.

Solution: First we draw the Free Body Diagram or the loaded beam

For equilibrium: $\Sigma F_x = 0$ ...(1)

$\Sigma F_y = 0$ ...(2)

$\Sigma M_B = 0$ ...(3)



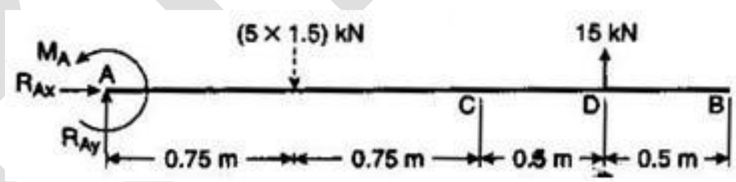
From equation (1), we get

$R_{Ax} = 0$

From equation (2), we get
$R_{Ay} – 5 \times 1.5 + 15 = 0$

$R_{Ay} – 7.5$ kN

$R_A = \sqrt{R_{Ax}^2 + R_{Ay}^2} = \sqrt{(0)^2 + (-7.5)^2} = -7.5$ kN



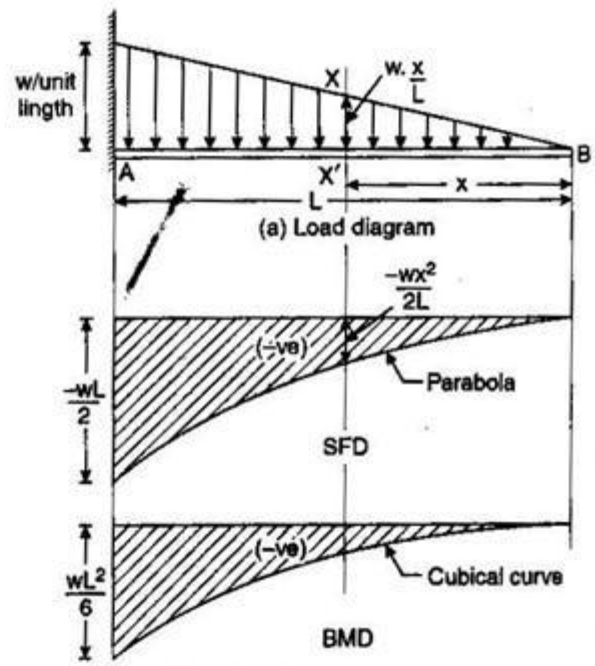
From equation (3), we get

$–M_A – 7.5 \times 2.5 – (5 \times 1.5) \times (1.5/2 + 1) + 15 \times 0.5 = 0$

$M_A = 24.37$ kNm.

36. Cantilever subjected to uniformly varying load over whole length

Let the cantilever AB, of length L meter, carrying load, as shown in figure. Then, at any section X – X' at a distance x from B, the intensity of load is



w/L.x = wx/L

Total load from B to X – X' = Area of the load diagram from B to X – X'.

= 1/2.x.w.x/L = $wx^2/2L$

Now,
Shear force at section (X – X') 'F_x'

= –1/2 . w.x/L . x = $-wx^2$ / 2L

(i.e., Parabola with concave curve)

At x = 0; $F_B = 0$

x = L, FA = –w.L / 2

Bending moment at section (X – X')

'M_x' = $-w.x^2$ / 2L × 1/3 x = $-$ $w.x^2$ / 6L

(i.e., Cubic cubic with concave surface)

At x = 0; $M_B = 0$

At x = L; $M_B = –wL^3 / 6L = –2L^2 / 6$

37. Simply supported beam subjected to uniformly varying load

Let a simply supported beam AB of length 'L' be subjected to a load uniformly from zero at support A and w/unit length at support B, as shown in figure. Then, at any section (X – X') at a distance x from A, the intensity of load = wx/L.

Since $d F_x / dx = –wx$

$F_x = - \int w_s\, dx = - \int w_x / L\, dx = - wx^2 / 2 L + C_1$

At x = 0, $F = R_A$

$\Rightarrow F_x = –wx^2 / 2L + R_A$ ...(1)

Further, $dMx / dx = F_x$

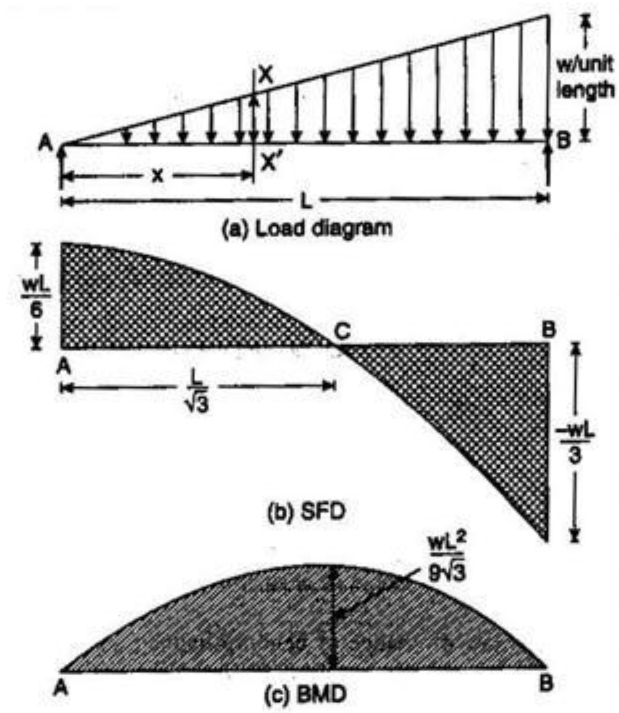
$\Rightarrow M_x = \int F_x.dx = - wx^2) / 2L + C_1$

At x = 0, M = 0 $\Rightarrow C_2 = 0$

$\Rightarrow M_x = wx^3 / 6L – R_A x$ ...(2)

Now, $R_A = (1/2\ wL)\ (1/3\ L) / L = 1/6\ wL$

$R_B = wL / 2 – wL/6 = wL/3$



$F_x = –wx_2/2L + 2L/6$

$M_x = –wx^3 / 6L + wL/6$

Shear force at x = 0;

$F_A = 0 + wL/6 = wL/6$

Shear force at x = L;

$F_B = –w.L^2 / 2L + wL/6 = –wL/3$

The shear force will have zero value at C = $wL/6 – wx^2 / 2L$

$\Rightarrow x= 1/\sqrt{3}$

Bending moment at x = 0; $M_A = 0$

Bending moment at x = L; $M_B = 0$

Bending will be maximum where shear force is zero, i.e., $x = L /\sqrt{3}$.

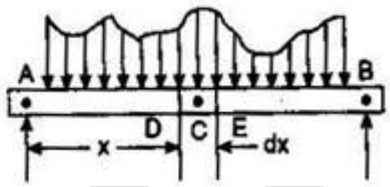
$M_{max} = wL / 6. L/\sqrt{3} - wL^3 / 6 \times 3 \sqrt{3}.L = wL^2 / (6 \sqrt{3}\ 1-1/3)$

$Mmax = wL^2 / 9\sqrt{3}$

38. RELATIONSHIP BETWEEN LOADING, SHEAR FORCE, AND BENDING MOMENT

The relationship developed between load, SF and BM is suitable for any type of distributed loading, but not for all cases of concentrated loading. Consider a differential element dx of a continuously loaded thin beam with a variable loading 'w' per unit length, such that

w = w(x)



Let the shear force changes from F to [F + d(F)] and Bending moment changes from M to [M + d(M)] over the length dx. For equilibrium;

(i) The sum of all the forces in y direction must be zero.

⇒ [F + d(F)] – F + w.dx = 0

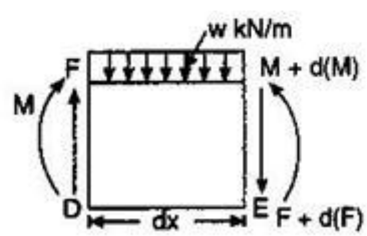
⇒ d(F)/dx = –w ...(1)

(ii) The sum of all the moments in the plane of the bending i.e., about Z axis must be zero. The load on the portion DE, w dx can be regarded as acting at mid point 'C'. Therefore, taking moment about E, we get

$M – [M + d(M)] + F.dx – w.dx. dx/2 = \Sigma M_E = 0$

$d(M) = F.dx – w . (dx)^2/2$

As dx -> 0, $(w.(dx)^2/2)$ can be neglected.



⇒ d(M)/dx = F ...(2)

Also, $d^2(M)/dx^2 = -w$ ...(3)

Thus, it can be concluded that:

(i) The rate of change of shear force along the length of the beam equals the leading with negative sign.

(ii) The rate of change of bending moment along the length of the beam equals the shear force.

(iii) The second derivative of BM with respect to the length of the beam equals the loading at that cross-section.

(iv) Integrating equation (2) between the values x = a and b we get

$(M)_b - (M)_a = \int_a^b w\, dx$ ...(3)

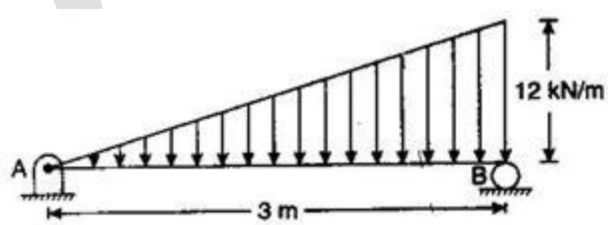
(v) Similarly, integrating equation (1), between the value x = a and b

(vi) The B.M. will be maximum when dM/dx = 0. Since, dM/dx = = F, hence, B.M. is maximum where S.F. is zero or changes sign.

39. Figure shows a beam pivoted at A and simply supported at B and carrying a load varying from O at A to 12 kN/m at B. Determine the reactions at A and B and draw the bending moment diagram.

Solution: At any section (X – X) at a distance x from A, the rate of loading,

$w_x = w. x/L$

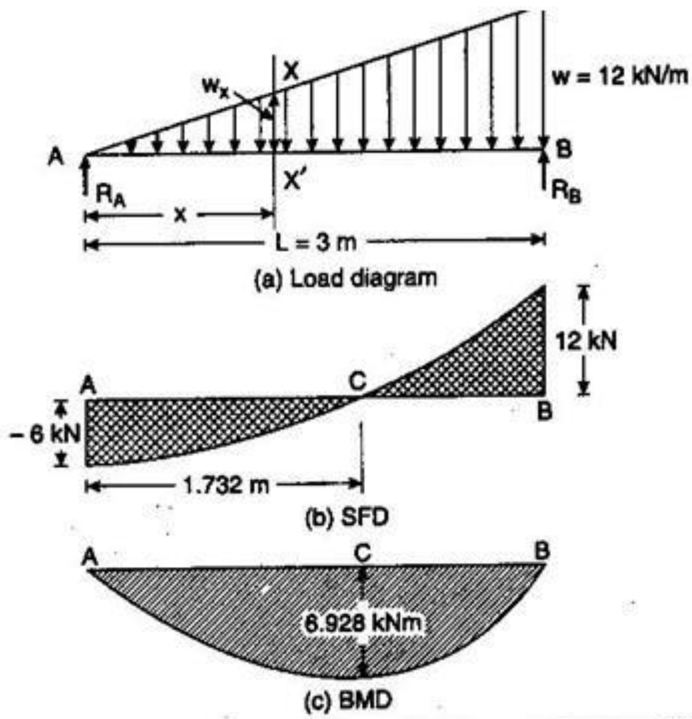


But $dF_x / dx = w_x$
$\Rightarrow F_x = \int w_s. d_s = \int wx/L. dx$

$\Rightarrow F_x = w/L . x^2/2 + C_1$

At x = 0 F = $-R_A$

⇒ $F_x = w/2L.x^2 - R_A$ ...(1)



(a) Load diagram

(b) SFD

(c) BMD

Further, we know that

Now, Bending moment at A = w/6L × 0 – 6 = 0

$M_x = \int f_x. d_x = \int [w/2L. x^2 - R_A]. dx$

$= w/2L.x^3/3 = R_A.x + C_2$

when x = 0, M = 0, $C_2 = 0$

Hence, $M_x = w/6L\ x^3 - R_A.x$ ...(2)

Now, R_A = (1/2 wL).(1/3L) / 2 = 1/6 wL = 1/6 × 12 × 3 = 6 kN

R_B = wL/2 – wL/6 = wL/3 = 12 × 3 / 3 = 12 kN

F_x = w/2L.0 –1/6 wL

⇒ Shear force at A = w/2L.0 –1/6 wL = –6 kN

Shear force at B = w/2L.L2 – 1/6 wL = 12/2 × 3 × (3)2 – 1/6 × 12 × 3 = 12 kN

Shear force will be zero at C,

$w/2L\ x^2 - 1/6\ wL = 0$

$\Rightarrow x^2 = 1/6\ wL \times 2L / w = L^2 / 3$

$x = L/\sqrt{3} = 0.577 = 1.732m$ from A.

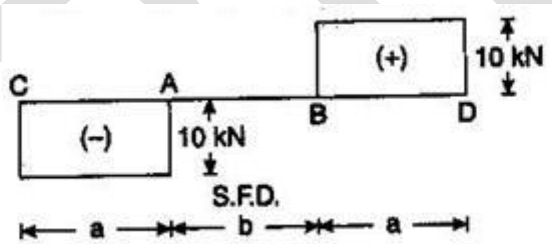
Now, Bending moment at A = $w/6L \times 0 - 6 = 0$

Bending moment at B = $w/6L \times L^3 - wL / 6.L = 0$

Bending moment at the point where shear force is zero, i.e., point C

$M_c = 12/ 6 X 3 X (1.732)^3 - 6 X (1.732) = -6.928$ kNm

40. The shear force diagram of a simply supported beam at A and B is given in figure, calculate the support reactions and draw the BMD of the beam.



Solution: Let R_A and R_B be the reactions at A and B respectively.

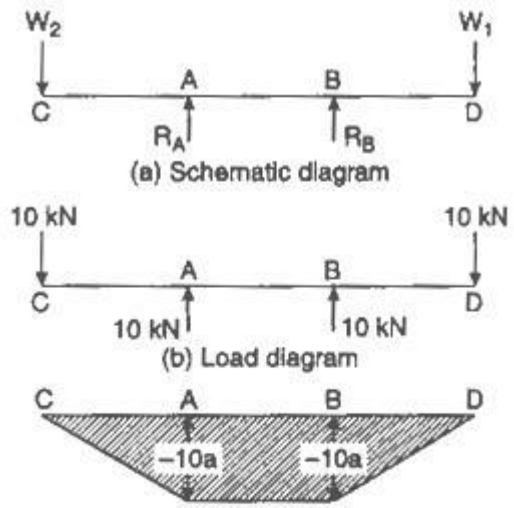
Shear force = Total unbalanced vertical force on either side of the section.

From S.F.D. load at D $W_1 = + 10$ kN

S.F. at B = 0

$W_1 - R_B = 0$

$\Rightarrow 10 - R_B = 0$



(a) Schematic diagram
(b) Load diagram
(c) BMD

$\Rightarrow R_B = 10$ kN

S.F. at A = – 10 kN

$W_1 - R_B - R_A = -10$ kN

$\Rightarrow R_A = 10$ kN

SF at C = 0

$W_1 - R_B - R_A + W_2 = 0$

$\Rightarrow 10 - 10 - 10 + W_2 = 0$

$\Rightarrow W_2 = 10$ kN

Bending moment = Algebraic sum of moments caused by vertical loads acting on either side of the section.

Bending moment between D and B; $M_x = -W_1x$

Bending moment at D = – 10 × 0 = 0

Bending moment at B = 10 × a = –10a

Bending moment between B and A; $M_x = -W_1x + R_B(x - a)$

Bending moment at B = –10.a + 10 (a – a) = –10 a

Bending moment at A = –10.(a + b) + 10 (a + b – a)

= –10 a – 10b + 10b = – 10 a

Bending moment at A and C;

$M_x = -W_1x + R_B(x - a) + R_A(x - a - b)$

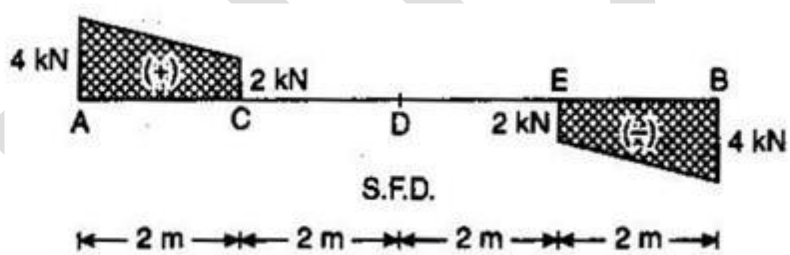
Bending moment at A = –10.(a + b) + 10 (a + b – a) + 10 (a + b – a – b)

= –10 a

Bending moment at C = –10 (a + b + c) + 10 (2a + b + c) + 10 (2a + b – a – b)

= –20a – 10b + 10a + 10b + 10a = 0

41. The shear force diagram (SFD) of a simply supported beam is given in figure. Calculate the support reactions of the beam and also draw the bending moment diagram of the beam.



Solution: Let the reactions at A and B be R_A and R_B respectively. Also, the point loads at C, D and E are W_1, W_2 and W_3 respectively.

Now, from S.F.D.

Shear force at A = $R_A \Rightarrow R_A = 4$ kN

Shear force at a section between A and C = $R_A - w_1.x$

Shear force at C = $R_A - w_1 \times 2 = 2$

$\Rightarrow 4 - w_1 \times 2 = 2$

$\Rightarrow w_1 = $ kN/m

Shear force at C due to load = $R_A - w_1 \times 2 - W_1 = 0 \Rightarrow W_1 = 2$ kN

Shear force at D = $R_1 - w_1 \times 2 - W_1 = 0 \Rightarrow W_1 = 2$ kN

$\Rightarrow 4 - 1 \times 2 - 2 - W_2 = 0$

$\Rightarrow W_2 = 0$

Shear force at E $R_A - w_1 \times 2 - W_1 - W_2 - W_3 = -2$ kN

$\Rightarrow 4 - 1 \times 2 - 2 - 0 - w_3 \times 2 = -2$

$\Rightarrow W_3 = 2$ kN

Shear force at B = $R_A - w_1 \times 2 - W_1 - W_2 - W_3 - w_2 \times 2 = -4$ kN

$\Rightarrow 4 - 1 \times 2 - 2 - 0 - w_2 \times 2 = -4$

$\Rightarrow w_2 = 1$ kN/m

For equilibrium $SF_y = 0$

$\Rightarrow R_A - (1 \times 2) - 2 - 2 - (1 \times 2) + R_B = 0$

$\Rightarrow 4 - 2 - 2 - 2 - 2 + R_B = 0$

$\Rightarrow R_B = 4$ kN

Now, bending moment at A = 0

Bending moment at a section between A and C = $R_A.x - w1.x.x/2$

$\Rightarrow (BM)_C = 4 \times 2 - (1 \times 2 \times 2/2) = 6$ kNm

Since, there is no load between C and E the bending moment will remain unchanged between C and E. For check. We can see that

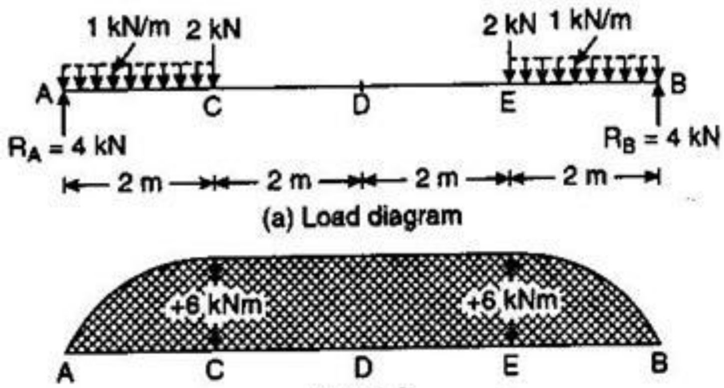
Bending moment at D = $4 \times 4 - 1 \times 2 \times (2/2 + 2) - 2 \times 2 = 6$ kNm

Bending moment at E = $4 \times 6 - 1 \times 2\ (2/2 + 4) - 2 \times 4 = 6$ kNm

Now, bending moment at a section between E and B at a distance x from A

⇒ = RA.x – 1 × 2 × (x – 1) – 2 × (x – 2) × (x – 6) – 1 × (x – 6).(x – 6)/2

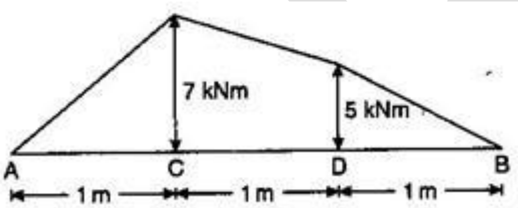
⇒ $(BM)_E$ = 4 × 8 – 1 × 2 × 7 – 2 × 6 – 2 × 2 – 1 × 2 × 2/2 = 0 kNm



(a) Load diagram

(b) BMD

42. The BMD of a simple supported beam is shown in figure. Calculate the support reactions of the beam.

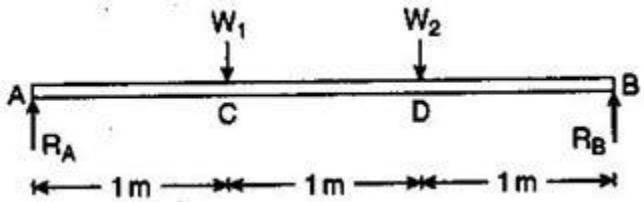


Solution: Since, we know that $d^2M/dx^2 = -w$

i.e., the second derivative of BM with respect to length of the beam equals the loading at that cross-section.

Thus, the load here will be point load type.

Let, the reactions at A and B be R_A and R_B respectively.



Now, bending moment at D = $R_B \times 1$

$\Rightarrow$ 5 kNm = $R_B \times 1$

$\Rightarrow R_B = 5$ kN

Now, bending moment at C = $R_B \times 2 - W_2 \times 1$

$\Rightarrow$ 7 kNm = $R_B \times 2 - W_2 \times 1$

$\Rightarrow 7 = 5 \times 2 - W_2 \times 1$

$\Rightarrow W_2 = 3$ kN

Now bending moment at A = $R_B \times 3 - W_2 \times 2 - W_1 \times 1$

$\Rightarrow 0 = 5 \times 3 - 3 \times 2 - W_1$

$\Rightarrow W_1 = 9$ kN

Now, $\Sigma F_y = 0$ for equilibrium

Therefore, $-R_A + W_1 + W_2 - R_B = 0$

$\Rightarrow -R_A + 9 + 3 - 5 = 0$

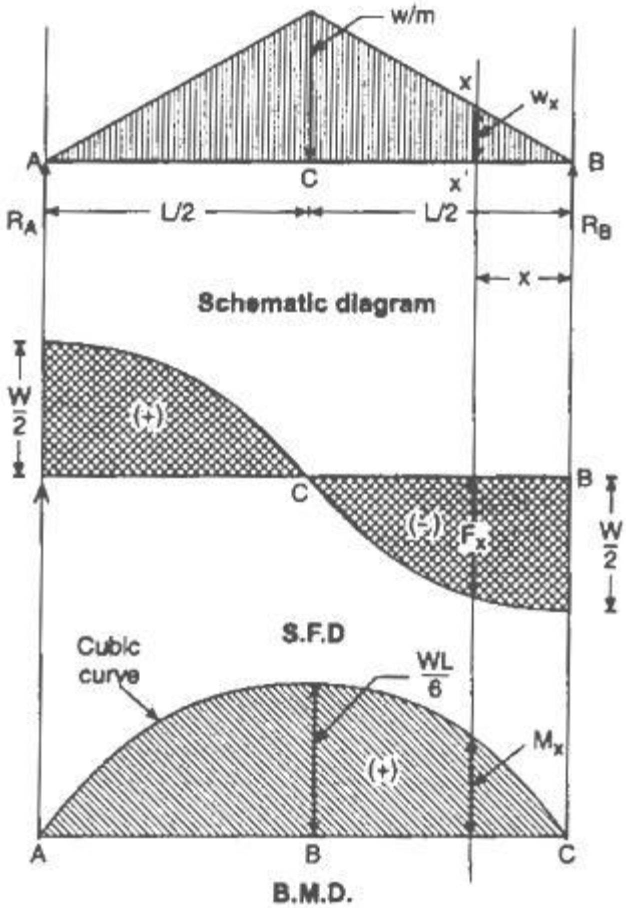
$\Rightarrow$ RA = 7 kN

43. A simply supported beam of span L is loaded with distributed load of intensity zero at the ends and w per unit length at the centre.

Solution: Since, the increase in load is symmetrical form both the ends, therefore reactions R_A and R_B will be equal.

Or $R_A = R_B$ = 1/2 .w.L/2 = wL/4 = w/2

Where W = total load on the beam



The shear force at any section (X – X') at a distance x from end B is:

$F_x = -R_B + wx^2/L$

$= - wL/4 + w.x^2/L$

$= -W/2 + wx^2/L$ ...(1)

We see that equation (1) represents a parabolic change in F_x

$F_B = -W/2$

$F_C = -W/2 + w.(L/2)^2/L$

$= -W/2 + w/L.(L^2/4)$

$= -W/2 + W/2 = 0$

Thus, we can say that S.F. is equal to –W/2 at B where x = 0 and increases in form of a parabolic curve to zero at C, beyond C it continue to + W/2 and A, where x = L.

The bending moment at any section (X – X') at a distance x from B.

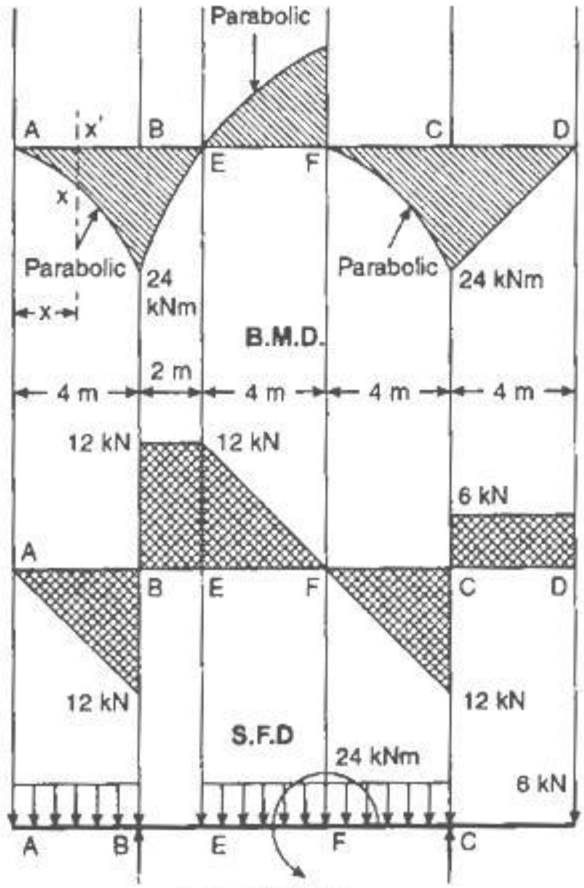
$M_x = R_B. X - wx/L * x/2 * x/3 = wL/4. x = wx^2/3L$ ...(2)

Thus, we can see that equation (2) represents a cubic curve with bending moment equal to zero at the ends A and B. The bending moment will be maximum at C, because S.F. changes sign at this point

$Mc = wL/4 (L/2) - w/3L (L/2)^3$

$= wL^2/12 = W.L/6$

44. A beam ABCD is simply supported at B and C. It has overhangs on both sides. The bending moment diagram for the beam is shown in figure. Draw the S.F.D. and load diagram for the beam.



Load diagram

Solution: Since we know that S.F. = dM_x / dx

We can calculate S.F. at all; salient points.

Part AB: At any section (X – X') between A and B at a distance x from A, the bending moment is giveb by;

$M_x = -x^2/(4)^2 \times 24 = -1.5\,x^2$

S.F. = $dM_x / dx = d/dx\,(-1.5\,x^2) = -3x$

At x = 0; $F_A = 0$

At x = 4 m; $F_B = -12$ kN

Part BE: At a section between B and E at a distance x from B, the bending moment is given by:

$24 - M_x = (4-x)^2 / 4^2 \times 24 = 24 - 1.5\,(4-x)^2$

$dM_x/dx = d/dx\,(24 - 1.5\,(4-x)^2) = 3(4-x)$

At E, $F_E = + 12$ kN

At F, $F_F = 3(4 – 4) = 0$

Part EC: At any section between F and C at a distance x from F the bending moment is given by:

$M_x = –x^2/(4)^2 \times 24 = –1.5\ x^2$

$F_x = dM_x / dx = –3x$

$\Rightarrow$ At F; $F_F = 0$

At C; $F_C = –3 \times 4 = –12$ kN

Part CD: At any section between C and D at a distance x from C, bending moment is given by

$M_x = –(4 – x)/4 \times 24 = –24 + 6x$

$F_x = d/dx\ (–24 + 6x) = + 6$ kN

Load Diagram:
If can be draw by observing the nature of the change of SFD.

Between A and B the S.F. varies uniformly, therefore a UDL acts between A and B.

$d/dx\ F_x = –d/dx\ (–3x) = + 3$ kN

since at F there is a sudden change in B.M. from 24 kNm to zero, there should be a couple of 24 kNm at F acting in anticlockwise direction.

At C there is a support and a sudden change in S.F. from –12 kN to + 6 kN. Hence the reaction at support C = 18 kN.

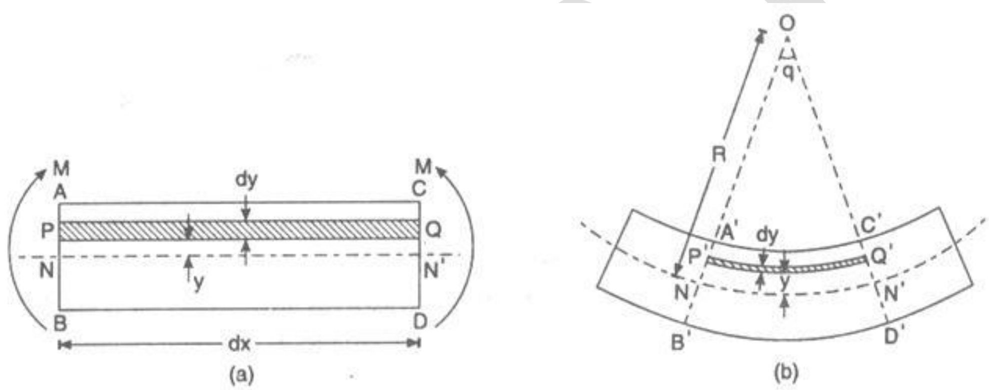
The S.F. is constant between C and D, therefore there is no load between C and D.

Now, at D there is a sudden change in SF from 6 kN to zero. Hence there should be a point load of kN at D.

45. SIMPLE BENDING THEORY

Bending stress is resistance offered by the internal stresses to the bending caused by bending moment at a section. The process of bending stops, when every cross-section sets up full resistance to the bending moment. It only couples are applied to the ends of the beam and no forces acted on the bar, the bending is termed as pure bending. For example, the portion of beam between the two downward forces is subjected to pure bending. The bending produced by forces that do not form couples is called ordinary bending. A beam subjected to pure bending has only normal stresses with no shearing set up in it. A beam subjected to ordinary bending has both normal and shearing stresses acting within it.

Now, it is a length of beam is acted upon by a constant bending moment (zero shearing force) as shown below in figure. The bending moment will be same at all point along the bar i.e., a condition of pure bending.



To determine the distribution of bending stress in the beam, let us cut the beam by a plane passing through it in a direction perpendicular to the geometric axis of the bar. Let the sections be AB and CD normal to the axis of the beam. Due to the action of bending moment, the beam as a whole bend as shown in figure.

Since, we are considering a small length of dx of the beam, the curvature of the beam could be considered to be circular. Because of it, top layer of the beam AC suffers compression and lowest layer BD suffers extension.

The amount of compression or extension depends upon the position of the layer with respect to NN'. The layer NN' is neither compressed nor stretched and has zero stress. It is called neutral surfaces of neutral plane.

From figure, we have

Length of layer PQ after applying moment = P'Q'

⇒ Decrease in length of PQ = PQ – P'Q'

Strain 'ε' = PQ – P'Q' / PQ

Also, from geometry of the curved beam, sections

OP'Q' and ONN' are similar

P'Q' / NN' = R – y / R

1 – P'Q' / N'N' = 1 – (R – y / R)

⇒ NN' – P'Q' / NN' = R – R + y / R

Since from figure

NN' = PQ

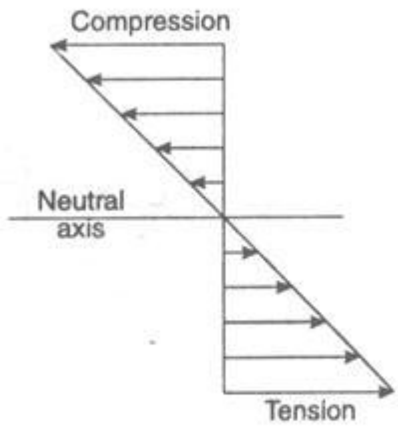
NN' = P'Q' / PQ = y/R

Using equation (1) and equation (2)

ε = y / R

Thus, the strain produced in a layer is proportional to its distance from neutral axis.

Now E = σ/ε ⇒ σ.ε.E



⇒ σ = y/R.E

Or σ / y = E / R ...(4)

Where, E is the modulas of elasticity

R is radius of curvature of neutral axis

y distance of layer under consideration from neutral axis

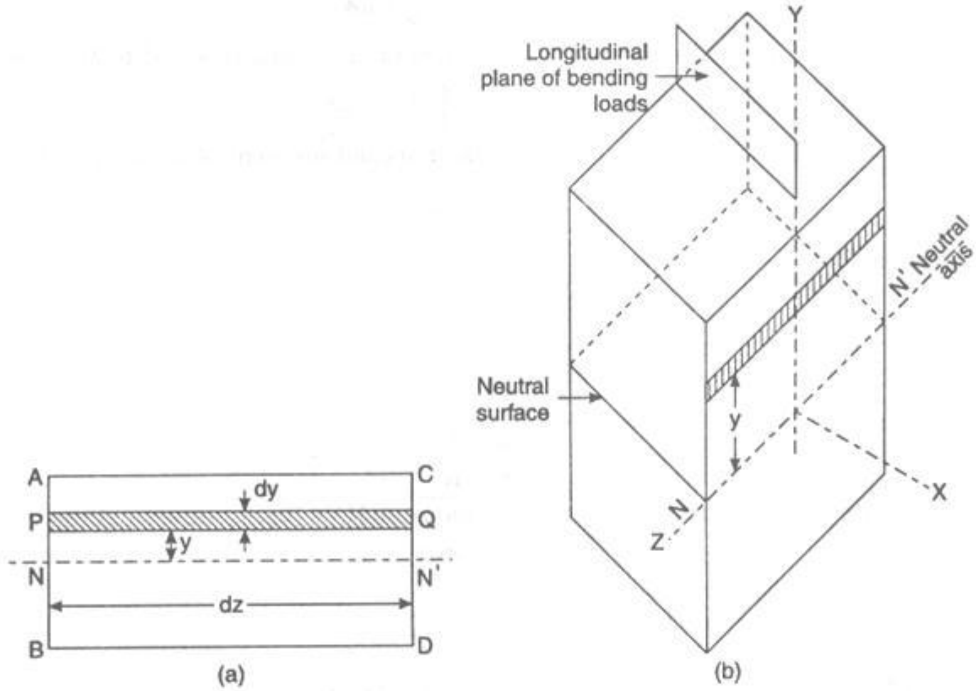
Since, E/R is constant, the stress is proportional to the distance from neutral axis (NN'). Thus, of the purpose of economy and weight reduction the material should be concentrated as much as possible at the greatest distance from neutral axis. Hence, I sections should be preferred.

46. Neutral Axis and Location Of Beam

The line of intersection of the neutral plane, with any normal cross-section of the beam, is known as neutral axis of that section.

Let NN' be neutral axis of the section. Then, for any layer PQ at a distance y from neutral axis.

σ/y = E/R



If the area of layer PQ be dA

Total stress on this laeyr = σ × Area = y × E/R × dA

Total stress of the section = ∫y.E/R.dA =E/R ∫y.dA

E/R ∫y dA = 0

For the section to be in equilibrium therefore total stress from top to bottom must be equal to zero, since E/R is constant it can't be zero, , for the section to be in equilibrium. The quantity y.dA is the moment of the dA about the neutral axis and is the moment of the entire area of the cross-section about the neutral axis. Thus, the neutral axis must be located in such way that the summation of moment of all elemental areas about it is zero.

Since, the moment of any area about an axis passing through its centroid, is also equal to zero. It means that that the neutral axis also passes through the centroid. Hence, the neutral axis of a section can be located by locating its centroid.

47. Moment of Resistance

We see the stresses on one side of neutral axis are compressive and on the other side are tensile in nature. These stresses form a couple, whose moment must be equal to the external moment M. The moment of this couple, which resists the external bending moment, is known as moment of resistance. Thus, the bending moment which can be carried by a given section for a limiting maximum stress is called the moment of resistance. From figure

Moment of section PQ about the neutral axis

$= y \times E/R \times dA \times y = E/R\, y^2 dA$

The algebraic sum of all such moments about the neutral axis must be equal to M Therefore,

$M = \int E/R\, y^2 .dA = E/Ry^2 .dA$ ...(1)

The quantity $\int y^2 dA$ is called moment of inertia or second moment of area. Let it be represented by I. Then

$I = \int y^2 dA$ ...(2)

From equation (1) and equation (2)

$m = E/R.I$

Or $M/I = E/R$

From equations above

$\sigma/Y = M/I = E/R$

where; s = bending or flexure stress due to bending moment M.

I = moment of Intertia or second moment of area

R = Radius of curvature of beam

y = Distance of point from neural axis

48. Assumptions in simple bending theory

(i) The material is homogenous, isotropic, and has the same value of modulus of elasticity from compression and tension.

(ii) The beam material is stressed within the elastic limit and thus follows Hooke's Law,

(iii) The transverse cross-section remain plane and perpendicular to the neutral surface after bending.

(iv) The beam is initially straight and all longitudinal filaments bend into circular arcs with a common center of curvature.

(v) The plane of loading must contain a principal axis of the beam cross-section and the loads must be perpendicular to the longitudinal axis of the beam.

49. Section Modulus

It can be defined s the ratio of moment of inertia of a section about the neutral axis to the distance of the outermost layer from the neutral axis. It is denoted by Z. Thus,

$Z = I/y_{max}$

Since, $M/I = \sigma/y$

$\Rightarrow$ $M/I =$ $M/I = \sigma_{max}/Y_{max}$ $Z = I/y_{max} = M/\sigma_{max}$

Hence, the moment of resistance (M) offered by section is maximum when section modulus is maximum. Thus, the section modulus represent the strength of the section.

50. Center of Gravity

Every body has only one point at which whole the weight of body can be supposed to be concentrated, this is called center of gravity (C.G.) of the body.

For plane figures (like circle, rectangle, quadrilateral, triangle etc.), which have only areas and no mass, the point at which the total area of plane is assumed to be concentrated, is known as the controid of that area.

The center of gravity and centroid are at the same point for plane body.

Determination of centroid of an area: The centroid of an area is defined by the equations.

$\bar{x} = \int x\, dA/A$, y ~= $\int t\, dA/A$

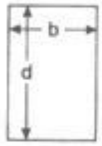
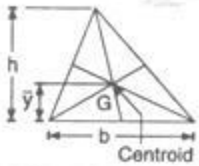
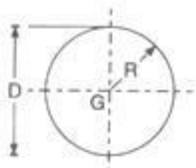
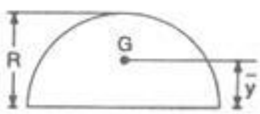
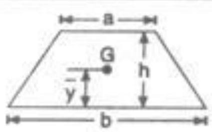
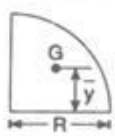
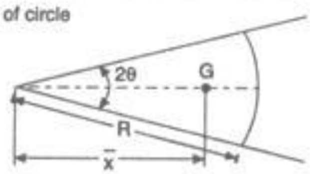
Centroid = $(\bar{x}, \bar{y})$ on x-y plane.

where A denotes the area. For a composite plane area, composed on N subareas Ai, each of whose centroidal coordinates $\bar{x}_i$ and $\bar{y}_i$ are known. The integral is replaced by summation.

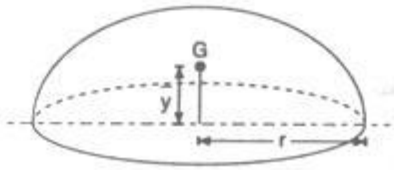
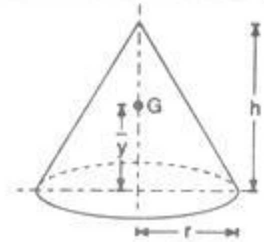
$$\bar{x} = \frac{\sum_{i=1}^{N} \bar{x}.A_i}{\sum_{i=1}^{N} A_i}, \qquad \bar{y} = \frac{\sum_{i=1}^{N} \bar{y}_i.A_i}{\sum_{i=1}^{N} A_i}$$

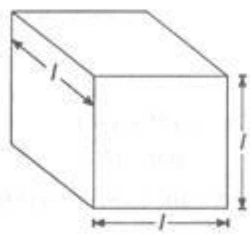
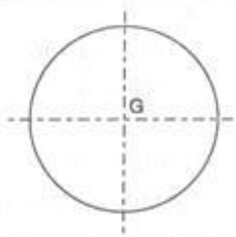
Centroid of few common figures are given in Table.

Table

Type of section		Area	Location of centroid
Rectangle or paralleogram		b . d	Geometric center where diagonal meet each other
Triangle		$\frac{1}{2}$ b . h	Point of intersection of median $\bar{y} = \frac{h}{3}$
Circle		πR^2 or $\pi/4\ D^2$	Geometric center
Semicircle		$\frac{1}{2}\pi R^2$ or $\pi/8\ D^2$	$\bar{y} = \frac{4R}{3\pi}$
Trapezium		$\frac{1}{2}(a+b) . h$	$\frac{h}{3}\left(\frac{b+2a}{b+a}\right)$
Quadrant of circle		$\frac{\pi R^2}{4}$	$\bar{y} = \frac{4R}{3\pi}$
Sector of circle		$\theta . R^2$ 'θ' in radian	$\bar{x} = \frac{2R \sin q}{3\theta}$ θ in radian

Table

	Type of section	Location of center of gravity
Hemisphere		$\bar{y} = \frac{3r}{8}$ from base along vertical radius.
Right circular solid cone		$\bar{y}$ = h/4 from base along the vertical axis

	Type of section	Location of center of gravity
Cube		l/2 from every face, where l is the length of each side
Sphere		d/2 from every point on the surface or geometrical center.

51. A steel bar 10 cm wide and 8 mm thick is subjected to bending moment. The radius of neutral surface is 100 cm. Determine maximum and minimum bending stresses in the beam.

Solution: Since it is a case of pure bending. Therefore using bending stress theory

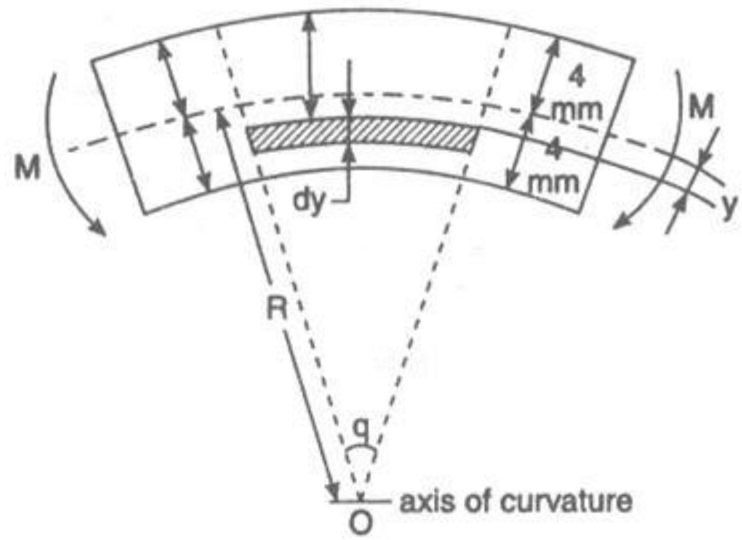
$\sigma/y = E/R$

Where E is the modulus of elasticity

R = radius of curvature of neutral axis

y = distance of layer under consideration from neutral axis.

σ = bending or flexure stress.



Given that R = 100 cm

And material of beam is steel, therefore $E = 2 \times 10^{11}$ N/m^2

Thus, the maximum and minimum bending stress can be given as:

$$\sigma_{max} = \frac{E \cdot y_{max}}{T}$$

$$= \frac{2 \times 10^{11} \times 4 \times 10^{-3}}{100 \times 10^{-2}}$$

$$8 \times 10^{8} N/m^{2} \;= 800\, MPa$$

$$\sigma_{min} = \frac{E\, y_{min}}{R}$$

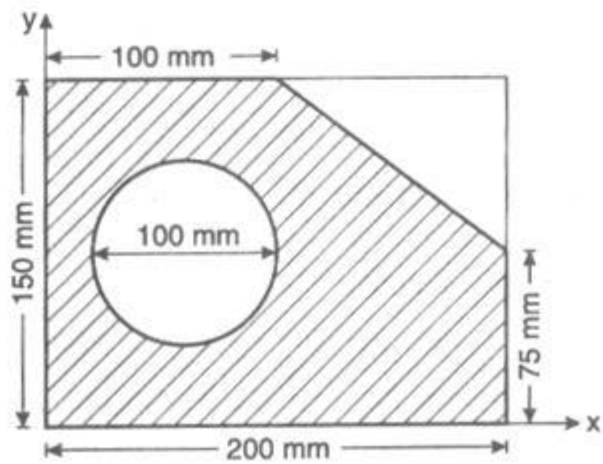
$$= \frac{2 \times 10^{11} \times (-4 \times 10^{-3'})}{100 \times 10^{-2}}$$

$$= \; -80\, MPa$$

⇒ σ_{max} = 800 MPa (Tensile)

σ_{min} = –800 MPa (comressive)

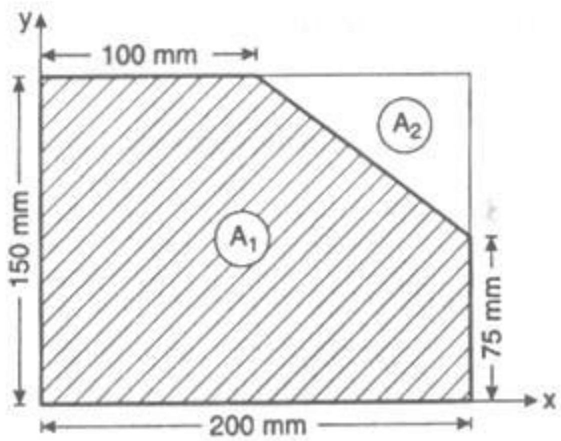
52. Determine the coordinates x_c and y_c of the centre of a 100 mm diameter circular hole cut in a thin plate, so that this point will be the centroid of the remaining shaded area shown in figure.



Solution:

If a circular hole is cut from the plate, in such a way that the centre of the circle coincide with the centre of gravity of the plate, the centre of gravity of the remaining plate will remain unchanged. Thus, we need to find out the centroid of the pentagonal solid plate, without taking the view of circular hole.

Now, from figure



$A_1 = 200 \times 150 = 30{,}000\ mm^2$

$A^2 = 1/2 \times 100 \times 75 = 3750\ mm^2$

$x_1 = 100$ mm

$_1 = 100$ mm

$y_1 = 75$ mm

$x_2 = 100 + 2/3\ (100) = 166/67$ mm

$y_2 = 75 + 2/3\ (75) = 125$ mm

$x_c = A_1x_1 – A_2x_2 / A_1 – A_2$

= (30000 × 100) – (3750 ×166.67) / 30000 – 3750 = 90.47 mm

$y_c = A1y_1 - A_2y_2 / A_1 - A_2$

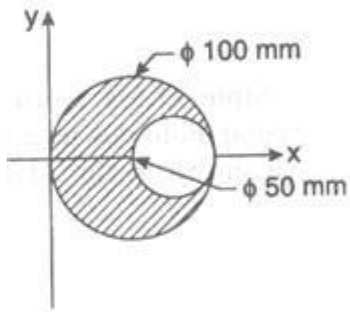
= (30000 × 75) – (3750 – 125) / 30000 – 3750 = 67.85 mm

[Ans: x_c = 90.47 mm from y axis. y_c = 67.85 mm from x axis]

53. From a circular plate of diameter 10 mm, a circular part is cut out whose diameter is 50 mm. Find the centroid of the remainder.

Solution: By definition

$$\bar{x} = \frac{A_1 x_1 = A_2 x_2}{A_1 = A_2}$$



$$\bar{y} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2}$$

Where,

A_1 = Area of the circle with Φ 100 mm

A_2 = Area of the circle with Φ 50 mm

x^- = Position of centroid on x axis

y^- = Position of centroid of y axis

Here

$A_1 = \pi/4\ (100)^2 = 7853.98\ mm^2$

$A_2 = \pi/4\ (50)^2 = 1963.5\ mm^2$

x_1 = 100/2 = 50 mm

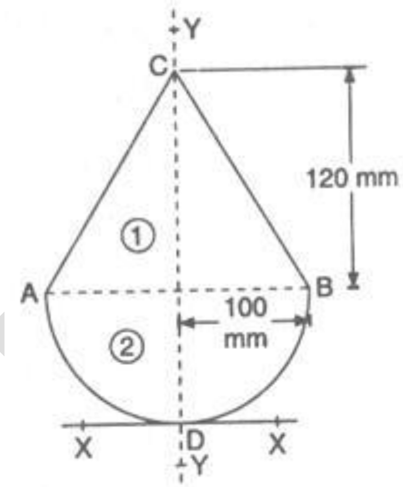
x_2 = 100 – 25 = 75 mm

X^- = (7853.98 X 50) - (1963.5 X 75)/(7853.98 - 1963.5) = 41.67mm

Since, the figure is symmetric about x axis the centroid will lie on x axis.

54. A body consists of a right circular solid cone of height 120 mm and radius 100 mm placed on a solid hemisphere of radius 100 mm of the same material. Find the position of center of gravity.

Solution: As the body is symmetrical bout Y – Y axis, the CG will lie on Y – Y axis.



Let be the distance between CG and point D (Point on XX axis)

Then

$$\bar{y} = \frac{v_1 . y_1 + v_2 . y_2}{v_1 + v_2}$$

$$= \frac{\left(\frac{\pi}{3} \times (100)^2 \times (120)\right) . \left(100 + \frac{120}{3}\right) + \left(\frac{2}{3}\pi (100)^3\right)\left(100 - \frac{3}{8}(100)\right)}{\left(\frac{\pi}{3} \times (100)^2 \times (120)\right) + \left(\frac{2\pi}{3} \times (100)^3\right)}$$

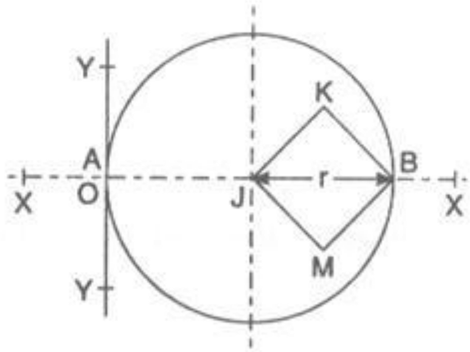
$= (4\pi \times 10^5)(140) + (6.67\pi \times 10^5)(62.5) / (4\pi \times 10^5) + (6.67\pi \times 10^5)$

$= (560\pi + 416.87\pi) \times 10^5 / (4\pi + 6.67\pi) \times 10^5 = 91.55$ mm

55. **A square hole is punched out a circular lamina, the diagonal of the square being the radius of the circle as shown in figure. Find the centroid of the remainder, if 'r' is the radius of circle.**

Solution: The given section is symmetric about diagonal AB, therefore its centroid will be on diagonal AB.

∴ Considering point A as reference on axis Y – Y



$$\bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2}$$

$$a_1 = \pi r^2; x_1 = r$$

$$a_2 = \frac{r \times r}{2} = \frac{r^2}{2}; x_2 = r + \frac{r}{2} \qquad \left(a_2 = \frac{JB \times kM}{2}\right)$$

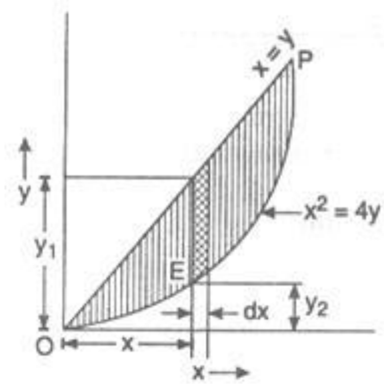
$$\Rightarrow \qquad a_2 = \frac{r^2}{2}; x_2 = 1.5r$$

$$\Rightarrow \quad \bar{x} = \frac{(\pi r^2 \times r) - \left(\frac{r^2}{2} \times 1.5r\right)}{\pi r^2 - \frac{r^2}{2}} = \frac{r^3(\pi - 0.75)}{r^2(\pi - 0.5)} = r\left(\frac{\pi - 0.75}{\pi - 0.5}\right)$$

∴ Centroid lies on diameter AB, at a distance 0.905r from A.

56. Determine the co-ordinates of the centroid of the shaded area between the parabola $4y = x_2$ and the straight line $x - y = 0$

Solution: Given the equations



$x^2 = 4y$ ...(1)

$x - y = 0$ ...(2)

Drawing both the curves on x – y plane.

At point P and O both the curves intersect each other. Thus, they wil have same co-ordinates.

Putting x = y in equation (1) $x^2 = 4x$

$\Rightarrow$ x = 4

From equation (2) y = 4

$\therefore$ co-ordinates of P are (4, 4)

Let there be two points D and E for same value of x on straight line and parabola respectively. So that,

$x - y_1 = 0$ ...(3)

$x^2 = 4y_2$ ...(4)

If we divide the shaded area into large number of small area each of height y and width dx, i.e.,

dA = y dX where $y = y_1 - y_2$

$\Rightarrow dA = (x - x^2/4).dx$ Using equations (3) and (4) ...(5)

Now, the co-ordinates of centroid for area dA will be

$x' = x$ ...(6)

$y' = y_2 + y/2 = y_2 + y_1 - y_2 / 2 = y_1 + y_2 / 2$

From equations (3) and (4)

$= (x+x^2/4)/2 = 1/2\ (x + x^2/4) \Rightarrow y' = 1/2\ (x + x^2/4)$ ...(7)

Now, the centroid of shaded region will be

G = (x¯, y¯)

where,

$$\bar{x} = \frac{\int x'\,dA}{\int dA};\ \bar{y} = \frac{\int y'\,dA}{\int dA}$$

Putting x' and y' from equations (6) and (7)

$$\bar{x} = \frac{\int_0^4 x\left(x - \frac{x^2}{4}\right)dx}{\int_0^4 \left(x - \frac{x^2}{4}\right)dx} = \frac{\int_0^4 \left(x^2 - \frac{x^3}{4}\right)dx}{\int_0^4 \left(x - \frac{x^2}{4}\right)dx} = \frac{\left[\frac{x^3}{3} - \frac{x^4}{4\times 4}\right]_0^4}{\left[\frac{x^2}{2} - \frac{x^3}{3\times 4}\right]_0^4} = \frac{\left(\frac{(4)^3}{3} - \frac{(4)^4}{4\times 4}\right)}{\left(\frac{(4)^2}{2} - \frac{(4)^3}{3\times 4}\right)} = \frac{16/3}{16/6} = 2$$

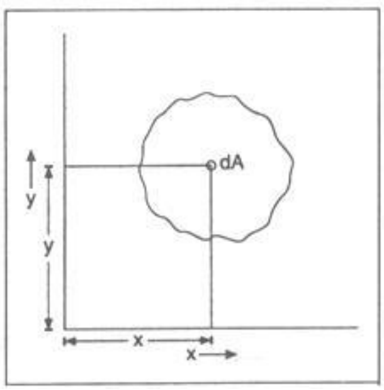
$$\bar{y} = \frac{\int_0^4 \frac{1}{2}\left(x + \frac{x^2}{4}\right)\left(x - \frac{x^2}{4}\right)dx}{\int_0^4 \left(x - \frac{x^2}{4}\right)dx} = \frac{\frac{1}{2}\int_0^4 \left(x^2 - \frac{x^4}{16}\right)dx}{16/6} = \frac{\frac{1}{2}\left[\frac{x^3}{3} - \frac{x^5}{5\times 16}\right]_0^4}{16/6} = \frac{\frac{1}{2}\left(\frac{(4)^3}{3} - \frac{(4)^5}{5\times 16}\right)}{16/6}$$
$$= \frac{64/15}{16/6} = \frac{8}{5}$$

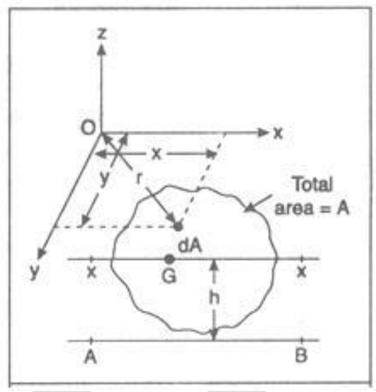
Centroid G = (2, 8/5)

57. MOMENT OF INERTIA

It is the product of the area (or mass) and the square of the distance of the centre of gravity of the area (or mass) from an axis. it is denoted by 1. Moment of inertia about x-axis is represented by I_{xx} whereas about y-axis by I_{yy}.

There are two theorems which are used to define moment of inertia about any other axis other than x and y.





58. THEOREM OF THE PERPENDICULAR AXIS

This theorem states that if I_{xx} and I_{yy} be the moment of inertia of a plane section about two mutually perpendicular axis x and y in the plane of section, then the moment of inertia of the section I_{xx}, about the axis Z, perpendicular to the plane and passing through the intersection of x and y axis is given by

Proof:

From figure

$$I_{zz} = \int r^2.dA = \int (x^2 + y^2)dA = \int x^2 dA + \int y^2 dA$$

$$I_{ZZ} = I_{XX} + I_{YY}$$

59. THEOREM OF THE PARALLEL AXIS

This theorem state that the moment of inertia of a plane area about an axis in the plane of area through the C.G. of the plane area be represented by IG, then moment of inertia of the given plane area about a parallel axis in the plane of area of a distance h from the C.G. of the area is given by

$I_{AB} = I_G + A.h^2$

Proof:
From figure

Let the axis in the plane of area 'A' and passing through the CG of the area be XX and AB is the axis in the plane of area A and parallel to axis X-X.

h be distance between XX and AB.

Then for a strip of area dA parallel to XX at a distance y from x-axis.

$$I_{XX} = \int y^2.dA \quad or \quad I_G = \int y^2 dA$$

Similarly, moment of inertia of the area dA about AB:

$$I_{AB} = \int (y+h)^2 dA = \int (y^2 + h^2 + 2hy)\, dA$$

$$= \int y^2.dA + \int h^2.dA + \int 2hy\, dA$$

$$= I_G + h^2 \int dA + 2h \int y\, dA$$

$$= I_G + h^2.A + 0 \quad \left(\int dA = A \text{ and } \int ydA = 0\right)$$

$\Rightarrow I_{AB} = I_G + h^2.A$

60. Moment Of Inertia of a Rectangular Section about an axis passing through the C.G. of the section.

Let
b = width of rectangular section

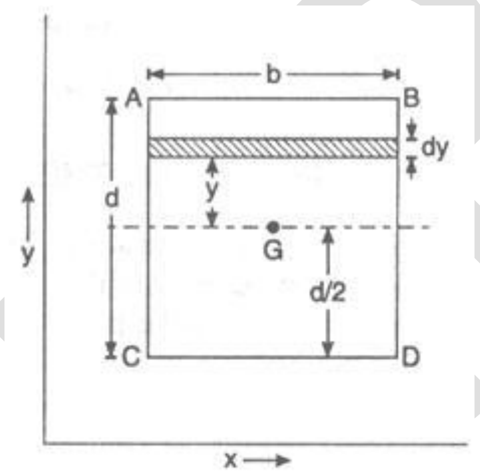
d = depth of rectangular section

$$I_{XX} = \int y^2.dA = \int y^2.b.dy = b\int_{-d/2}^{d/2} y^2.dy = b\left[\frac{y^3}{3}\right]_{-\frac{d}{2}}^{+\frac{d}{2}} = \frac{bd^2}{12}$$

⇒ $I_{XX} = bd^2 / 12$

Similarly, the moment of inertia about YY axis passing through the CG of the section is given by

$I_{YY} = db^3/12$



And, the moment of inertia about a line passing through the base parallel to x-axis:

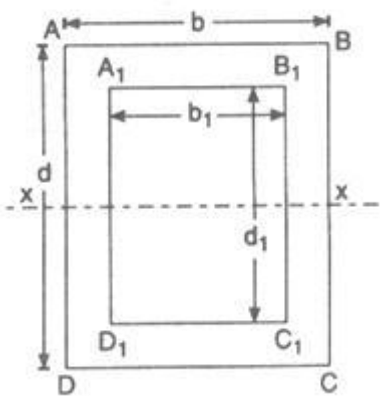
$$I_{CD} = \int y^2.dA = \int_0^d y^2.b\,dy$$

$$= b\left[\frac{y^3}{3}\right]^d = \frac{bd^3}{3}$$

$I_{CD} = bd^3 / 3$

61. Moment Of Inertia Of A Hollow Rectangular Section:

The moment of inertia of the main section



ABCD about axis X-X is given by

$I = bd^3 / 12$

Similarly, for area $A_1B_1C_1D_1$

$I_1 = b_1d_1^3 / 12$

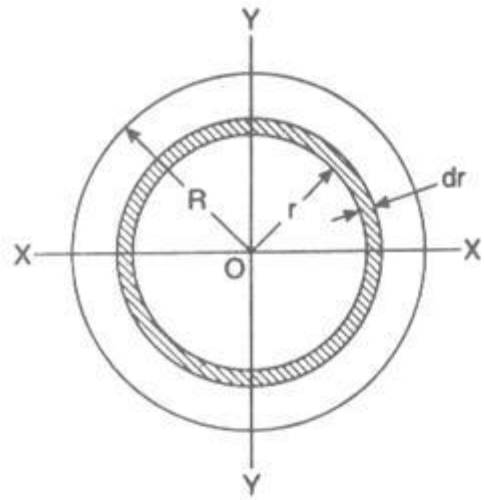
∴ Moment of inertia of hollow rectangular section about X-X axis.

$I_{XX} – I – I_1$

$= (bd^2/12) - (b_1d_1^2/12$

62. Moment of inertia of a circular section

Considering an element ring of radius



'r' and thickness 'dr'

Area of circular ring = 2πr.dr

Moment of inertia about an axis passing through O, normal to plane X – Y in Z direction = I_{ZZ}.

$$I_{ZZ} = \int (Area\ of\ ring) * (radius\ of\ ring)^2$$

$$I_{ZZ} = \int_0^R (2\pi r.dr).r^2$$

$$= \int_0^R 2\pi r^3 dr = 2\pi \left[\frac{r^4}{4}\right]_0^\pi = 2\pi \frac{R^4}{4}$$

I_{ZZ} = p/R^4 = pD^4/32

From the theorem of perpendicular axis:

$I_{ZZ} = I_{ZZ} + I_{yy}$

But from symmetry:
$I_{XX} = I_{YY}$

∴ $I_{XX} = I_{YY} (I_{ZZ} / 2)$

⇒ $I_{XX} = I_{YY}$ = pD^4/64

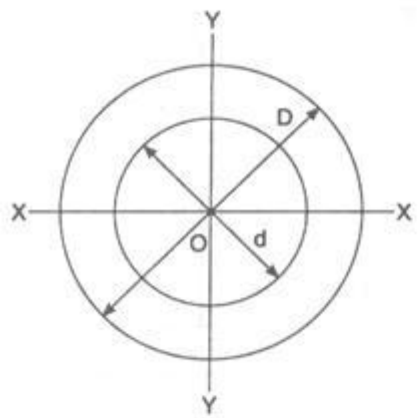
63. Moment of inertia of a hollow circular section:

I_{XX} = (Moment of inertia of outer circle) – (Moment of inertia of cutout circle)

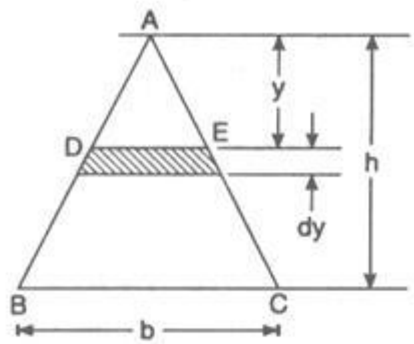
$I_{XX} = I_{YY} = = \pi/64\ D^4 - \pi/64\ d^4 = \pi/64\ (D^4 - d^4)$

64. Moment of inertia of triangular section

(a) MI about its base



Moment of inertia of the strip about the base



$I_{BC} = \int (\text{Area of strip}) \times (\text{Distance of strip from base})^2$

$\Rightarrow \quad I_{BC} = \int_0^h (DE.dy)(h-y)^2$

DE / BC = y/h

$\Rightarrow$ DE = BC.y/h = b.y/h

$\Rightarrow$ DE = BC.y/h = b.y/h

$\Rightarrow$ $I_{BC} = \int_0^h \left(\frac{b.y}{h}.dy\right)(h-y)^2 = \int_0^h \frac{by}{h}(h-y)^2.by$

$\Rightarrow$ $I_{BC} = b/h \int_0^h y(h-y)^2 dy = \frac{b}{h}\int_0^h (yh^2 + y^3 - 2hy^2)dy$

$$= \frac{b}{h}\left[\frac{y^2h^2}{2} + \frac{y^4}{4} - \frac{2hy^3}{3}\right]_0^h = \frac{b}{h}\left[\frac{h^2.h^2}{2} + \frac{h^4}{4} = \frac{2h.h^3}{3}\right]$$

$$= \frac{b}{h}.h^4\left[\frac{6+3-8}{12}\right] = \frac{1}{12}bh^3$$

$I_{BC} = bh^3 / 12$

(b) M.I. about an axis passing through center of gravity and parallel to base

From parallel axis theorem;

$I_G + A.(h/3)^2 = I_{BC}$

$$I_G = I_{BC} - A\left(\frac{h}{3}\right)^2 = \frac{bh^3}{12} - \left(\frac{b\times h}{2}\right)\left(\frac{h}{3}\right)^2$$

$I_G = bh^3 / 36$

65. Moment Of Inertia Of Composite Sections

It can be found out as discussed below:

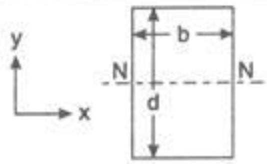
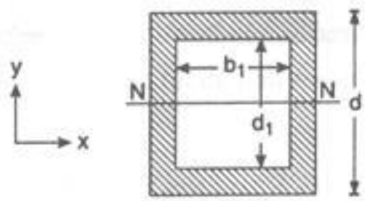
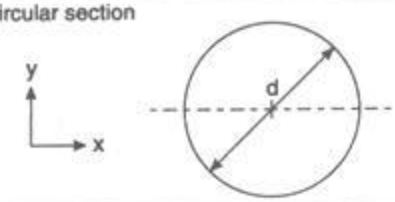
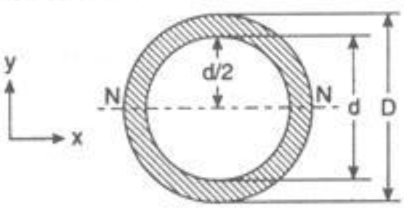
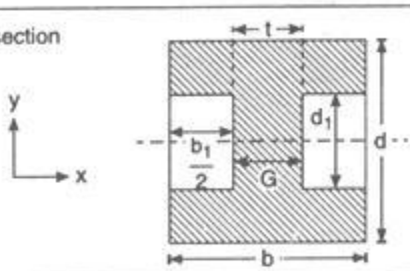
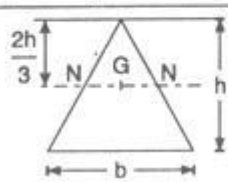
1. Split up given section into regular lanes likes circles, rectangles and triangles etc.

2. Find out the moments of inertia of the these area about their respective centers of gravity.

3. Now, transfer these moments of inertia about the required axis using parallel axis theorem or perpendicular axis theorem.

4. The moment of inertia of composite section may now be obtained by the algebraic sum of moments about the required axis.

Table 11.2.1

Type of section	Moment of Inertia	y_{max}	Section modulas (Z)
Rectangle or paralleogram 	$I_{xx} = \frac{bd^3}{12}$ $I_{yy} = \frac{db^3}{12}$	$\frac{d}{2}$ $\frac{b}{2}$	$Z_{xx} = \frac{bd^2}{6}$ $Z_{yy} = \frac{db^2}{6}$
Hollow rectangular section 	$I_{xx} = \frac{bd^3}{12} - \frac{b_1d_1^3}{12}$ $I_{yy} = \frac{db^3}{12} - \frac{d_1b_1^3}{12}$	$\frac{d}{2}$ $\frac{b}{2}$	$Z_{xx} = \frac{1}{6d}(bd^3 - b_1d_1^3)$ $Z_{yy} = \frac{1}{6b}(db^3 - d_1b_1^3)$
Circular section 	$I_{xx} = \frac{p}{64} d^4$ $I_{yy} = \frac{p}{64} d^4$	$\frac{d}{2}$ $\frac{d}{2}$	$Z_{xx} = \frac{p}{32} d^3$ $Z_{yy} = \frac{p}{32} d^3$
Hollow curcular section 	$I_{xx} = I_{yy} = I$ $I_{yy} = \frac{p}{64}(D^4 - d^4)$	$\frac{D}{2}$	$Z_{xx} = Z_{yy} = Z$ $Z = \frac{p}{32D}(D^4 - d^4)$
I-section 	$I_{xx} = \frac{bd^3}{12} - \frac{b_1d_1^3}{12}$ $I_{yy} = \frac{db^3}{12} - \frac{d_1b_1^3}{12}$ or $I_{xx} = \frac{1}{12}(bd^3 - (b - t) d_1^3)$	$\frac{d}{2}$ $\frac{b}{2}$	$Z_{xx} = \frac{1}{6d}(bd^3 - b_1d_1^3)$ $Z_{yy} = \frac{1}{6b}(db^3 - d_1b_1^3)$
Triangle 	$I_G = \frac{bh^3}{36}$	$\frac{2}{3}h$	$Z_G = \frac{bh^2}{24}$

66. POLAR MOMENT OF INERTIA

The moment of inertia with respect to an axis extending normal to the plane of area is known as polar moment of inertia and is denoted by J. The expression of inertia is therefore:

$J = \int r^2.dA$

where r is the distance of elementary area from Z axis.

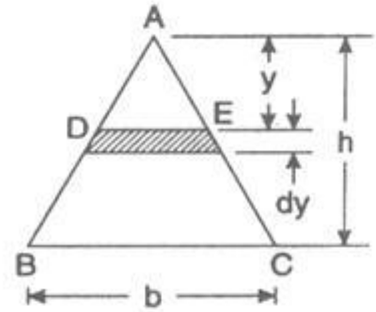
Now $r^2 = x^2 + y^2$

Hence $J = \int (x^2 + y^2).\ dA = \int x^2\ dA + \int y^2.\ dA$

$= I_x + I_y$

67. Show that moment of inertia of a triangular section about its base is equal to $bh^3/12$, where b and h are base and height of the triangle respectively.

Solution: Moment of inertia of the strip about the base



$\Rightarrow \quad I_{BC} = \int_0^h (DE.dy)\,(h-y)^2$

$\Rightarrow \quad DE/BC = y/h$

$\Rightarrow \quad DE = CB.y/h = b.y/h$

$\Rightarrow \quad I_{BC} = \int_0^h \left(\frac{b.y}{h}.dy\right)(h-y)^2 = \int_0^h \frac{by}{h}(h-y)^2.dy$

$\Rightarrow \quad I_{BC} = \frac{b}{h}\int_0^h y(h-y)^2 dy = \frac{b}{h}\int_0^h (yh^2 + y^3 - 2hy^2)dy$

$\Rightarrow \quad = \frac{b}{h}\left[\frac{y^2h^2}{2} + \frac{y^4}{4} - \frac{2hy^3}{3}\right]_0^{h'} = \frac{b}{h}\left[\frac{h^2.h^2}{2} + \frac{h^4}{4} - \frac{2h.h^3}{3}\right]$

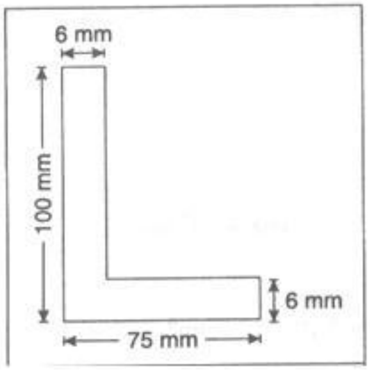
$= \frac{b}{h}.h^4\left[\frac{6+3-8}{12}\right] = \frac{1}{12}bh^3$

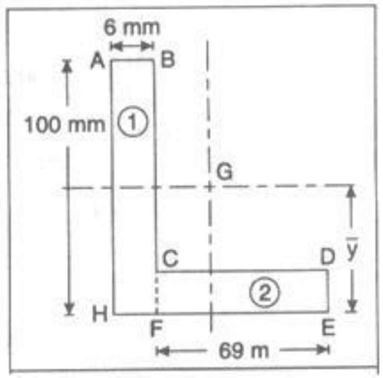
$I_{BC} = bh^3 / 12$

68. Solved Examples Based On Moment Of Inertia About The Centroidal Axis

Solution: Let us divide the area in two parts (1) and (2)

Area A_1 = 100 × 6 = 600 mm2





$A_2 = 69 \times 6 = 414\ mm^2$

$x_1 = 3$ mm from AH

$y_1 = 50$ mm from HE

Co-ordinates of centroid (X¯, y¯) are

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2} \text{ and } \bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$\Rightarrow \bar{x} = \frac{(600\times3)+(414\times40.5)}{600\times414} = 18.37\ mm$$

$$\bar{y} = \frac{(600\times50)+(414\times3)}{600\times414} = 30.81\ mm$$

Now, moment of inertia about centroidal xx axis (I_{xx}):

$$I_{XX_1} = \frac{b_1 d_1^3}{12} + A_1 h_1^2 \qquad where; h_1 = y_1 - \bar{y}$$

$$= \frac{6(100)^3}{12} + 600(50 - 30.81) = 7.21 \times 10^5 mm^4$$

$$I_{XX_2} = \frac{b_2 d_2^3}{12} + A_2 h_2^2 \qquad where; h_2 = y_2 - \bar{y}$$

$$= \frac{69 \times 6^3}{12} + 414(3 - 30.81) = 3.21 \times 10^5 \times 10^5 mm^4$$

Similarly ,

Therefore,

$$I_{XX} = I_{XX_1} + I_{XX_2}$$

$$I_{XX} = (7.21 \times 10^5) + (3.21 \times 10^5)$$

$\Rightarrow I_{XX} = 10.42 \times 10^5 \ mm^4$

Now, moment of inertia about centroid yy axis (I_{yy}):

$Iyy_1 = d_1 b^2{}_1/12 + A_1 h^2{}_1$

where, $h_1 = x_1 - x^-$

Iyy_1 = 100 X 6^3 + 600 (3 - 18.31) = 1.42 X 10^5 mm^4

Similarly, $Iyy^2 = d_2 \, b_2^3 /12 \; A2h_2^2$ where; $h_2 = x_2 - x^-$

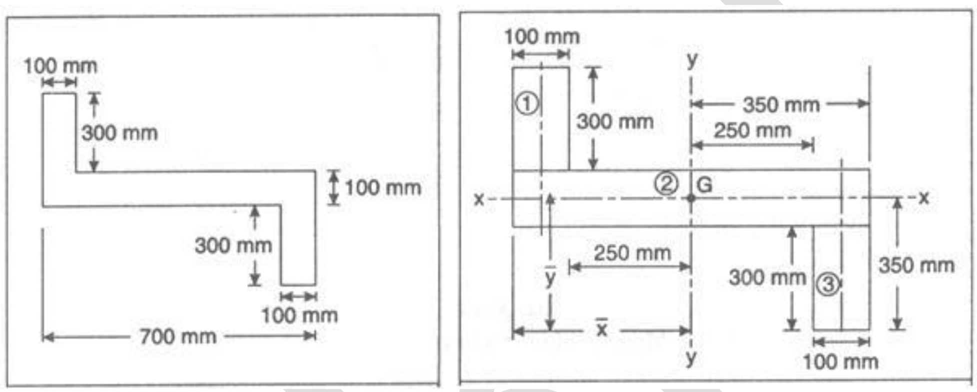
= 6 X $(69)^2$/12 + 414 (40.5 - 18.31)^2 = 3.68 X 10^5 mm^4

Therefore,

$Iyy = Iyy_1 + Iyy_2$

$\Rightarrow Iyy = (1.42 \text{ X } 105) + (3.68 \text{ X } 10^5) = 5.10 \text{ X } 10^5 \text{ mm}^4$

69. Find the moment of inertia of the section shown in figure about x-x and y-y axes.



Solution: Let us divide the complete section is three parts (1), (2) and (3) as shown in figure.

Then,
Area $A_1 = 300 \times 100 = 30{,}000 \text{ mm}^2$

Area $A_2 = 700 \times 100 = 70{,}000 \text{ mm}^2$

Area $A_3 = 300 \times 100 = 30{,}000 \text{ mm}^2$

$x_1 = 50$ mm, $y_1 = 400 + 300/2 = 550$ mm

$x_2 = 350$ mm, $y_2 = 300 + 100/2 = 350$ mm

$x_3 = 600 + 100/2 = 650$ mm, $y_3 = 300/2 = 150$ mm

where y_1, y_2 and y_3 are the distances of centroid for areas A_1, A_2 and A_3 respectively, from the base of area (3).

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3} = \frac{(30{,}000 \times 50) + (30{,}000 \times 650)}{30{,}000 + 70{,}000 + 30{,}000} = 350mm$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2} = \frac{(30{,}000 \times 550) + (70{,}000 \times 350) + (30{,}000 \times 150)}{30{,}000 + 70{,}000 + 30{,}000} = 350mm$$

Now, moment of inertia about centroidal x-x axis is I_{xx}.

∴ I_{xx} = (Moment of inertia of rectangle 1 about xx axis) + (Moment of inertia of rectangle 2 about xx axis) + (Moment of inertia of rectangle 3 about xx axis)

= $(I_{G1} + A_1 h_1^3) + (I_{G2} + A_2 H_2^2) + (I_{G2} + A_3 H_3^2)$

where $h_1 = y_1 - y^-$

$h_2 = y_2 - y^-$

$h_3 = y_3 - y^-$

$$I_{G_1} = \frac{b_1 d_1^3}{12}, I_{G_2} = \frac{b_2 d_2^3}{12}, I_{G_3} = \frac{b_3 d_3^2}{12}$$

$$\Rightarrow \quad I_{xx} = \left[\frac{100 \times (300)^3}{12} + 30{,}000\,(550 - 350)^2\right] + \left[\frac{700 \times (100)^2}{12} + 70{,}000(350 - 350)^2\right] +$$

$$\left[\frac{100 \times (300)^3}{12} + 30{,}000(150 - 350)^2\right]$$

= 2.9×10^9 mm^4.

Now moment of inertia about centroidal y-y axis is I_{yy}.

∴ $I_{yy} = (I_{G1} + A_1 h_1^2) + (I_{G2} + A_2 H_2^2) + (I_{G2} + A_3 H_3^2)$

where $h_1 = x_1 - x^-$

$h_2 = x_2 - x^-$

$h_3 = x_3 - x^-$

$$I_{G_1} = \frac{d_1 b_1^3}{12}, I_{G_2} = \frac{d_2 b_2^3}{12}, I_{G_3} = \frac{d_3 b_3^3}{12}$$

$$I_{yy} = \left[\frac{3000 \times (100)^3}{12} + 30{,}000\,(50 - 350)^2\right] + \left[\frac{100 \times (700)^3}{12} + 70{,}000(350 - 350)^2\right]$$

$$+ \left[\frac{300 \times (100)^3}{12} + 30{,}000\,(650 - 350)^2\right] = 8.3 \times 10^9 mm^4$$

70. Determine the moment of inertia 'I' around the horizontal axis for the area shown in figure.

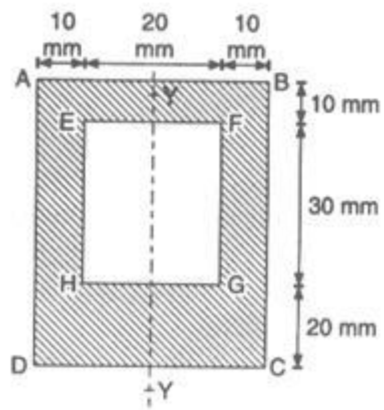
Solution: The section is symmetric about a vertical axis Y-Y. Therefore the centroid of the section will lie on this axis.

Let the centroud be at a distance from base DC.

Then

$\bar{y} = a_1 y_1 + a_2 y_2 / a_1 + a_2$ = (60 X 40) X 30 - (20 X 30) X 35 /(60 X 40) - (20 X 30)

Or $\bar{y}$ = 28.3mm



Moment of inertia about horizontal axis through centroid for ABCD is

$Izz_{(1)} = I_2 + a_1 h_1^2$

$= 40 X (60)^3/12 + (60 X 40) X (30 - 28.3)^2$

$= 72.69 X 10^4 mm^3$

Similarly for hollow section

$$I_{zz_{(2)}} = I_2 + a_2 h_2^2$$

$$= \frac{20 \times (30)^3}{12} + (20 \times 30) \times (35 - 28.3)^2 = 7.19 \times 10^4 mm^4$$

$$I_{zz} = I_{zz_{(1)}} - I_{zz_{(2)}} = 72.69 \times 10^4 - 7.19 \times 10^4 = 65.50 \times 10^4 mm^4$$

Note:
In case of composite area, each element contributes two terms to the total L. One term is the moment of inertia of an area around its won centroidal axis, and other is due to transfer of its axis to the centroid of whole area.

71. Obtain the expressions for section modulus for a hollow circular section (inside diameter d_1, outside diameter d_2), and a rectangular section (width b, depth w).

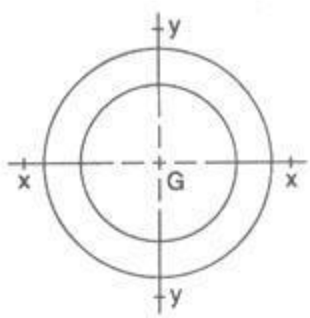
Solution: Consider a hollow circular section with inside diameter 'd_1' and outside diameter 'd_2' then we have

$Ixx = Iyy = \pi/64 [(d_2)^4 - (d_1)^4]$

And $x^- = y^- = d/2$

∴ Section modulus about x-axis $Z_{XX} – I_{XX} / y_{max}$

⇒ $Z_{XX} = \pi/64 [(d_2)^4 – (d_1)^4] / (d^2/2)$



$= \pi/32 [(d_2)^4 - (d_1)4 /d_2]$

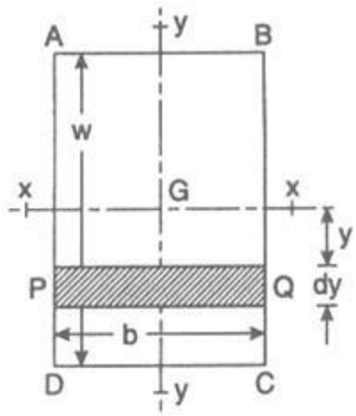
By symmetry; $Z_{yy} = \pi/32 [(d_2)^4 – (d_1)^4 / d_2]$

Rectangular section:
Consider a rectangular section ABCD, as shown in figure, with width 'b' and depth 'w'. Consider a strip PQ of thickness d, parallel to x-x axis at a distance y from x-x axis.

∴ Area of strip = b.dy

∴ MI of strip about x-x = Area × y^2 = b.y^2.dy

$$\Rightarrow I_{xx} = \int_{-w/2}^{w/2} b.y^2.dy = b\left[\frac{y^3}{3}\right]_{-\frac{w}{2}}^{w/2}$$



= b.w^2/12

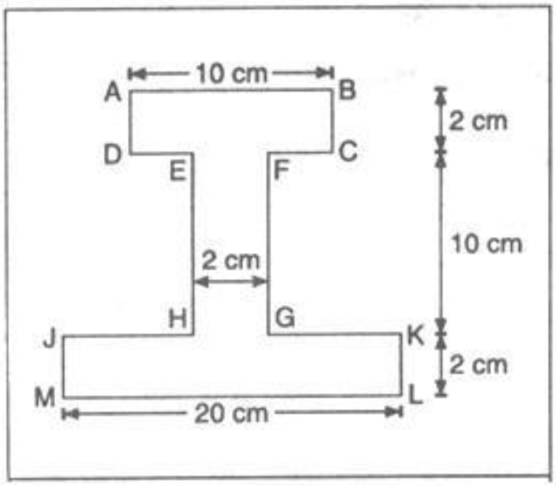
Similarly I_{yy} = w.b^3/12

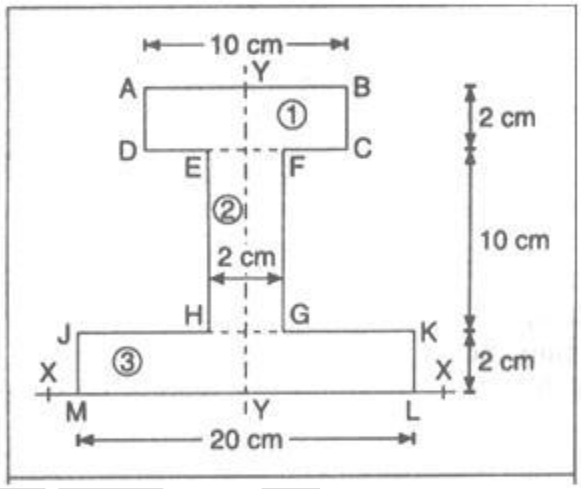
⇒ $Z_{xx} = I_{xx}$ / y = (bw^3/12) / (w/2) = bw^2 / 6

And $Z_{yy} = I_{yy}$/x = (wb^3/12) (b/2) = wb^2/6

72. Find the moment of inertia of the section shown in figure about the centroidal axis X-X perpendicular to the web.

Solution: Redrawing the figure as shown in figure





The given section is symmetrical about Y – Y axis and hence CG of the section will be on Y-Y axis.

Splitting into three rectangles ABCD, EFGH and JKLM the centroid of the section can be obtained as

$\bar{y} = a_1 y_1 + a_2 y_2 + a_3 y_3 / a_1 + a_2 + a_3$

where a_1, a_2, a_3 are areas of sections ABCD, EFGH and JKLM respectively, $y_1 y_2$ and y_2 are distances of centroids of ABCD, EFGH and JKLM from bottom lime MI.

$\therefore \bar{y} = [(10 X 2)(2 + 10 + 1)] + [(10 X2)(2 + 5) +] [(20 X 2)(2)]] / (10 X 2) + (10 X 2) + (20 X 2)$

$= 260 + 140 + 40 / 80 = 5.5$ cm

Thus, centroid or CG of given section lies at a distance of 5.5 cm from bottom line ML on central axis Y-Y.

To find the moment of inertia of the section.

Let IG_1, IG_2, IG_3 as moment of inertia of rectangle ABCD, EFGH and JKLM respectively about the horizontal axes passing through their respective centroids.

h_1, h_2 and h_3 are distances between the centroids of ABCD, EFGH and JKLM and the centroid of given section.

$\therefore h_1 = y_1 - \; = 13 - 5.50 = 7.50$ cm

$h_2 = y_2 - \; = 7.0 - 5.50 = 1.50$ cm

$h_3 = \; - y_3 = 5.50 - 1.0 = 4.5$ cm

Now, $IG_1 = 10 \times (2)^3 / 12 = 6.667\ cm^4$

$IG_2 = 2 \times (10)^3 / 12 = 166.667\ cm^4$

$IG_3 = 20 \times (12)^3 / 12 = 13.333\ cm^4$

From the theorem of parallel axes, the moment of inertia of ABCD about the horizontal axis passing through the CG of the give section.

$I_1 = IG_1 = a_1h_1^2 = 6.67 + (10 \times 2)\ (7.50)^2 = 1131.67\ cm^4$

Similarly $I_2 = IG_2 + a_2h_2^2 = 166.667 + 20 \times (1.5)^2 = 211.67\ cm^4$

$I_3 = IG_3 \times a_3h_3^2 = 13.333 + (40)\ (4.5)^2 = 823.333\ cm^4$

∴ Moment of inertia of the given section about the horizontal axis, passing through centroid or CG of given section is

$I = I_1 + I_2 + I_3$

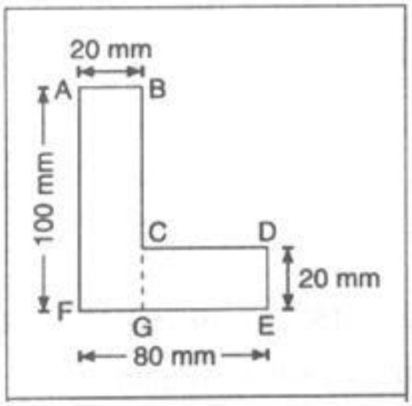
$= 1131.67 + 211.67 + 823.33$

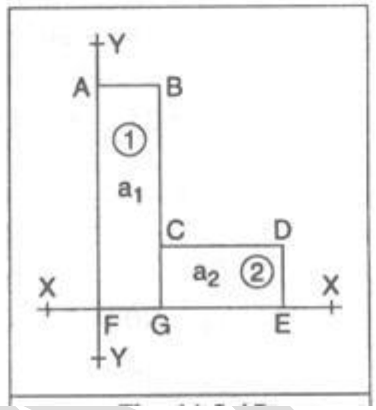
$= 2166.67\ cm^4$

73. Find the moment of inertia about the centroid X-X and Y-Y axes of the angle section shown in figure.

Solution: As the given section is not symmetric about any axis, both X-X and Y-Y should be considered.

Let the distance of centroid from X-X be and from Y-Y be





Then

$\bar{y} = a_1 y_1 + a_2 y_2/a_1 + a_2$

$\Rightarrow \bar{y}$ = (100 X 20) (100/2) + (80 - 20) X 20 X (20/2)/(100 X 20) + ((80 - 20) X 20)

$\Rightarrow$ $\bar{y}$ = 35mm

Similarly,

$\bar{Y} = a_1 x_1 + a_2 x_2/a_1 + a_2$ = (20 X 100) X (20/2) + (90 - 20) X 20 X (20 + (60/2))/(20 X 100) + (80 - 20) X 20

$\bar{Y}$ = 25mm

Now, moment of inertia about centroidal X-X axis is I_{XX}

I_{XX} = (Moment of inertia of rectangle 1 about X-X axis) + (Moment of Inertia of rectangle 2 about X-X axis)

$= (IG_1 + a_1h_1^2) + (I_{G2} + a_2H_2^2)$

where I_G is moment of inertia about an axis through the CG of section and parallel to X-X axis and h is the distance of centroid of rectangle from centroidal axis.

$I_{G1} + 20 \text{ X } (100)^3/12 = 1.67 \text{ X } 10^6 \text{ mm}^4$

h_1 = 50 - 35 = 15mm

$I_{G2} = 20 \text{ X } (100)^3/12 = 0.04 \text{ X } 10^6 \text{mm}^4$

h_2 = 35 - 10 = 25mm

$\Rightarrow I_{XX} = [(1.67 \times 10^6) + (100 \times 20 \times (15)2)] + [(0.04 \times 10^6) + (60 \times 20 \times (25)^2]$

$I_{XX} = (2.12 \times 10^6) + (0.79 \times 10^6) = 2.91 \times 106\ mm^4$

Similarly, the moment of inertia about centroidal Y-Y axis is I_{YY}

$Iyy = (IG1 + a1\ h^2{}_1) + (IG2 + a2\ H_2{}^2)$

where, I_G is moment of inertia of rectangle about at axis through the centroid of the section parallel to Y-Y axis, and h is the distance of centroid of rectangle from centroidal axis.

Now,

$I_{G1} = 100 X 20^6/12 = 0.067 X 10^6\ mm^4$

$h_1 = 25 - 10 = 15mm$

$IG2 = 20 X 60^3/12 = 0.36 X 10^6\ mm^4$

$h_2 = 50 - 25 = 25mm$

$\therefore I_{YY} [(0.067 \times 10^6) + (100 \times 20 \times (15)^2)] + [(0.36 \times 10^6) + (60 \times 20 \times (25)^2)]$

$I_{YY} = (0.517 \times 10^6) + (1.11 \times 10^6) = 1.627 \times 10^6\ mm^4$

$I_{XX} = 2.91 \times 10^6\ mm^4$

$I_{YY} = 1.627 \times 10^6\ mm^4$

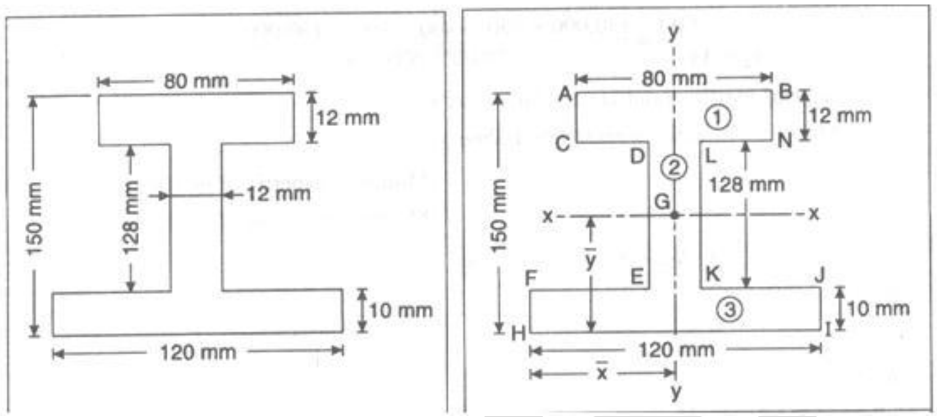
74. Determine the polar moment of inertia of I-section shown is figure. (All dimensions are in mm).

Solution: Let us divide the complete section in three parts (1), (2) and (3) as shown in figure. Then,

Area $A_1 = 80 \times 12 = 960\ mm^2$

$A_2 = 128 \times 12 = 1536\ mm^2$

$A^3 = 120 \times 10 = 1200\ mm^2$



Distance of centroids for areas A_1, A_2 and A_3 from the base HI of area (3)

$y_1 = 138 + 12/2 = 144$ mm

$y_2 = 10 + 128/2 = 74$ mm

$y_3 = 10/2 = 5$ mm

Since, the problem is symmetric about yy axis $\bar{x}$ = 60 mm from the leftmost fibre FH of area (3).

Now, $\bar{y}$ = A1 y1 + A2y2 + A3 y3/ A1 + A2 + A3 = (960 X 144) + (1536 X 74) + (1200 X 5) / 960 + 1536 + 1200 = 69.78mm

Now, moment of inertia about centroidal x-x axis is I_{xx}.

$\therefore I_{xx}$ = (Moment of inertia of rectangle 1 about xx axis) + (Moment of inertia of rectangle 2 about xx axis) + (Moment of inertia of rectangle 3 about xx axis).

$= (I_{G1} + A_1 h_1^2) + (I_{G2} + A_2 h_2^2) + (I_{G3} + A_3 h_3^2)$

where $I_{G1} = b_1 d_1^3/12$, $h_1 = y_1 - \bar{y}$

$I_{G2} = b2\, d_2^3/12$, $h_2 = y_2 - \bar{y}$

$I_{G3} = b_3 d_3^3/12$, $h_3 = y_3 - \bar{y}$

$$I_{xx} = \left[\frac{80 \times (12)^3}{12} + 960(144 - 69.78)^2\right] + \left[\frac{12 \times (128)^3}{12} + 1536(74 - 69.78)^2\right]$$

$$+ \left[\frac{120 \times (10)^3}{12} + 1200(5 - 69.78)^2\right]$$

$$= 12.46 \times 10^6 \text{ mm}^4.$$

Now, moment of inertia about centroidal y-y axis is I_{yy}

$$I_{yy} = (I_{G_1} + I_{G_2}) = \left(\frac{d_1 b_1^3}{12} + \frac{d_2 b_2^3}{12} + \frac{d_3 d_3^3}{12}\right)$$

$$= \left[\frac{12 \times (80)^3}{12} + \frac{128 \times (12)^3}{12} + \frac{10 \times (120)^3}{12}\right] = 1.97 \times 10^6 mm^4$$

Polar moment of inertia = J = $I_{xx} + I_{yy}$

= $(12.46 \times 10^6) + (1.97 \times 10^6) = 14.43 \times 10^6$ mm^4

75. Three beams have the same length, same allowable stress and the same bending moment. The cross-sections of the beams are a square, a rectangle with depth twice the width and a circle. Find the ratios of weights of rectangular and circular beams with respect to square beams.

Solution: Let the square have each side of length 'a' mm the rectangle has width of 'b' mm and depth '2b' mm the circle has diameter of 'd' mm.

Since, the allowable stress (σ) and bending moment (M) remains the same for all three sections, then from equation.

M/I = σ/y

$\Rightarrow Z = I/y = M/\sigma = \text{constant}$

$\Rightarrow Z_1 = Z_2 = Z_3$

$Z_1 = b.d^2/6 = a \times a^2 / 6 = a^3 / 6$ ...(1)

$Z_2 = b.d^2/6 = b.(2b)^2 / 6 = 2/3\ b^3$ ...(2)

$Z_3 = \pi/32\ d^3$

Equating equations (1) and (2) on right hand side

$a^3 / 6 = 2b^3 / 3 \Rightarrow b = 0.63a$

Similarly from equations (1) and (3)

$A_3/6 = I9d^3/32 \Rightarrow d = 1.19a$

Now

$$\frac{Weight\ of\ square\ beam}{weight\ of\ rectangular\ beam} = \frac{\rho \times Area \times L}{\rho \times Area \times L} = \frac{a^2 \times L \times \rho}{(b \times 2b) \times L \times \rho}$$

$$= \frac{a^2}{2b^2} = \frac{a^2}{2(0.63a)^2} = \frac{1}{0.794}$$

and

$$\frac{Weight\ of\ square\ beam}{weight\ of\ circular\ beam} = \frac{Density \times volume}{Density \times volume} = \frac{\rho \times a^2 \times L}{\rho \times \frac{\pi d^2}{4} \times L}$$

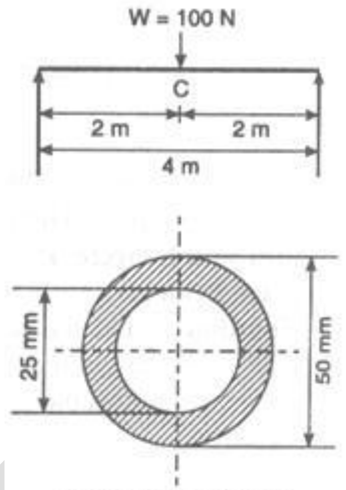
$$= \frac{4a^2}{\pi d^2} = \frac{4a^2}{\pi(1.19a)^2} = \frac{1}{1.112} = 0.899$$

Thus, the ratio of their weights is:
Square : rectangle = 1 : 0.794

Square : circular = 1 : 1.112

76. A beam made of C.I. having a section of 50 mm external diameter and 25 mm internal diameter is supported at two points 4 m apart. The beam carries a concentrated load of 100 N at its centre. Find the maximum bending stress induced in the beam.

Solution: Given that



Cross section of beam

Outer diameter of cross-section D_0 = 50 mm

Inner diameter of choss-section D_i = 25 mm

Length of span 'L' = 4 m

Load applied W = 100 N

For a simply supported beam with point load at center

For AC $M_x = R_A.X = W/2\ x$

when x = 0, MA = 0

at x = L/2 $M_C = W/2.L/2 = WL/4$

By symmetry $M_B = 0$

⇒ Maximum bending moment occurs at center and its value is WL/4

⇒ $M = WL/4 = 100 \times 4 / 4 = 100$ Nm

From bending equation $\sigma/y = M/I$

where s is bending or flexure stress

M is bending moment M

I is moment of inertia

y is the distance of point from neutral axis.

$\sigma = M.y/I = M(D_0/2) / \pi/64\ [D_0^4 - D_1^4]$

$\Rightarrow \sigma = (100 \times 100) \times (50/2) / \pi/64\ ((50^4 - (25)^4))$

$\Rightarrow \sigma = 8.692\ N/mm^2$

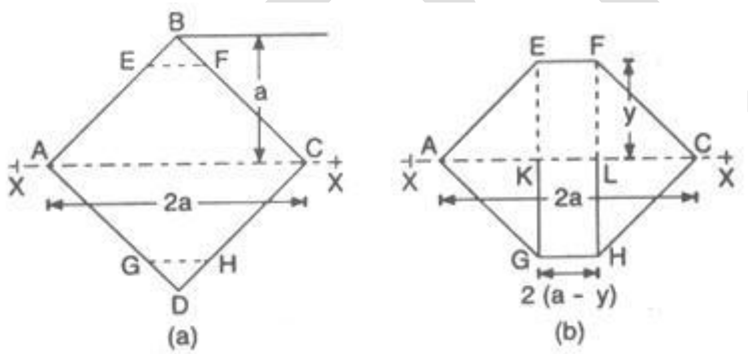
77. Prove that the moment of resistance offered by a beam of square section, with its diagonal in the plane of bending is increased by flattering the top and bottom corners, as shown in figure and its value is maximum when y = 8a/9. Also, find the percentage increase in moment of resistance offered.

Solution: Let, the moment of inertia of square ABCD, about axis X-X be I_1, its section modulas be Z_1, and the moment of resistance offered be M_1.

Similarly, for new section with cut portion (i.e., AEFCHG) the moment of inertia about axis X-X be I_2, section modulus be Z_2, and moment of resistance offered be M_2.

Given that,

AC = BD = 2a



Now, I_1 = 2 × (moment of inertia of triangle ABC) about X-X

$I_x = 2 \text{ X } (2a \text{ X } a^3/12) = a^4/3$

$Z_1 = I_1/y_{max} = a_4/3/a = a^3/3$ ...(1)

Now I_2 = 4 (moment of inertia of triangle AEK about X-X axis) + (moment of inertia of rectangle EFHG about X-X axis)

$\Rightarrow I_2 = 4.(y.y^3/12) + 2(a–y)(2y)^3 / 12$

$= y^4/3 + 4/3 (a – y).y^3$

$= y^4/3 + 4a.y^3/3 – 4y^4/3 = 4ay^3/3 – y^4$

$\Rightarrow Z_2 = I_2 / y_{max} = 4/3\ ay^3 – y^4 / y = 4/3ay^2 – y^3$

$\Rightarrow M_2 = \sigma.Z_2 = \sigma.[4/3a.y^2 – y^3]$ ...(2)

∴ The moment of resistance offered by new section will be maximum, if $dM^2/dy = 0$

∴ Differentiating equation (2) w.r.t. y and equating to zero, we get

$d/dy\ (\ \sigma\ (\ 4/3\ ay^2 - y^3))= 0$

$\Rightarrow \sigma.\ (4/3.\sigma.2y - 3y^2)= 0$

$\Rightarrow 8/3\ a.y – 3y^2 = 0$

$\Rightarrow$ y = 8/9 a ...(3)

Hence the moment of resistance offered is maximum when

y = 8d/9

$\therefore (M_2)max = \sigma[4/3.a.(8a/9)^2 – (8a/9) = \sigma[4 \times 64 / 3 \times 81\ a^3 – 512 / 729\ a^3]$

$= \sigma/a^3\ [356/243 – 512/729] = 0.351\ \sigma.a^3$

∴ Percentage increase in moment of resistance offered is

$= M_2 – M_1 / M_1 \times 100 = 0.351.\sigma.a^3 – \sigma.a^3/3 / \sigma.a^3/3 \times 100$

$= (0.351 – 0.333 / 0.333) \times 100 = 5.40\%$

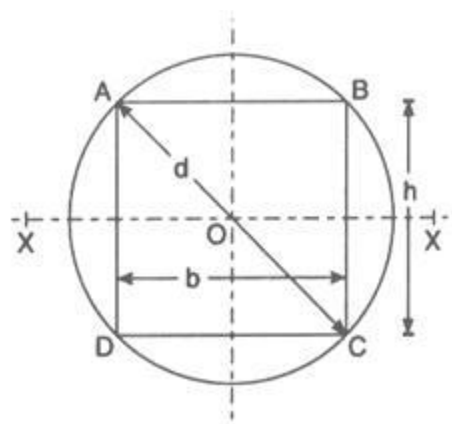
78. Prove that the ratio of depth to width to the strongest beam that can be cut from a circular log a diameter 'd' is . Hence, calculate the depth and width of the strongest beam that can be cut of a cylindrical log of wood whose diameter is 200 mm.

Solution: Given that, the diameter of log is 'd'

Let the strongest section which can be cut out of cylindrical log be ABCD as shown in figure.

Let the width of strongest section be 'b' and depth be 'd'.

Now, moment of inertia of ABCD about axis X-X is



$I = b.h^3/2$

$y_{max} = h/2$

$\therefore Z = 1/y_{max} = bh^3/12 / h/2 = bh^2/6$ ...(1)

Since, 'b' and 'h' are dependent variables, we can write

$b^2 + h^2 = d^2$ (from triangle ABC)

$h_2 = d_2 – b_2$ ...(2)

Substituting the value of h^2 from equation (2) in equation (1)

$Z = b(d^2-b^2)/6$ ...(3)

In equation (3)b is the only variable, since d is constant.

Now, for a beam to be strongest, its section modulus 'Z' should be maximum

$\Rightarrow dZ/db = 0$

$\Rightarrow d/db\ (db^2 – b^2 / 6) = 0$

Or $d^2 – 3b^2 = 0$

$\Rightarrow d^2 = 3b^2$ ...(4)

Putting in equation (2)

$h^2 = 3b^2 – b^2 = 2b^2$

$\Rightarrow h = \surd 2b$

$\therefore h/b = \surd 2$, hence proved.

For $d = 200$ mm

From equation (4) $d^2 = 3b^2$

$\Rightarrow (200)^2/3 = b^2$

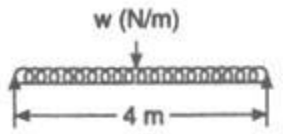
$b = 115.47$ mm

$h = \sqrt{2}b = 163.30$ mm

79. A rectangular beam 300 mm deep is simply supported over a span of 3 meters. Determine the uniformly distributed load per meter which the beam may carry, if the bending stress should not exceed 120 N/mm^2. Take I = 8 × 10^6 mm^4.

Solution: Given that

Depth of beam 'd' = 300 mm



Span of beam 'L' = 4 m

Bending stress = σ_b' = 120 N/mm^2

Moment of inertia 'I' = 80 × 10^5 mm^4

Let w = uniformly distributed load per meter length over the beam in N/m.

The bending stress will be maximum, where bending moment is maximum.

Since, a simply supported beam carrying u.d.l. have maximum bending moment at the center of the beam. The maximum bending moment is

∴ M = 2w × 2 – 2w × 1 $(R_A + R_B = 4.w, R_A = R_B = 2w)$

Now $\sigma_b = M/Z$

$Z = I/y_{max} = 8 \times 10^6/150$

⇒ $120 = 2w \times 1000 \times 150 / 8 \times 10^6$

w = 3200 N/m^3

80. A beam of square cross-section of the side 'a' has permissible bending stress σ. Find the moment of resistance when the beam is placed such that

(a) Two sides are horizontal

(b) One diagonal is vertical

Also, determine the ratio of the moments of the resistance of the section in the two positions.

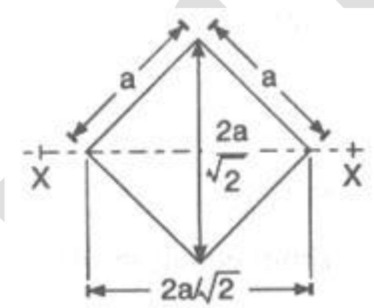
Solution: (a) Let the moment of resistance of square beam, when two of its faces are horizontal be 'M_1'

Since, $M/I = \sigma/y$

$\Rightarrow M = \sigma.(I/y) = \sigma Z$; Z is section modulus

$\Rightarrow M_1 = \sigma .Z_1$

$Z_1 = I/y_{max} = (a.a^3/12)/(a/2) = a^3 / 6$



$\Rightarrow M_1 = \sigma .a^3/6$

(c) Let the moment of resistance of square beam when one of its diagonal is vertical be M_2.

Then

$M_2 = \sigma .Z_2$

$Z_2 = I_2 / y_{max}$

Moment of inertia of both triangles formed by XX can be obtained by using $bh^3/12$

$\therefore$ I2 = 2/12 [2a/√2. (a/√2)^3] = a^4/12

Z_2 = a^4 /12.a/√2 = a^3 /6√2 $\therefore$ M_2 = σ . σ^3 / 6√2

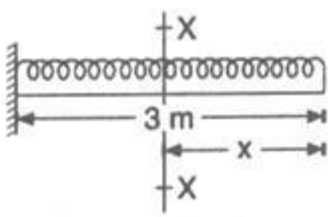
Ratio of moment of resistance of the section in the two positions is (M_1/M_2)

M_1/M_2 = (σ(σ^3/6) / (σ(a^3/6√2) = √2 = 1.414

81. A cantilever beam 3m long is subjected to a u.d.l. of 30 kN/m. The allowable working stress in either tension or compression is 150 MN/m^2. If the cross-section is to be rectangular; determine the dimensions if the height is twice as that of width.

Solution: Considering a section X-X at a distance x from free end

M_x = w.x.(x/2) N.m



Thus, the maximum bending moment will occur at fixed end, at x = 3 m

$\therefore$ M = 30 × 3 × 3/2 = 135 KN.m

Let the width of rectangular cross-section be 'b'

⇒ its height 'h' = 2b

Now, moment of inertia about the neutral axis, which passes through the centroid of the section

I bh^3/12 = b(2b)^3/12 = 2/3b^4

From M/I = σ /y

σ = M/I.y

$150 \text{ X } 10^6 = (13^5 \text{ X } 10^3) \text{ X } b / 2/3 \text{ X } b^4$

$150 \times 10^6 = 3(135 \times 10^3) / 2b^3$

b = 110.5 mm

h = 221 mm

82. A wooden beam of rectangular cross-section is to support a load of 2 kN uniformly distributed over a span of 3.6 meters. If the depth of the section is to be twice the breadth, and the stress in wood is not to exceed 70 N/cm^2, find the dimensions of the cross-section.

How would you modify the cross-section of the beam, if it was a concentrated load placed at the centre with the same ratio of breadth to depth?

Solution: Given that

Total load 'W' = 2 KN

Length 'L' = 3.6 m

Maximum stress σ = 70 N/cm^2

Let the breadth and depth of beam be 'b' and 'd' respectively.

Now, $I = bd^3 / 12 = b.(2b)^3 / 12$

$y_{max} = d/2 = 2b/2 = b$

$Z = I/y_{max} = 8b^4 / 12.b = 2/3\ b^3$

When the beam carries a u.d.l. the bending moment at the center of the beam

M = w.L2/8 = W.L./8 = 2 X 3.6 /8 = 0.90kN.m

Also, M = σ.Z

$\Rightarrow M = 70 \times 2/3\ b^3 = 0.90 \times 10^3 \times 10^2$

$\Rightarrow 70 \times 2/3\ b^2 = 0.90 \times 10^3 \times 10^2$

$\Rightarrow b = 12.5$ cm

and d = 2(12.5) = 25 cm

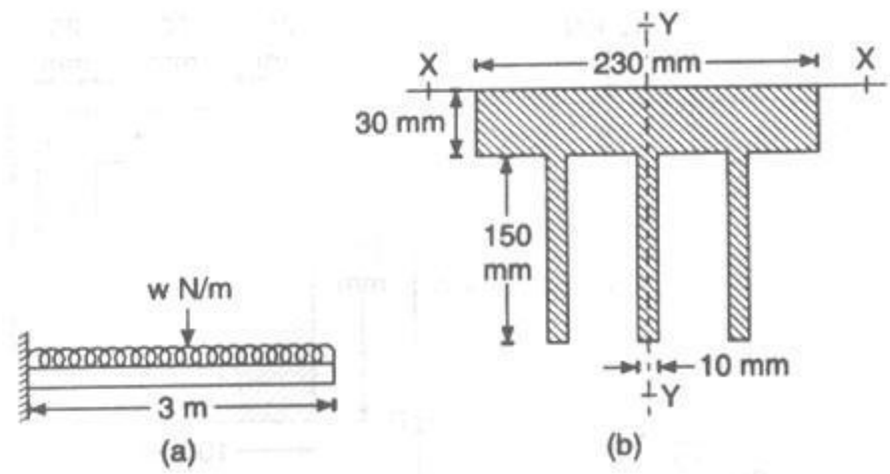
For concentrated load at the center

M = W.L/4 = 2 X 3.6 /4 = 1.8kN.m

$\Rightarrow 70 \times 2/3\ b^3 = 1.8 \times 10^3 \times 10^2$

$\Rightarrow b = 15.7$ cm and d = 31.4 cm

83. An extruded beam of aluminium alloy having an allowable working stress of 90 MPa is shown in figure. The beam is a cantilever, subjected to a uniform vertical load. Determine the allowable intensity of uniform loading.



Solution: The section is symmetric about Y-Y axis therefore, centroid will be on Y-Y axis

Let the centroid be at a distance from top face.

Then,

$\bar{y}$ (200 X 30) 15 + 3 X (180 X 10) X 90/(200 X 30) + 3 X 150 X 10

$\bar{y}$ = 50.5mm

Now, the moment of inertia of entire section about X-X axis is

$I_x = 1/3 \times 200 \times (30)^3 + 3\ (1/2 \times 10\ (180)^3) = 60.12 \times 10^6\ mm^4$

⇒ The moment of inertia about an axis passing through the centroid parallel to X-X axis

$I_G = (60.12 \times 10^6) - a\ (50.5)^2$

$a = (30 \times 230) + (150 \times 10 \times 3) = 11400 m^2$

⇒ $I_G = (60.12 \times 10^6) - (11400)(50.5)^2 = 31.05 \times 10^6\ mm^4$

For a cantilever beam subjected to u.d.l. maximum bending moment is $w.L^2/2$

⇒ $M = w.(3)^2/2$

Using relation $M/I = \sigma / y$

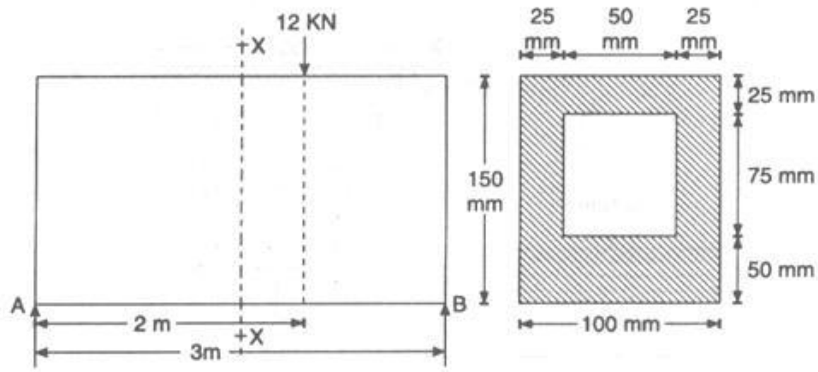
$\sigma = M/I.y$

⇒ Putting the values we get

$90 \times 10^6 = w \times (3)^2 \, (180 - 505) \times 10^{-3}/2 \times (31.05 \times 10^6) \times (10^{-3})^4$

⇒ $w = 4.86$ kN/m

84. A simply supported beam of length 3 m carries a point load of 12 kN at a distance of 2 m from left support. The cross-section of beam is shown in figure. Determine the maximum tensile and compressive stress at X-X.



Solution: Taking moment about A, we get

$R_B \times 3 - 12 \times 2 = 0$

$R_B = 24/3 = 8$ kN

$R_A + R_B = W$

$R_A = 12 - 8 = 4$ kN

Bending moment at X-X = $R_A \times 1.5$

$= 6 \times 10^3 \times 10^3$ Nmm $= 4 \times 1.5$

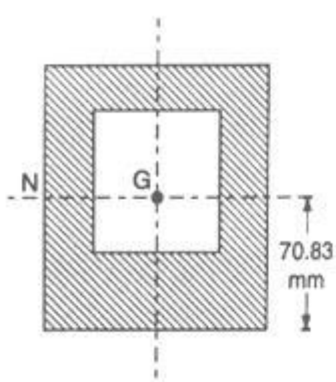
$= 6$ kNm $= 6 \times 10^6$ Nmm

The position of neutral axis of the section of the beam can be obtained if the position of center of gravity is obtained.

Let $\bar{y}$ = Distance of the center of gravity of the section from the bottom edge.

Since the section is symmetric about X-X, $\bar{x}$ is not required.
$\bar{y} = A_1 y_1 - A_2 y_2/A_1 - A_2$ = (150 X 100) X 75 - (75 X 50)X (50 + 75/2)/(150 X 100) - (75 X 50)

$\bar{y}$ = 70.83mm



Thus, the position of CG and normal axis are as in figure, and moment of inertia of the section about neutral axis is

$I = I_1 - I_2$

$I_1 = 100 \times (150)^3 / 12 + 100 \times 150 \times (75 - 70.83)^2$

$= 28.38 \times 10^6 \text{ mm}^4$

$I_2 = 50 \times (75)^3 / 12 + 50 \times 75 \times (50 + 75/2 - 70.83)^2$

$= 28.0 \times 10^5 \text{ mm}^4$

$I = (28.38 - 2.8) \times 10^6 \text{ mm}^4$

$= 25.58 \times 10^6 \text{ mm}^4$

The bottom edge of the section will be subjected to tensile stress whereas the top edge will be subjected to compressive stresses.

Therefore, using

$M/I = \sigma/y$

The tensile stress at bottom edge

$\sigma = (6 \times 10^6) \times 70.83 / 25.58 \times 10^6$

$\Rightarrow \quad \sigma = 16.61 \text{ N/mm}^2$

The compressive stress at top edge

$\sigma' = (6 \times 10^6) \times (150 - 70.83)$

$\Rightarrow \quad \sigma' = (6 \times 10^6) \times 79.17 / 25.58 \times 10^6 = 18.57 \text{ N/mm}^2$

85.
86.
87.
88.

89.
90.
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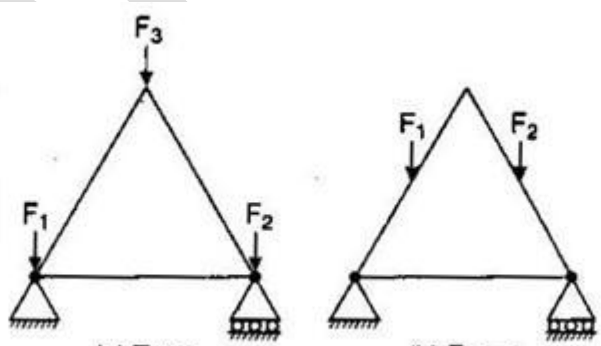
99.

1. **Plane truss**

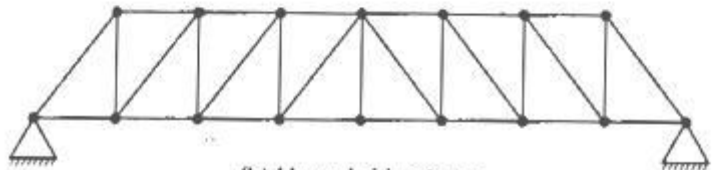
A plane truss is defined as a system of bars all lying in one plane and joined together at their ends in such a way that they form a rigid (non-collapsible) framework. It can be observed that it forms a coplanar system of forces. The basic element of plane truss is triangle. The three bars are joined by pins at their

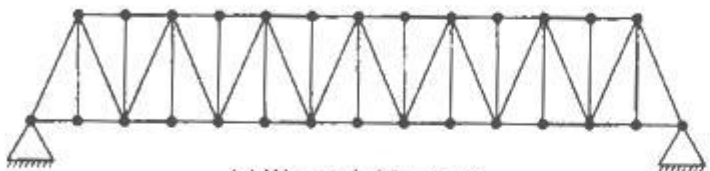


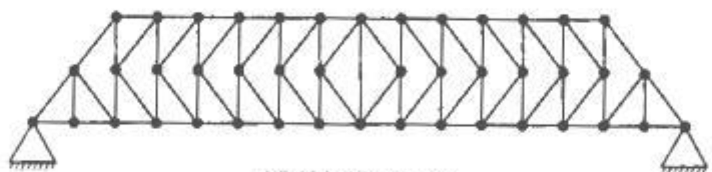
(a) Truss (b) Frame

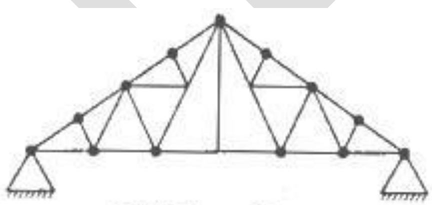
ends to constitute a rigid frame. It is notable that four or more bars pin jointed to form a polygon constitute a non-rigid frame. The non-rigid frame can be made rigid by adding diagonal bars. The commonly used examples of plane trusses include Pratt. Howe, Warren, K, Baltimore, and Fink Trusses etc.

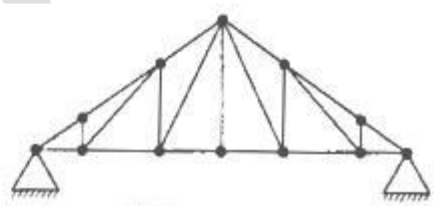

(a) Pratt bridge truss

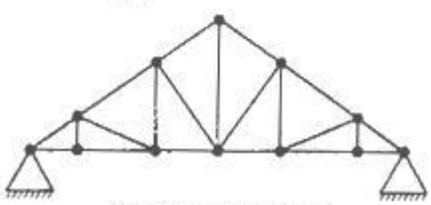

(b) Howe bridge truss

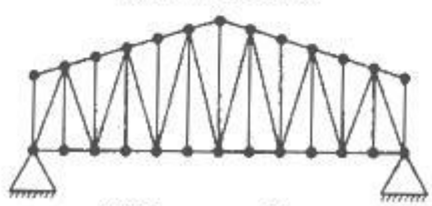

(c) Warren bridge truss


(d) K-bridge truss


(e) Fink roof truss


(f) Pratt roof truss


(g) Howe roof truss


(h) Warren roof truss

2. Space Truss

A three-dimensional system of members which is three dimensional counterpart of plane truss is called a space truss. It requires six bars joined at their ends to form the edges of a tetrahedron for the basic rigid unit. An ideal space truss is formed by point supports to avoid the bending in the members. In case of riveted and welded connections if the centerlines of jointed members intersect at a point, they can be treated as two force members.

Note:
If the weight of the member is not negligible, half the weight of the member is applied to each of its two joints. The idealization of two force member thus can be sustained.

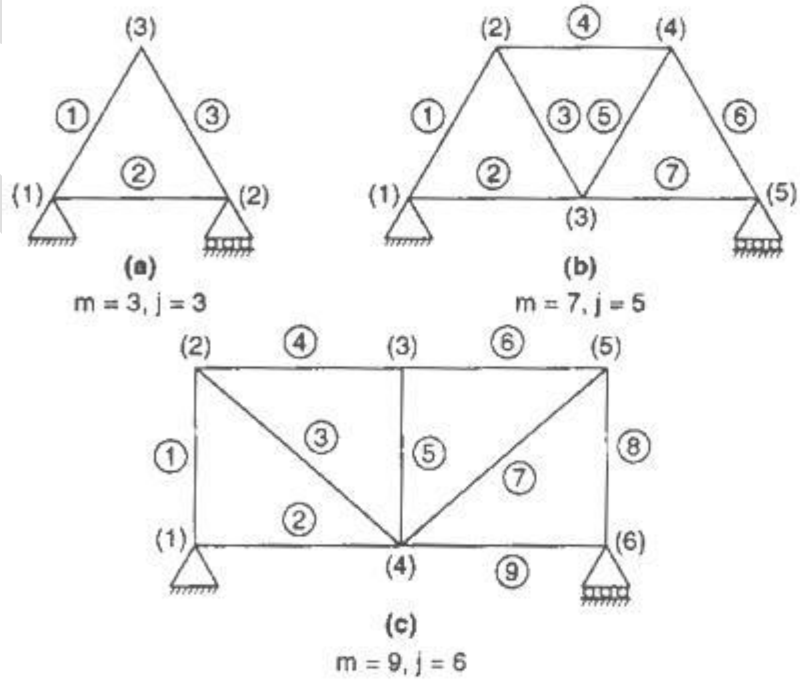
3. SIMPLE TRUSSES (PERFECT TRUSSES)

A simple truss is an idealized truss which is in just – rigid state and removal of any of its members destroys its rigidity. It is also known as perfect truss. The most simple truss is formed by three members connected to form a triangle. We can classify simple trusses as follows:

(1) The simple plane truss is built up from an elementary triangle by adding two new members for each new pm. A simple plane truss is just rigid and fulfills the condition

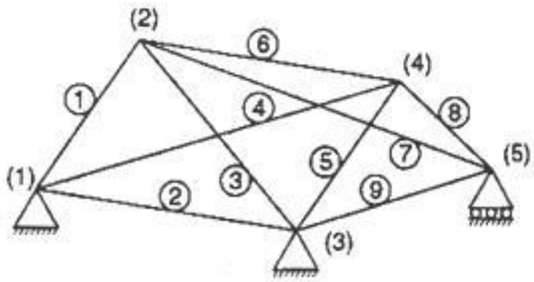
$m = 2j - 3$

where m is the number of members in a simple truss and j is the number of joints.



(2) The simple space truss is built up from an elementary triangle by adding three new members for each new joint. The simple space truss should also be just rigid and it fulfills the condition.

m = 3j – 6



m = 9, j = 5

The conditions explained give only a necessary condition for a truss to be perfect but not a sufficient condition. The truss will no more be a perfect one if it can't retain its shape on application of loads. The basic assumptions for a truss to be perfect are as follows:

(1) The joints are pin connected and frictionless. They can't resist moments.

(2) The loads are applied only at the joints.

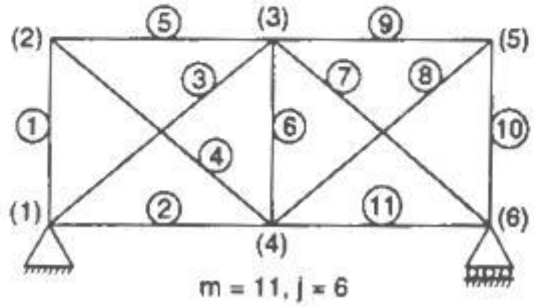
(3) The members of any truss are two force members, i.e., two equal and opposite collinear forces are applied at the ends of the member.

(4) The truss is statically determinate. It means that the equations of static equilibrium alone are sufficient to determine the axial forces in the members.

4. IMPERFECT TRUSSES

If the truss member do not fulfill the condition of just rigid situation, it will either be a deficient structure or a redundant structure.

Case (i) : Redundant trusses : m > 2j – 3

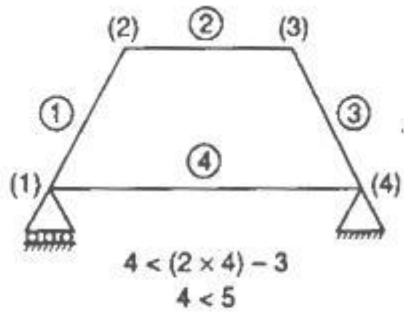


⇒ The truss contains more members then required for a perfect truss. It is therefore a redundant truss. This truss can not be analysed completely by simply making the use of equations of static equilibrium. The condition is therefore statically indeterminate and it requires consideration of deformation for analysis. Each extra member adds one degree of indeterminacy. Fig. 8.6 represent 2 degree of indeterminacy.

If a simple truss has more external supports than are necessary to ensure a stable equilibrium, the situation is termed as external redundancy. On the other hand, if the truss has more number of internal members then are necessary to prevent collapse, the condition is termed as internal redundancy.

Case (ii) : Deficient trusses : $m < 2j - 3$

The truss contains less number of members in it than required for a perfect frame. These trusses can't retain their shape when loaded and get distorted. These are also called as under rigid trusses. For example:
The truss shown below in Fig. 8.7 has one degree of deficiency.



$4 < (2 * 4) - 3$

$4 < 5$

5. DETERMINATION OF FORCE IN SIMPLE TRUSS MEMBERS

Two types of forces are to be determined in a truss. These are as follows :

(1) The reactions at the supports.

(2) The forces in the members of the truss.

The forces are determined from the conditions of equilibrium. But, before the determination of forces we should settle certain assumptions in finding out the forces in the members of the truss. These are as follows :

(1) The truss is statically determinate or perfect one in nature.

(2) The truss carries load only at the joints and loads lie in the plane of the truss.

(3) All the members of the truss are pin-connected and joint is frictionless. The joints thus can't resist moment.

The members of the truss are straight and uniform. Each member makes two force system.

(5) The weights of the members are negligibly small unless and otherwise mentioned. If the weight is to be considered the common practice is to apply half the weight of the member to each of its two joints.

6. Determination of reactions of the supports

The trusses are generally supported on hinged or roller supports. If it is supported on a roller support, the line of action will be at the right angles to the roller base. In case of hinged support the line of action of the reaction will depend upon the load system. The magnitude of reactions is determined from equations of equilibrium.
Determination of forces in the members of the truss:
There are two methods for the force analysis of simple trusses. These

(1) Analytical methods :
(a) Method of joints
(b) Method of sections

(2) Graphical method

7. Method of joints

This method consists of satisfying the equations of equilibrium for the forces acting on the connecting pin of each joint.The forces acting at a joint constitute a system of concurrent forces. This method therefore deals with the equilibrium of concurrent forces.

Two independent equations of equilibrium can be formed at each joint. The analysis begins from a joint where at least one force is known and number of unknowns is not more than two. The process of determination of forces can be concluded through following steps.

Step 1:
Determine the inclination of all inclined members, if not known. Otherwise escape this step.

Step 2:
Draw the free body diagram of joint under analysis. The free body diagram should clearly indicate the mechanism of the action and reaction. Tension should be represented by an arrow away from the pin and the compression should be indicated by an arrow towards the pin.

Note:
Two arrow heads facing each other in the member shows the internal tensile force.

Step 3:
Members of the truss should be named by Bow's method. According to Bow's notation each external force setup in members is denoted by two capital letters, placed on its either side in the space diagram.

Step 4:
Determine the reactions at the supports, using equations :

$\Sigma F_x = 0$ and

$\Sigma F_y = 0$ and

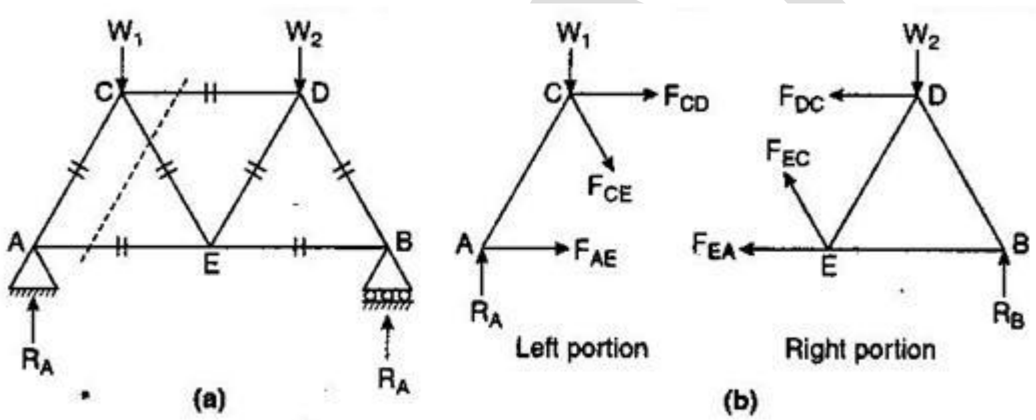
Step 5:
Consider the free body diagram of a joint under consideration. The two unknown forces should be determined at the joint under consideration using two equations of equilibrium. If the value of the force obtained from calculation is negative the assumed direction of force should be reversed.

Step 6:
Steps 2 and 5 should be repeated for next joint which has two unknown forces. Steps 2 and 5 should be repeated till the forces in all the members are known.

8. Method of sections

This method finds wide applications due to its advantages over method of joints. In this method the force in any desired member can be determined directly from the analysis of a section which cut that member. In this method the entire portion of the truss on either side of the section is considered a single body in equilibrium.

The section for analysis should be so selected that it does not cut more than three members, since there are only three available equilibrium equations which are independent.



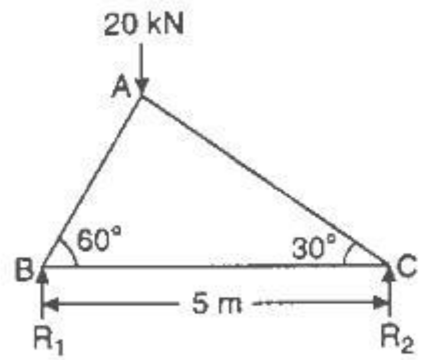
This method deals with non-concurrent force system and is preferred when we need to calculate the force in only a few members or method of joint fails to initiate the solution. The equilibrium of entire truss ensures that

$\Sigma F_x = 0$ ...(1)

$\Sigma F_y = 0$ ...(2)

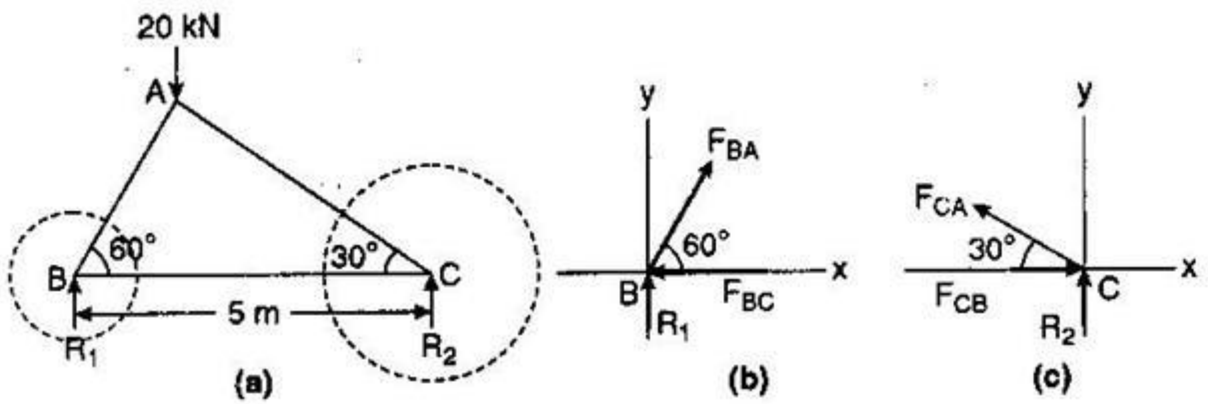
And $\Sigma M = 0$ ...(3)

9. **T**he truss ABC shown in the figure has a span of 5 meter. It is carrying a load of 20 kN at its apex. Find the forces in the members AB, AC and BC.



Solution:

Step 1: Free body diagrams figure and represent the forces at joints B and C respectively.



Step 2: Name the members by Bow's method.

Step 3: The reactions at the supports can be obtained from equations of equilibrium.

From the geometry of the truss, the load of 20 kN is acting at a distance 1.25 m from the left hand support, i.e., B and 3.75 m from C.

$\Sigma F_x = 0 \Rightarrow$ There is no force acting in horizontal direction

$SF_y = 0 \Rightarrow R_1 + R_2 = 20$ kN ...(1)

$SM_B = 0 \Rightarrow R_2$ ´ $5 = 15$ kN ...(2)

$R_2 = 5$ kN

$R_1 = 20 – R_2 = 30 – 5 = 15$ kN

Step 4: Consider the free body diagram of joint B.

$\Sigma \ F_y = 0$ Þ $F_{BA} \cos 60^o – F_{BC} = 0$ ...(3)

Σ $F_y = 0$ Þ $F_{BA} \sin 60^o + R_1 = 0$ ...(4)

$F_{BA} \sin 60^o = -R_1$

$F_{BA} \sin 60^o = -15$

(Negative sign shows that the direction of force is opposite to the direction assumed).

$F_{BA} = -15 / \sin 60^o = -17.32$ kN

From equation (3)

$F_{BA} \cos 60^o - F_{BC} = 0$

$F_{BA} \cos 60^o = F_{BC}$

$\Rightarrow F_{BC} = F_{BA} \cos 60^o = -17.32 \cos 60^o$

$\Rightarrow F_{BC} = 8.66$ kN (Tensile)

Step 5: Consider the free body diagram of next joint C:

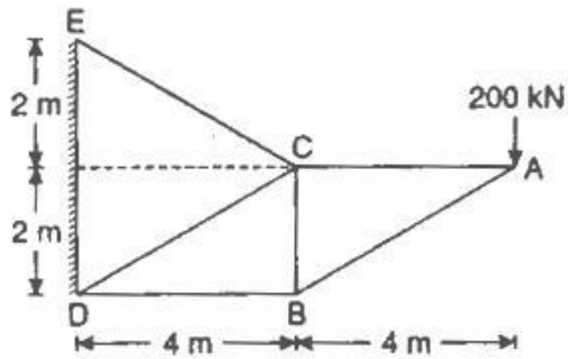
S$F_x = 0$ Þ $F_{CA} \cos 150^o - F_{CB} = 0$ ...(5)

S$F_y = 0$ Þ $F_{CA} \sin 150^o + R_2 = 0$ ...(6)

$F_{CA} \cos 150^o = F_{CB}$

$\Rightarrow F_{CA} \cos 150^o = 8.66$

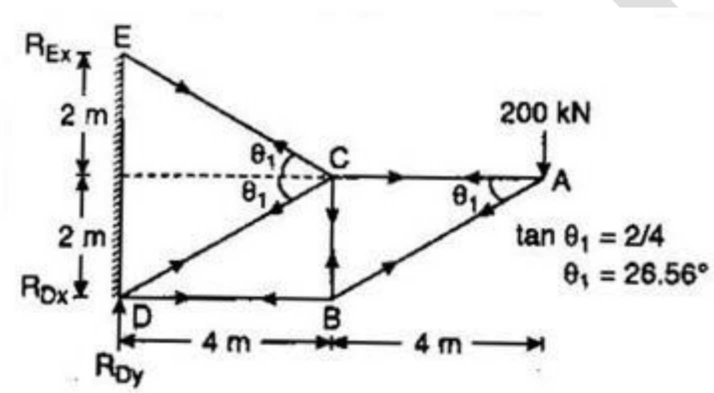
10. **Determine the support reactions and nature and magnitude of forces in members of truss shown in figure.**



Solution: Let us consider that all the members of the truss are under tension.

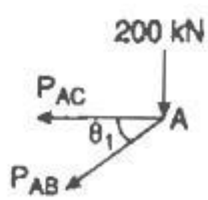
Using the method of joints for joint A.

S F_y = 0 Þ – P_{AC} – P_{AB}.cos q_1 = 0 ...(1)



S F_y = 0 Þ –200 – P_{AB} sin θ_2 = 0

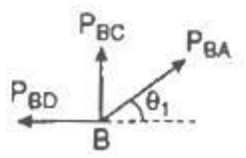
⇒ P_{AB} = –447.21 kN ...(2)



Putting the value of PAB in equation (1), we get

$-P_{AC} + 447.21 \cos (26.56^o) = 0$

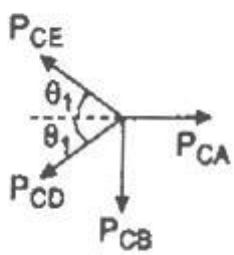
$\Rightarrow P_{AC} = 400$ kN



For joint B;

$SF_x = 0$ Þ \ P_{CE}.cos q_1 + P_{CD} cos q_1 = P_{CA} ...(3)

P_{BD} = –447.21 cos (26.56^o)= –400 kN

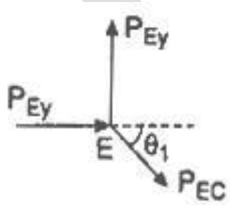


$SF_y = 0$ Þ P_{BC} + P_{BA}.sin q_1 = 0 ...(4)

P_{BC} = + 447.21 sin (26.56^o) = 200 kN

For joint C;

S F_x = 0 Þ P_{CE}.cos q_1 + P_{CD} cos q_1 = P_{CA} ...(5)



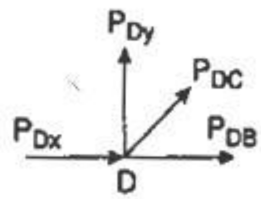
S F_y = 0 Þ P_{CE} sin q_1 – P_{CD}.sin q_1 = P_{CB} ...(6)

From equations (5) and (6), we get

P_{CE} = 447.24 kN and P_{CD} = 0 kN

For joint E; S F_x = 0 Þ R_{Ex} + P_{EC} cos q_1 = 0

⇒ R_{Ex} = –447.24 cos (26.56^o)



⇒ $R_{Ex} = -400$ kN

S $F_y = 0$

⇒ $R_{Ey} - P_{EC} \sin q_1 = 0$

⇒ $R_{Ey} = P_{EC} \sin q_1 = 447.24 \sin (26.56^o)$

⇒ $R_{Ey} = 200$ kN

For joint D; $S_x = 0$

⇒$E_{Dx} + P_{DB} = P_{DC} \cos q_1 = 0$

$R_{Dx} = 400 - 0$ ´ $\cos (26.56^o)$

= 400 kN.

S $F_y = 0$ Þ $R_{Dy} + P_{DC} = 0$

⇒ $R_{Dy} = -P_{DC} = 0$

∴ Reaction of D; R_{Dx} 40 = kN

$E_{Dy} = 0$ kN

Reaction at E;

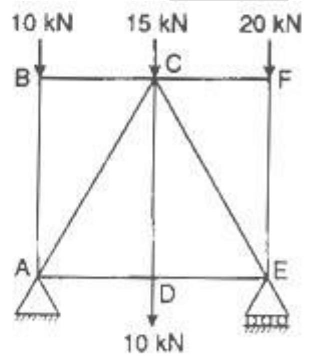
$R_{Ex} = -400$ kN

$R_{Ey} = 200$ kN

Member	**Force**	**Nature**
AB	447.21	Compressive
AC	400	Tensile
BD	400	Compressive
BC	200	Compressive

CE	447.24	Tensile
CD	00	—

11. **A truss is shown in figure. Find the forces in all the members of truss and indicate whether it is tension or compression.**



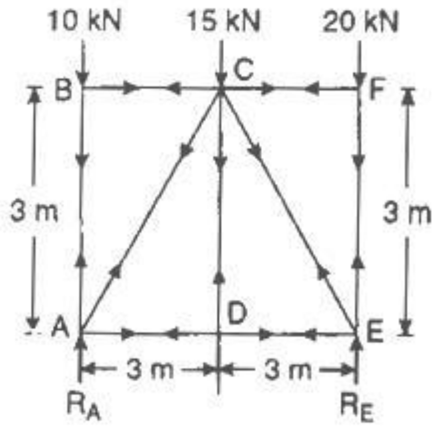
Solution: Since the length of members is not given. The data is assumed as shown in figure. Let the reactions at the joints A and E be R_A and R_E respectively.

$SM_A = 0$ ...(1)

$-R_A \times 0 + 15 \times 3 + 20 \times 6 - R_E \times 6 = 0$

$45 + 30 + 120 = 6.R_E$

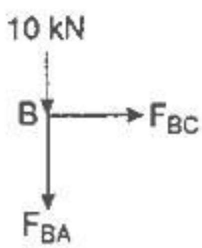
$R_E = 32/5$ kN



For equilibrium

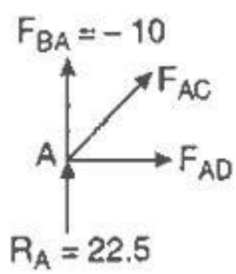
$SF_x = 0$ ...(2)

$SF_y = 0$ ...(3)



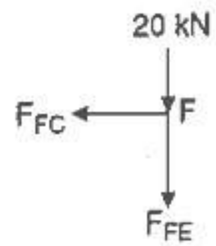
From equilibrium (3), we get $– R_A + 10 + 15 + 10 + 20 – R_B = 0$

$\Rightarrow R_A = 22.5$ kN



Form study of free body diagram of joint B:

$SF_x = 0$ Þ $F_{BC} = 0$ $F_{BC} = 0$



$SF_y = 0$ Þ $F_{BA} + 10 = 0$ Þ $F_{BA} = -10$ kN

Due to negative sign the direction F_{BA} will be reverted.

From study of joint free body diagram of A:

$SF_x = 0$

$\Rightarrow F_{AD} + F_{AC}.\cos 45^o = 0$

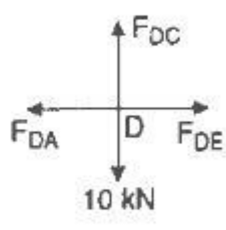
$\Rightarrow F_{AD} = -F_{AC} \cos.45^o$

$SF_y = 0$

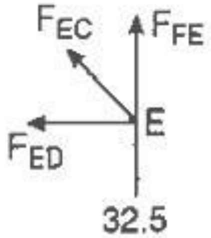
$\Rightarrow F_{AC}.\ \text{Sin } 45^o + 22.5 - 10 = 0$

$\Rightarrow F_{AC} = -12.5 / \sin 45^o = -12.5\sqrt{2}$kN

$\Rightarrow F_{AD} = (-12.5$



Due to negative sign the direction FAC will be reverted.
From study of free body diagram of joint F:



$\Sigma F_x = 0$

$\Rightarrow F_{FC} = 0$

$\Sigma SF_C = 0$

$\Sigma F_y = 0$

$\Rightarrow F_{FE} + 20 = 0$

$\Rightarrow F_{FE} = -20$ kN

From study of free body diagram of joint D:

$\Sigma F_x = 0$

$\Rightarrow -F_{AD} + F_{DE} = 0$

$F_{DE} = F_{AD} = 121.5$ kN

$\Sigma F_y = 0$

$\Rightarrow F_{DC} - 10 = 0$

$\Rightarrow F_x = 10$ kN

From study of free body diagram of joint E;

$\Sigma F_x = 0 \Rightarrow -F_{ED} - F_{EC} \cos 45^o = 0$

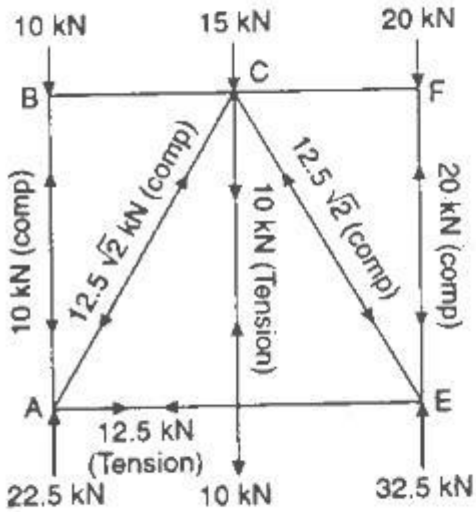
$-12.5 - F_{EC}.\cos 45^o = 0$

$\Rightarrow F_{EC} = -12.5.\sqrt{2}$

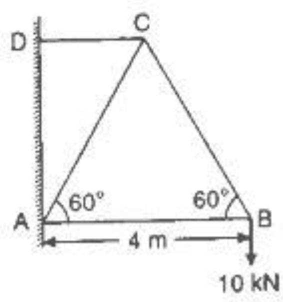
$\Sigma F_y = 0 \Rightarrow F_{FE} + F_{EC}.\sin 45^o + 32.5 = 0$

$\Rightarrow F_{FE} + (-12.5\sqrt{5}$

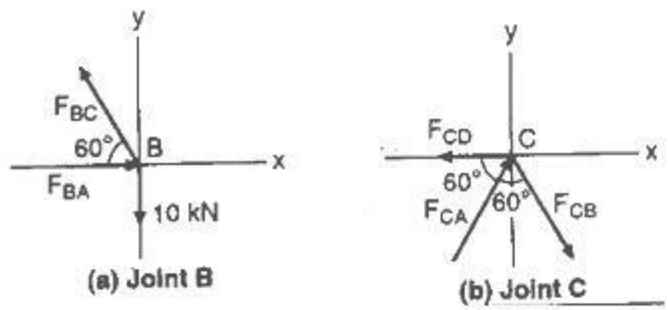
$F_{FE} = -20$ kN.



12. Find the forces in the members of the truss shown in figure.



Solution:



Step 1: Consider the free body diagram of joint B

$\Sigma F_x = 0$

$\Rightarrow F_{BA} - F_{BC} \cos 60^o = 0$ ...(1)

$F_{BC} \sin 60^o - 10 = 0$ ...(2)

$F_{BC} \times 0.866 = 10$

$F_{BC} = 11.54$ kN (Tensile)

From equation (1)

$F_{BA} = F_{BC} \cos 60^o = 11.54 \times 0.5$

$F_{BA} = 5.77$ kN (Compressive)

Step 2: Consider the free body diagram of joint C

$\Sigma F_y = 0$ Þ $F_{CA} \sin 60^o - F_{CB} \sin 60^o = 0$.,.(3)

$F_{CA} = F_{CB} = 11.54$ kN (Compressive)

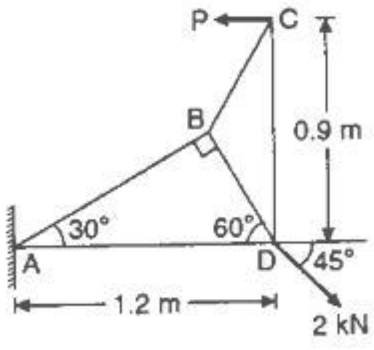
$\Sigma F_x = 0$ Þ $F_{CD} - F_{CA} \cos 60^o - F_{CB} \cos 60^o = 0$ ...(4)

$F_{CD} = F_{CA} \cos 60^o + F_{CB} \cos 60^o$

$= 11.54 \times 0.5 + 11.54 \times 0.5$

$= 5.773 + 5.773 = 11.54$ kN (Tensile).

13. A pin-jointed frame shown in figure is hinged at A and B loaded at D.A. horizontal pull is applied at C so that AD is horizontal. Determine the pull on the chain and also the force in each member. Tabulate the results.



Solution:

Step 1: Determine the R_A and P

$\Sigma F_x = 0 \Rightarrow 2 \cos 45^o + R_{AH} - P = 0$ ...(1)

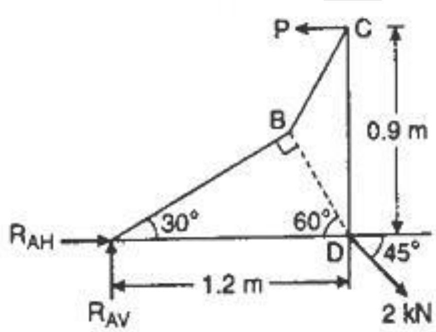
$\Sigma F_y = 0 \Rightarrow R_{AV} - 2 \sin 45^o = 0$ ...(2)

$\Sigma M_A = 0 \Rightarrow P \times 0.9 = 2 \cos 45^o \times 1.2$ ...(3)

P = 2 X 0.707 X 1.2 0.9 = 1.889kN

From equation (2)

$R_{AV} = 2 \sin 45^o$



$= 2 \times 0.707 = 1.414$ kN

From equation (1)

$2 \cos 45^o + R_{AH} - P = 0$

$R_{AH} = P - 2 \cos 45^o$

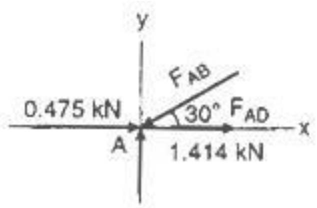
$= 1.889 - (2 \times 0.707)$

= 1.889 – 1.414

= 0.475 kN

Step 2: Consider the free body diagram of joint A

$\Sigma F_x = 0$ Þ $F_{AD} + 0.475 - F_{AB} \cos 30^o = 0$



(b) Joint A

$\Sigma F_y = 0$ Þ $F_{AB} \sin 30^o - 1.414 = 0$ ...(5)

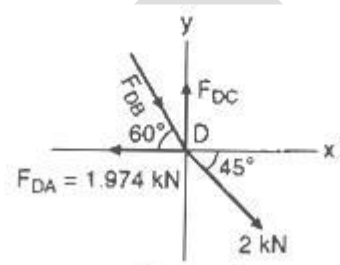
$F_{AB} \times 0.5 = 1.414$

From equation (4)

$F_{AD} = F_{AB} \cos 30^o - 0.475$

$= (2.828 \times 0.566) - 0.475$

= 1.974 kN (Tensile)



(c) Joint D

From eq. (4)

$F_{AD} = F_{AB} \cos 30^o - 0.475$

$= (2.828 \times 0.866) - 0.475$

= 1.974 kN (Tensile)

Step 3: Consider the free body diagram of joint D

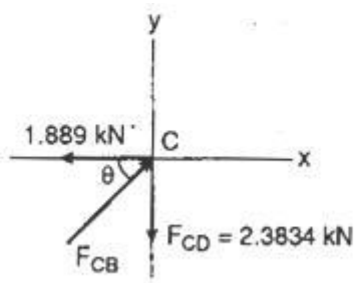
$\Sigma F_y = 0 \Rightarrow 2 \cos 45^o + F_{DB} \cos 60^o - 1.974 = 0$ ...(6)

$F_{DB} \times 0.5 = 0.559$

$F_{DB} = 0.559 / 0.5 = 1.1195$ kN (compressive)

$\Sigma F_y = 0 \Rightarrow F_{DC} - F_{DB} \sin 60^o - 2 \cos 45^o = 0$

$F_{DC} = F_{DB} \sin 60^o + 2 \cos 45^o$



(d) Joint C

$F_{DC} = (1.495 \times 0.866) + (2 \times 0.707)$

$F_{DC} = 0.9694 + 1.414$

$F_{DC} = 2.3824$ kN (Tensile)

Step 4: Consider the free body diagram of joint C

From ΔBCD

$BD = AD \sin 30^o = 1.2 \times 0.5 = 0.6$ m

$BE = BD \sin 30^o = 0.6 \times 0.5 = 0.3$ m

$DE = BD \cos 40^o = 0.6 \times 0.866 = 0.52$ m

$CE = DC - DE = 0.9 - 0.52 = 0.38$ m

$\therefore \tan \angle BCE = BE / CE = 0.3 / 0.38 = 0.7895$

$\angle BCF = 38^o 17'$

$\theta = 90^o - \angle BCF = 90^o - 38^o 17' = 51^o 42'$

$\Sigma F_x = 0$

$F_{CB} \cos 51^{o} 42' - 1.889 = 0$

$F_{CB} = 1.889 / \cos 51^{o} 42' = 3.0485$

F_{CB} = 3.0485 kN (Compressive)

Step 5: Now tabulate the results as given below:

S.No.	Member	Magnitude of force in kN	Nature of force
1.	AB	2.828	Compressive
2.	AD	1.974	Tensile
3.	BD	1.1195	Compressive
4.	CD	2.3834	Tensile
5.	BC0	3.0485	Compressive

14. A truss shown in figure to carrying a point load of 1000 N at E. Find the forces in all the members of the truss and indicate the results in a tabular form.

Solution:
Step 1: Find the inclination of all the forces.

∠CED = ∠CBD = 0

$\tan \theta = 2/4 = 0.5$

$\theta = 26^{o} 34'$

Step 2: Free body diagram and are drawn to represent the forces, at the joint E, D and C respectively.

Step 3: Name the structure by Bow's method.

Step 4: Consider the free body diagram of joint E

$\Sigma F_x = 0 \Rightarrow F_{ED} \cos 26' 34' - F_{EC} = 0$ ...(1)

$\Sigma F_y = 0 \Rightarrow F_{ED} \sin 26' 34' - 1000 = 0$ ...(2)

$F_{ED} = 1000 / \sin 26o\ 34' = 1000 / 0.4472$ N

= 2236.136 N (Compressive)

From equation (1)

$F_{EC} = F_{ED} \cos 26^o 34' = 2236.136 \times 0.894$

= 2000 N (Tensile)

Step 5: Consider the free body diagram of joint D:

$\Sigma F_x = 0 \Rightarrow F_{DE} \cos 36^o 34' - F_{BD} = 0$ ...(3)

$\Sigma F_y = 0 \Rightarrow F_{DE} \sin 26^o 34' - F_{DC} = 0$ ...(4)

$F_{DC} = F_{DE} \sin 26^o 34'$

$= 2236.136 \times 0.4472$

= 1000 N (Tensile)

From equilibrium (3) $F_{DB} = F_{DE} \cos 46^o 34'$

$= 2236.136 \times 0.894$

= 2000 N (Compressive)

Step 6: Consider the free body diagram of joint C

$\Sigma F_x = 0 \Rightarrow -F_{CA} \cos 26^o 34' + F_{CB} \cos 26^o 34' + F_{CE} = 0$ ...(5)

$\Sigma F_y = 0 \Rightarrow F_{CA} \cos 26^o 34' + F_{CB} \cos 26^o 34' + F_{CD} = 0$ ...(6)

$\therefore F_{CE} = F_{EC}$

$\therefore F_{CD} = F_{DC}$

On solving equations (5) and (6) we get

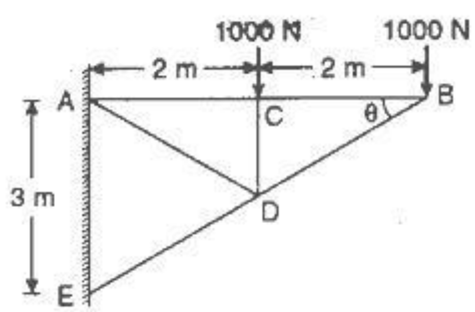
F_{CB} = 2236.66 N (Compressive)

F_{CA} = 0

Step 7: Now tabulate the results as given below:

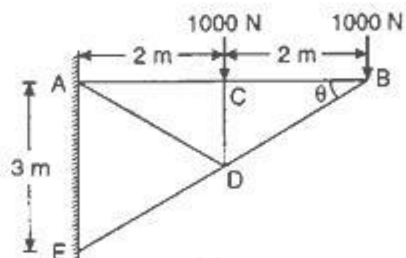
S.No.	Member	Magnitude of force in N	Nature of force
1.	AC	0	—
2.	BC	2236.66	Compressive
3.	BD	2000	Compressive
4.	DC	1000	Tensile
5.	CE	2000	Tensile
6.	DE	2236.136	Compressive

15. Find the forces in the members of the truss shown in figure.



Solution:
Step 1: Determine of inclination

tan θ = 3/4 ⇒ θ $\tan^{-1}$ (3/4) = 36.86^{o}



(a)

⇒ sin θ = 0.6 and cos = 0.8

Step 2: Considering the free body diagram of joint B

$\Sigma F_x = 0 \Rightarrow F_{BC} - F_{BD} \cos\theta = 0$ ...(1)

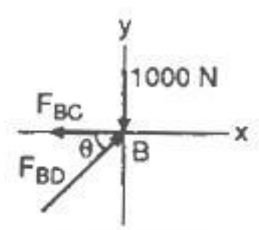
$\Sigma F_y = 0 \Rightarrow F_{BD} \sin\theta - 1000 = 0$ ...(2)

$F_{BD} = 1000 / \sin\theta = 1000 / 0.6 = 1666.67$ N (Compressive)

From eq. (1)

$F_{BC} - F_{BD} \cos\theta = 0$

$F_{BC} = F_{BD} \cos\theta = 1666.67 \times 0.8$



(b) Joint B

$F_{BC} = 1333.34$ N (Tension)

Step 3: Consider the free body diagram of joint C

$\Sigma F_y = 0 \Rightarrow 1000 - F_{CD} = 0$ ...(3)

$F_{CD} = 1000$ N (Compressive)

$\Sigma F_x = 0 \Rightarrow F_{CA} = F_{CB}$ ...(4)

$F_{CA} = 1333.43$ N (Tensile)

Step 4 Consider the free body diagram of joint D

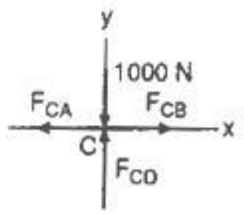
$\Sigma F_y = 0 \Rightarrow F_{DE} \sin\theta + F_{BA} \sin\theta - F_{DC} - F_{BD} \sin\theta = 0$

$0.6\ F_{DE} + 0.6\ F_{DA} - 1000 - 1666.67 \times 0.6 = 0$

$F_{DE} + F_{DA} = 333.34$ ...(5)

$\Sigma F_x = 0 \Rightarrow F_{DA} \cos\theta - F_{DE} \cos\theta + F_{DB} \cos\theta = 0$

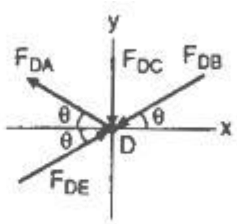
$F_{DA} - F_{DE} = -F_{DB} = -1666.67$ ...(6)



(c) Joint C

On solving (5) and (6)

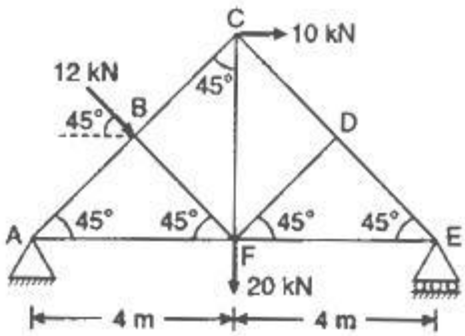
$F_{DA} = 833.33$ N (Tensile)
$F_{DE} = 3333.33 - F_{DA}$



(d) Joint D

$= 3333.33 - 833.33$
$= 2500$ N (Compressive)

16. Determine the forces in all members of truss shown in figure. Tabulate the results.

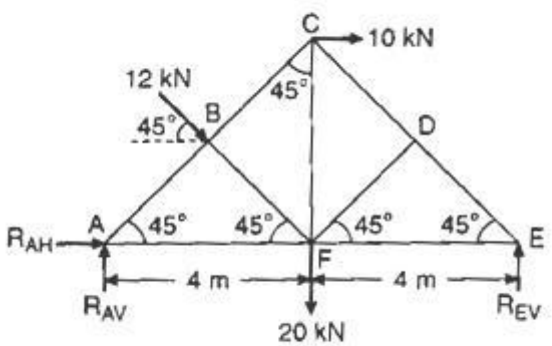


Solution:

Step 1: Determine the reactions at A and E

$\Sigma F_x = 0 \Rightarrow R_{AH} + 10 + 12 \cos 45^o = 0$ ...(1)

$R_{AH} = -18.485$ kN

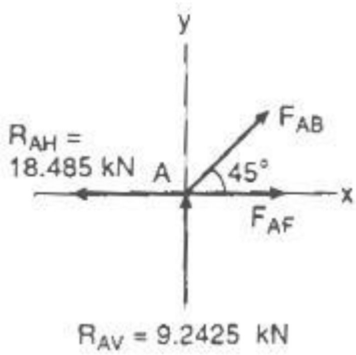


$\Sigma F_y = 0 \Rightarrow R_{AV} + R_{EV} = 20 + 12 \sin 45^o = 28.485$

$\Rightarrow R_{AV} \times 8 - 12 \sin 45^o \times 6 + 12 \cos 45^o \times 2 + 10 + 4 - 20 \times 4 = 0$

$R_{AV} = 9.2425$ kN

$R_{EV} = 19.2425$ kN



Joint A

Step 2: Consider the free body diagram of joint A

$\Sigma Fx = 0 \Rightarrow F_{AF} + F_{AB} \cos 45^o = 18.485$ ...(3)

$\Sigma F_y = 0 \Rightarrow F_{AB} \sin 45^o = -9.2425$ ...(4)

From eq. (3)

$F_{AF} = 18.485 - 13 \cos 45^o = 9.293$ kN

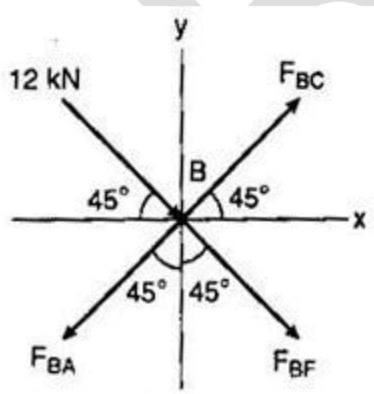
$F_{AF} = 9.293$ kN (Tensile)

Step 3: Consider the free body diagram or joint B

$\Sigma F_x = 0 \Rightarrow 12 \cos 45^o + F_{BC} \cos 45^o + F_{BF} \sin 45^o - F_{BA} \sin 45^o = 0$

$12 + F_{BC} + F_{BF} - (-13) = 0$

$F_{BC} + F_{BF} = -25$ ...(5)



Joint B

$\Sigma F_x = 0 \Rightarrow 12 \cos 45^o - F_{BA} \cos 45^o - F_{BF} \cos 45^o + F_{BC} \sin 45^o = 0$

$-12 + 13 + F_{BC} - F_{BF} = 0$

$F_{BC} - F_{BF} = -1$ ...(6)

On solving eq. (5) and (6)

$F_{BC} = -13$ kN = 1.3 kN (Compressive)

$F_{BC} = -12$ kN = 12 kN (Compressive)

Step 4: Consider the free body diagram of joint C:

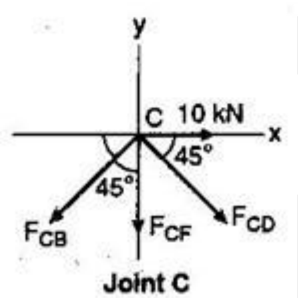
$\Sigma F_x = 0 \Rightarrow F_{CD} \cos 45^o + 10 - F_{CB} \sin 45^o = 0$

$F_{CD} - F_{CB} = 10 / 0.707 = 14.144$

$F_{CD} - F_{CB} = 14.14$

$F_{CD} = 14.14 + 13 = 27.14$ kN (Tensile) ...(7)

$\Sigma F_y = 0 \Rightarrow F_{CD} \sin 45^o + F_{CB} \cos 45^o + F_{CF} = 0$ ...(8)



$0.707\ F_{CD} + F_{CB} \times 0.707 + F_{CF} = 0$

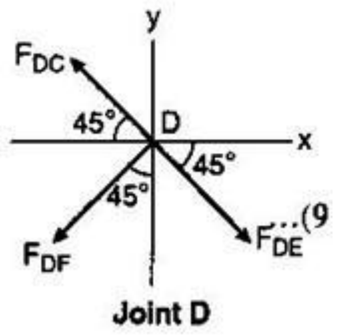
$F_{CF} = -0.707 \times 27.14 - 13 \times .707$

$= -28.4$ kN

$F_{CF} = 28.4$ kN (Compressive)

Step 5: Consider the free body diagram of joint D

$\Sigma F_x = 0 \Rightarrow F_{DE} \cos 45^o - F_{DC} \cos 45^o - F_{DF} \sin 45^o = 0$



$F_{DE} - F_{DC} - F_{DF} = 0$

$F_{DE} - 27.1 - F_{DF} = 0$

$F_{DE} - F_{DE} = 27.14$ ...(9)

$\Sigma F_y = 0 \Rightarrow -F_{DE} \sin 45^o - F_{DF} \cos 45^o + F_{DC} \sin 45^o = 0$

$-F_{DE} - F_{DF} + F_{DC} = 0$

$-F_{DE} - F_{DF} = -27.14$

$F_{DE} + F_{DF} = 27.14$ ...(10)

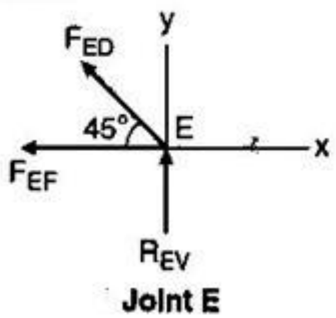
On solving equations (9) and (10), we get

$F_{DE} = 27.14$ (Tensile)

$F_{DF} = 0$

Step 6: Consider the free body diagram of joint E

$\Sigma Fx = 0 \Rightarrow F_{EF} + F_{ED} \cos 45^o = 0$ ...(11)



$F_{EF} = -F_{ED} \cos 45^o$

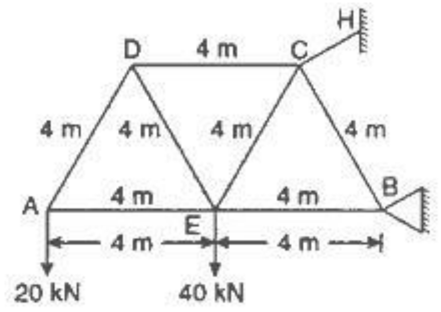
$= -27.14 \times 0.707 = -19.187$

F_{EF} = 19.187 kN (Compressive)

Step 7: Now we represent the resultants in tabular form

S.No.	**Member**	**Magnitude of force in N**	**Nature of force**
1.	AB	13	Compressive
2.	BC	13	Compressive
3.	CD	27.14	Tensile
4.	DE	27.14	Tensile
5.	EF	19.187	Compressive
6.	CE	28.4	Compressive
7.	DF	0	—
8.	BF	12	Compressive
9.	AF	9.293	Tensile

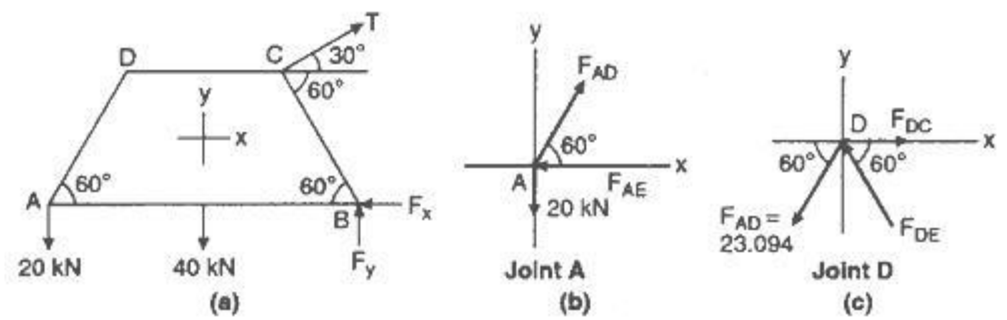
17. Determine the force in each in each member of the loaded cantilever truss by the method of joints.



Solution:

Step 1: The inclination of CH is obtained from geometry

Step 2: Free body diagrams (a) and (b) are drawn to represent the forces.



Step 3: Name the members by Bow's method.

Step 4: The reactions at the supports can be obtained from equations of equilibrium:

$\Sigma F_x = 0 \Rightarrow T \cos 30^o - F_x = 0$ ...(1)

$\Sigma F_y = 0 \Rightarrow T \sin 30^o + F_y - 40 - 20 = 0$ ...(2)

$\Sigma M_B = 0 \Rightarrow 4 \times T - 40 \times 4 - 20 \times 8 = 0$ ...(3)

From equation (3), we get $T = 80$ kN

From equation (1), we get $F_x = 80 \cos 30^o = 69.282$ kN

From equation (2), we get $F_y = 60 - 80 \sin 30^o = 20$ kN

Step 5: Calculation of unknown forces F_{AD} and F_{AE}

$\Sigma F_x = 0 \Rightarrow F_{AE} - F_{AD}.\cos 60^o = 0$ ...(4)

$\Sigma F_y = 0 \Rightarrow F_{AD} \times \sin 60^o - 20 = 0$ ...(5)

From equation (5), we get

F_{AD} = 20 / sin 60^o = 23.094 kN (Tensile)

From equation (4), we get

F_{AE} = 23.094 × cos 60^o = 11.547 kN (Compressive)

Step 6: Considering the free body diagram of joint D, we repeat steps (2) and (5).

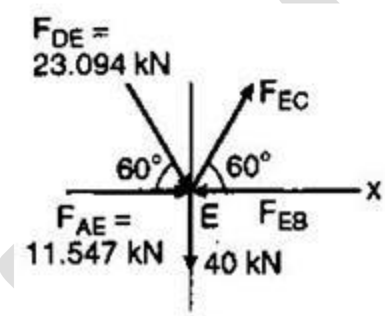
$\Sigma F_x = 0 \Rightarrow F_{DE}.\cos 60^o - F_{DC} + F_{AD} \cos 60^o = 0$ ...(6)

$\Sigma F_y = 0 \Rightarrow 20 - F_{CB} \sin 60^o = 0$ ...(7)

From equation (7), we get

$F_{DE} = F_{AD} \sin 60^o / \sin 60^o = F_{AD}$ = 23.094 kN

From equation (6), we get



(d)

23.094 cos 60^o + 23.094 60^o = 0

$\Rightarrow F_{DC}$ = 23.094 cos 60^o + 23.094 cos 60^o

$\Rightarrow F_{DC}$ = 23.094 kN (Tensile)

Considering the free body diagram of joint E, we see

$\Sigma F_x = 0 \Rightarrow -F_{EB} + F_{AE} + F_{EC} \cos 60^o + F_{DE}.\cos 60^o = 0$ ...(8)

$\Sigma F_y = 0 \Rightarrow F_{DE} \sin 60^o + 40 - D_{EC} \sin 60^o = 0$ ...(9)

From equation (9), we get

$F_{EC} = 40 + 23.094 \sin 60^o / \sin 60^o = 69.282$ kN (Tensile)

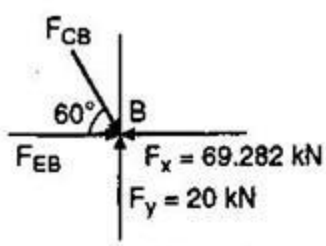
From equation (9), we get

$F_{EB} = 11.547 + 67.282 \cos 60^o + 23.094 \cos 60^o = 57.734$ kN (Compressive)

Finally, considering free body diagram of joint B, we see

$\Sigma F_x = 0 \Rightarrow F_{EB} – F_x + F_{CB} \cos 60^o = 0$ ...(10)

$\Sigma F_y = 0 \Rightarrow 20 – F_{CB} \sin 60^o = 0$ ...(11)



(e)

From equation (11), we get

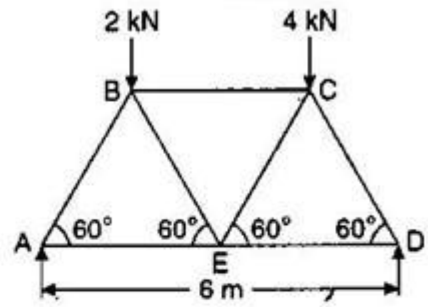
$\Rightarrow F_{CB} = 20 / \sin 60^o = 23.094$ kN

From equation (10), we get

$\Sigma F_x = 57.74 – 69.282 + 23.094 \cos 60^o = 0$

Equation (10) provides the check from correct solution.

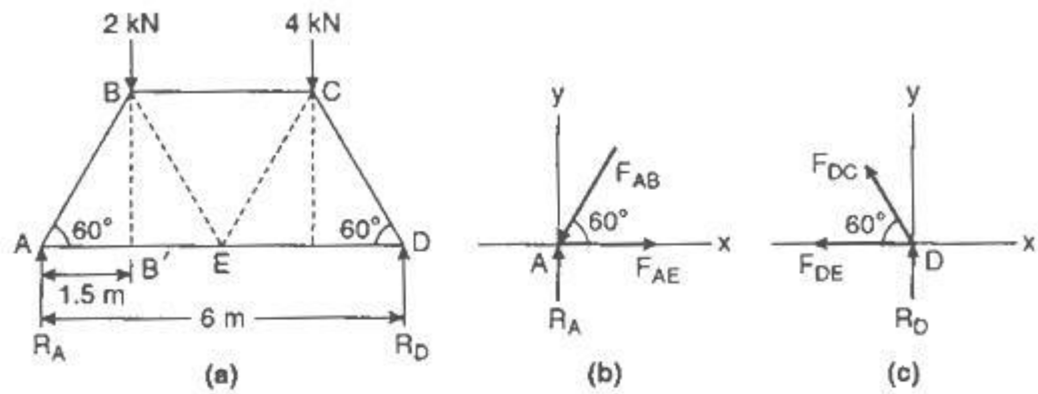
18. Figure shows a Warren girder consisting of seven members each of 3 m length freely supported at its end points. The girder is loaded at B and C as shown. Find the forces in all the member of the girder, indicating whether the force is compressive or tensile.



Solution:

Step 1: The distance AB' is obtained from the geometry.

Step 2: Free body diagrams are drawn to represent the forces.



Step 3: Name the members by Bow's method.

Step 4: The reactions at the supports can be obtained from equations of equilibrium:

$\Sigma F_x = 0$

$\Sigma F_y = 0 \Rightarrow R_A + R_D = 6$ ...(1)

$\Sigma M_A = 0 \Rightarrow 2 \times 1.5 + 4 \times 4.5 - R_D \times 6 = 0$ ...(2)

$R_D = 3.5$ kN

$R_A = 6 - 3.5 = 2.5$ kN

Step 5: Consider the free body diagram of joint A

$\Sigma F_x = 0 \Rightarrow F_{AE} - F_{AB} \cos 60^o = 0$ ...(3)

$F_{AB} \sin 60^o = R_A$ ...(4)

$R_A = 2.5$ kN

$F_{AB} \sin 60^o = 2.5$

$F_{AB} = 2.886$ kN (Compressive)

From equation (3)

$F_{AE} - F_{AB} \cos 60^o = 0$

$F_{AB} = F_{AB} \cos 60^o = 2.886 \times \cos 60^o$

= 1.44 kN (Tensile)

Step 6: Now consider the free body diagram of joint D:

$\Sigma F_x = 0 \Rightarrow F_{DC} \cos 120^o - F_{DE} = 0$ ...(5)

$\Rightarrow F_{DE} = F_{DC} \cos 120^o$

$F_{DE} = F_{DC} \cos 120^o$

$\Sigma F_y = 0 \Rightarrow R_D + F_{DC} \sin 120^o = 0$ ...(6)

$\therefore R_D = 3.5$

$\Rightarrow F_{DC} = -3.5 / \sin 120^o = -4.042$ kN

$F_{DC} = 4.042$ kN (Compressive)

$F_{DE} = F_{DC} \cos 120^o = 40.042 \times 0.5 = 2.021$ kN (Tensile)

Step 7: Consider the free body diagram at joint B

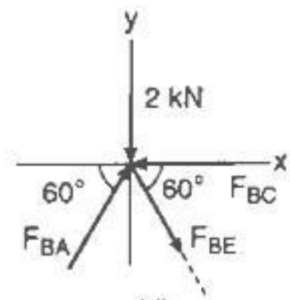
$F_{BA} = F_{AB} = 2.887$ kN

$\Sigma F_x = 0 \Rightarrow - F_{BA} \cos 240^o + F_{BE} \cos 300^o - F_{BC} = 0$ ...(7)

$\Sigma F_y = 0 \Rightarrow F_{BE} \sin 300^o - 2 - F_{BA} \sin 240^o = 0$ ...(8)

$-F_{BA} \cos 240^o + F_{BE} \cos 300^o - F_{BC} = 0$

$F_{BE} \times 0.5 - F_{BC} = -2.887 \times 0.5$



(d)

$F_{BE} - 2\,F_{BC} = -2.887$ ...(9)

$F_{BE} \sin 300^o - 2 - 2.887 \times \sin 240^o = 0$

$F_{BE} \sin 300^o = 2 + 2.887 \times (0.866)$

$F_{BE} = -2\,F_{BC} = -2.887$

$0.577 + 2.887 = 2\,F_{BC}$

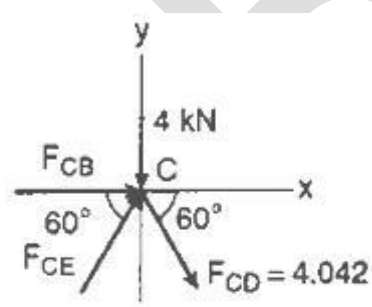
$F_{BC} = 1.732$ kN (Compressive)

Step 8: Consider the free body diagram of joint C

$\therefore F_{CD} = F_{DC} = 4.042$

$\Sigma F_x = 0 \Rightarrow F_{CB} - F_{CE} \cos 240^o + F_{CD} \cos 300^o = 0$ ...(10)

$\Sigma F_y = 0 \Rightarrow F_{CE} \sin 240^o - 4 - F_{CD} \sin 300^o = 0$ ...(11)



(e)

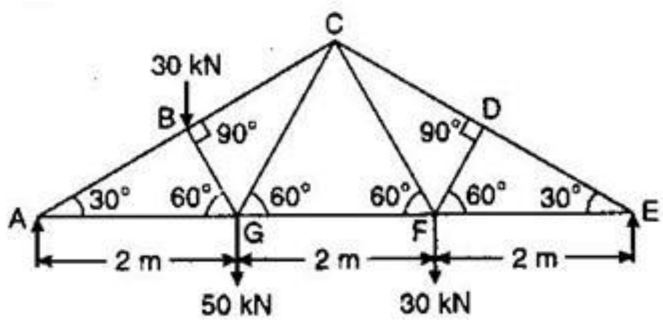
$F_{CE} \times (-0.866) - 4 - 0.042 \times (-0.866) = 0$

$-0.866\,F_{CE} - 0.4995 = 0$

$F_{CE} = -0.577$ kN

F_{CE} = 0.577 kN (Compressive)

19. An inclined truss is loaded as shown in figure. Determine the nature and magnitude of the forces in a all the members of the truss.



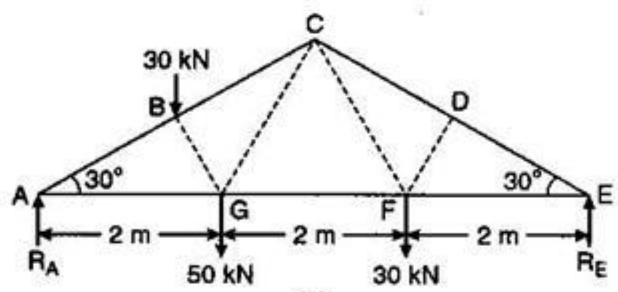
Solution:

Step 1: The inclination of AB and DE is obtained from the figure.

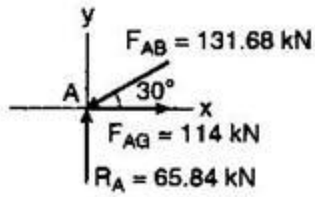
Step 2: Free body diagram and are drawn to represent the forces.

Step 3: Name the members by Bow's method.

Step 4: The reactions the supports can be obtained from equations of equilibrium.



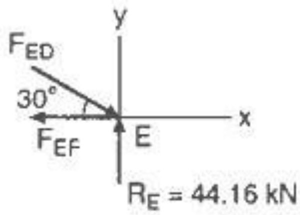
(a)



(b) Joint A

$\Sigma F_x = 0$

$\Sigma F_y = 0 \Rightarrow R_A + R_E = 30 + 50 + 30 = 110$ kN



(c) Joint E

$R_A + R_E = 110$ kN ...(1)

$\Sigma M_A = 0 \Rightarrow 30 \times 1.5 + 50 \times 2 + 30 \times 4 - RE \times 6 = 0$ ...(2)

$R_E = 44.16$ kN

$R_A = 110 - 44.16 = 65.84$ kN

$R_A = 65.84$ kN

Step 5: Consider the free body diagram of joint A

$\Sigma F_x = 0 \Rightarrow F_{AG} - F_{AB} \cos 30^o = 0$ ...(3)

$\Sigma F_y = 0 \Rightarrow F_{AB} \sin 30^o - R_A = 0$ ...(4)

$F_{AB} \sin 30^o - 63.84 = 0$

$\Rightarrow F_{AB} = 131.68$ kN (Compressive)

From eq. (3)

$F_{AG} - 131.68 \times \cos 30^o = 0$

$F_{AG} = 131.68 \times 0.866$

$F_{AG} = 114$ kN

Step 6: Consider the free body diagram of joint E

$\Sigma F_x = 0 \Rightarrow F_{ED} \cos 30^o - F_{EF} = 0$ ...(5)

$\Sigma F_y = 0 \Rightarrow F_{ED} \sin 30^o - R_E = 0$ ...(6)

$F_{ED} \sin 30^o - 44.16 = 0$

$\Rightarrow F_{ED} = 88.32$ kN (Compressive)

$F_{EF} = 88.32 \times \cos 30^o$

$F_{EF} = 88.32 \times (0.866) = 76.48$

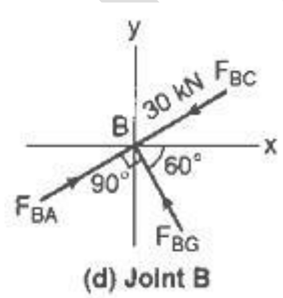
$\Rightarrow F_{EF} = 76.48$ (Tensile)

Step 7: Consider the free body diagram of joint B

$\therefore F_{AB} = F_{BA} = 131.68$ kN

$\Sigma F_x = 0 \Rightarrow F_{BC} \cos 30^o - F_{BA} \cos 30^o + F_{BG} \cos 60^o = 0$ ...(7)

$\Rightarrow \Sigma F_y = 0$



(d) Joint B

$F_{BC} \sin 30^o + 30 - F_{BA} \sin 30^o - F_{BG} \sin 60^o = 0$ ...(8)

$F_{BC} \times 0.866 + F_{BG} \times 0.5 = 131.68 \times 0.866$

$0.866\ F_{BC} + 0.5\ F_{BG} = 114.03$

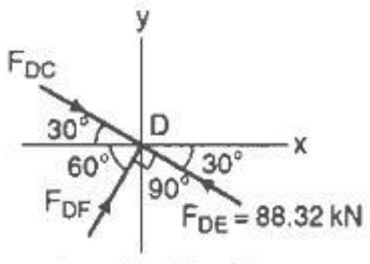
$0.866\ F_{BC} + 0.5\ F_{BG} = 114.03$

$0.5\ F_{BC} - 0.866\ F_{BG} = 35.84$

On solving above equations, we get

$F_{BC} = 116.6$ kN (Compressive)

$F_{BG} = 26$ kN (Compressive)



(e) Joint D

Step 8: Consider the free body diagram of joint D

$\Sigma F_x = 0 \Rightarrow -F_{DE}\cos 30^o + F_{DC}\cos 30^o + F_{DF}\cos 60^o = 0$ ...(9)

$\Sigma F_y = 0 \Rightarrow F_{DC}\sin 30^o - F_{DE}\sin 30^o - F_{DF}\sin 60^o = 0$ ...(10)

$F_{DC} \times 0.866 + 0.5\ F_{DF} = 76.49$

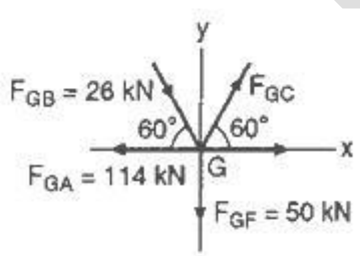
$0.5\ F_{DC} - 0.866\ F_{DF} = 44.16$

On solving above equations, we get

$F_{DC} = 88.32$

$F_{DF} = 0$

Step 9: Consider the fre body diagram of Joint G



(f) Joint G

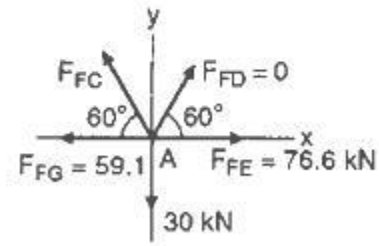
$\Sigma F_x = 0 \Rightarrow F_{GC} \cos 60^o - F_{GB} \cos 60^o + F_{GA} + F_{GF} = 0$ ...(11)

$\Sigma F_y = 0 \Rightarrow F_{GC} \sin 60^o - F_{GB} \sin 60^o - 50 = 0$ ...(12)

On solving above equations, we get

F_{GC} = 83.7 kN (Tensile)

F_{GF} = 59.1 kN



(g) Joint A

Step 10: Consider the free body diagram of joint F

$\Sigma F_x = 0 \Rightarrow F_{FC} \cos 60^o - F_{ED}\ 60^o + 76.6 - 59.1 = 0$ ...(13)

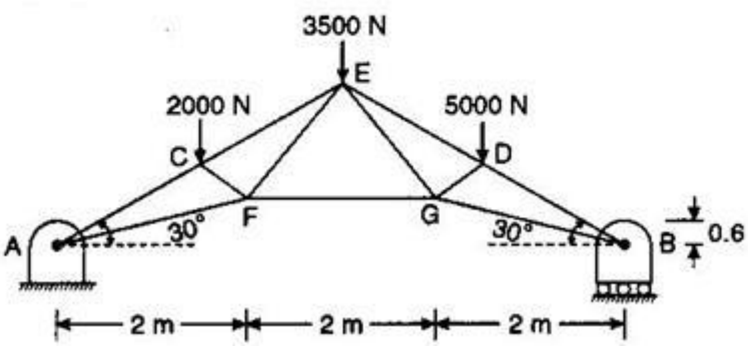
$\Sigma F_y = 0 \Rightarrow F_{FC} \sin 60^o + F_{FD} \sin 60^o - 30 = 0$ ...(14)

On solving equations (13) and (14) we get

F_{FC} = 34.6 kN

20. The roof truss shown in figure is supported at A and B carries vertical loads at each of the upper chord points.

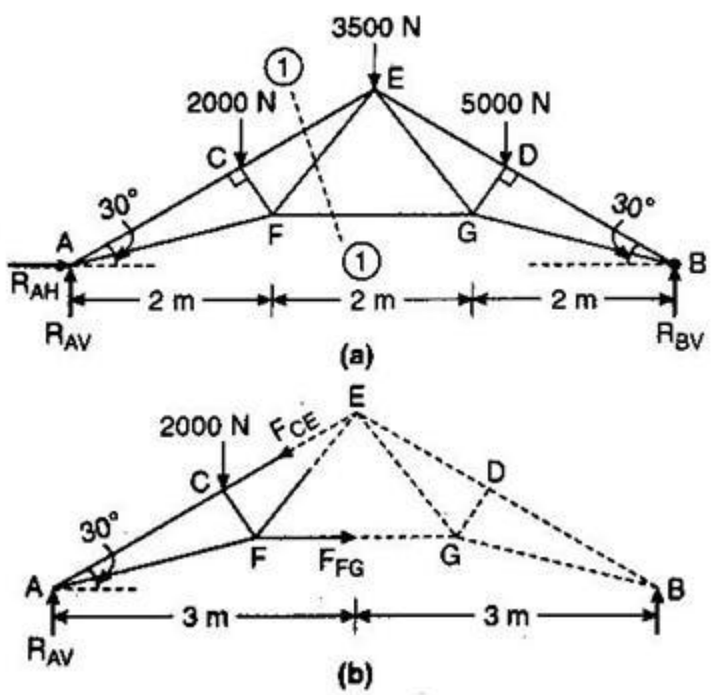
Using the method of sections, determine the forces in the member CE and FG of truss, starting whether they are in tension or compression.



Solution:

Step1: Indicating all the reaction forces on the supports

Step 2: Free body diagrams shown in figure and are drawn to represent the forces.



Step 3: Name the members by Bow's method.

Step 4: There is no horizontal load on the truss

$\Rightarrow \Sigma F_x = 0$

$\therefore R_{AH} = 0$

$\Sigma MA = 0 \Rightarrow RBV \times 6 = (2000 \times 1.5) + (3500 \times 3) + (5000 \times 4.5)$

$R_{BV} = 6000$ N

$\Sigma F_y = 0 \Rightarrow R_{AV} + R_{BV} = 2000 + 3500 + 5000$

$R_{AV} + R_{BV} = 10500$ ...(1)

$R_{AV} + 6000 = 10500$

$R_{AV} = 4500$ N

Step 5: From the geometry of the figure, we find that the length,

$AE = 3/\cos 30^o = 3/0.866 = 3.46$ m

∴ Vertical distance between the joint E and the member

$F_G = 3 \tan 30^o – 0.45 = (3 \times 0.577) – 0.45 = 1.28$ m

And length of the member $AF= \sqrt{(2)^2+ (0.45)^2} =2.05$ m

$\tan \angle FAB = 0.45/2 = 0.225$ or $\angle FAB = 12^o\ 41'$

and $\angle CAF = 30^o – 12^o\ 41' = 17^o\ 19'$

∴ Length of member $CF = AF \sin 17'\ 19' = 2.05 \times 0.2977 = 0.61$ m

And length of the member $AC = AF \cos 17^o\ 19' = 2.05 \times 0.9547 = 1.96$ m

And length of the member $CE = 3.46 – 1.96 = 1.5$ m

∴ Distance between the line of action of load at C and joint F

$= 2 – AC \cos 30^o = 2 – 1.96 \times 0.866 = 0.3$ m

Distance between the line of action of load at C and joint E

$= CE \cos 30^o = 1.5 \times 0.866 = 1.3$ m

Step 6: Pass section (1 – 1) cutting the member CE, FE and FG as shown in figure. Consider the equilibrium of the left part of the truss. Let the directions of F_{CE} and F_{FG} be assumed as shown in figure.

Taking moment about E and equating to zero, we get

$\Sigma ME = 0$

$F_{FG} \times 1.28 = R_{AV} \times 3 – 2000 \times 1.3$

$F_{FG} \times 1.28 = 4500 \times 3 - 2000 \times 1.3$

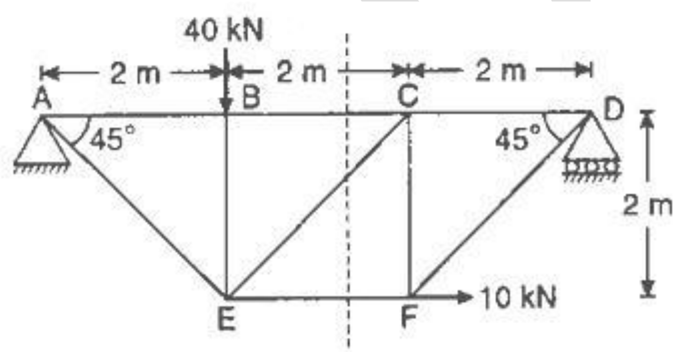
$F_{FG} = 8515.625$ N (Tensile)

Step 7: Now taking moments about the joint F,

$F_{CE} \times 0.61 = (4500 \times 2) - (2000 \times 0.3) = 8400$

$F_{CE} = 13770.49$ N (Compressive)

21. Determine support reactions and nature and magnitude forces in members BC and EF of the diagonal truss shown in figure.



Solution: For equilibrium

$\Sigma F_x = 0$ ...(1)

$\Sigma F_y = 0$ ...(2)

$\Sigma M_A = 0$ ...(3)

From equation (1), we get

$R_{Ax} + 10 = 0$ Þ $R_{Ax} = -10$ kN ...(4)

From equation (2), we get

$R_{Ax} + R_D - 40 = 0$ ...(5)

From equations (3), we get

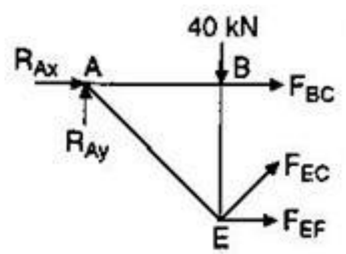
$-R_D \times 6 + 40 \times 2 - 10 \times 2 = 0 \Rightarrow R_D = 10$ kN ...(6)

Putting value of RD in equilibrium (5), we get

$R_{Ay} = 30$ kN

For calculating force in member BC and EF of the truss we can use the method of sections.

Let us consider a vertical section which cuts member BC, EC and EF. Draw F.B.D. or left portion of the section as shown in figure.



For equilibrium of F.B.D.,

$\Sigma F_x = 0 \Rightarrow -10 + F_{BC} + F_{EC} \cos 45^o = 0$ ...(7)

$\Sigma F_y = 0 \Rightarrow 30 - 49 - F_{BC}.\sin 45^o = 0$

$\Rightarrow F_{EC} = 10\sqrt{2}$ kN ...(8)

$\Sigma M_E = 0$

$\Rightarrow -10 \times 2 + 30 \times 2 + F_{BC} \times 2 = 0$

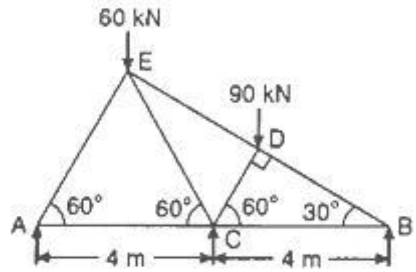
$\Rightarrow F_{BC} = -20$ kN ...(9)

Putting the values of F_{EC} and F_{BC} in equation (7), we get

$F_{EF} = 20$ kN

Thus, BC will be under compression while EF m Tension

22. Determine the forces in the member ED, CD and BC of the truss shown in figure.

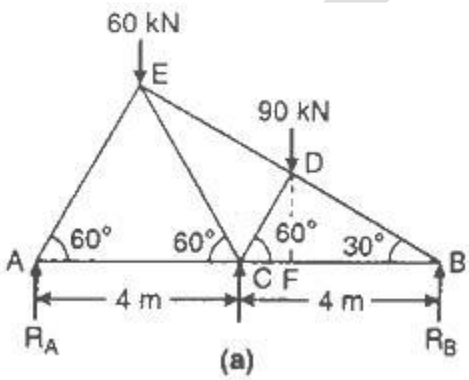


Solution:

Step 1: Determine the reactions R_A and R_B

$\Sigma F_y = 0 \Rightarrow R_A + R_B = 60 + 90$

$R_A + R_B = 150$ ...(1)



$\Sigma M_A = 0 \Rightarrow R_B \times 8 = 60 \times 2 + 90 \times (4 + CF)$

$CD = BC \sin 30_o = 4 \times 1/2 = 2$ m

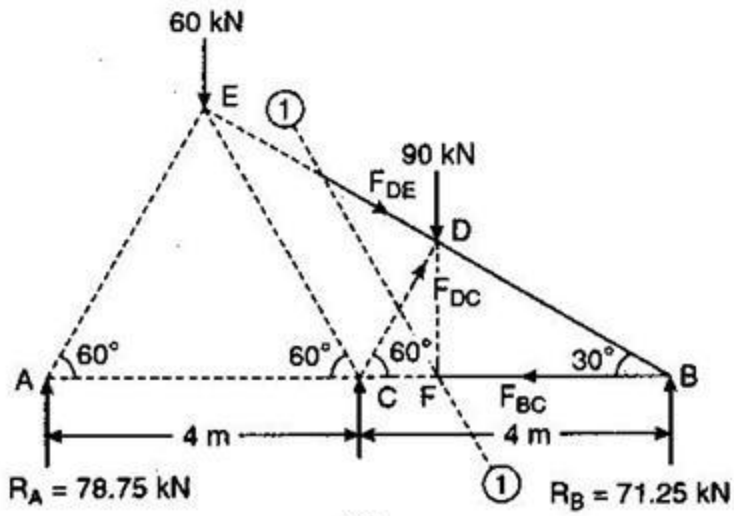
$CF = CD \times \cos 60^o = 2 \times 1/2 = 1$ m

$\therefore 8\ R_B = 120 + 90 \times 5$

$\Rightarrow R_B = 570/8 = 71.25$ kN

$\Rightarrow R_A = 150 – 71.25 = 78.75$ kN

Step 2: Draw a section line 1 – 1 as shown in figure and consider the equilibrium of the right part of the section line.



(b)

Force in member ED;

$\Sigma M_C = 0 \Rightarrow F_{DE} \times 4 \sin 30^o + 90 \times 1 = R_B = 4$

$F_{DE} \times (4 \times 0.5) = 71.25 \times 4 - 90 = 195$

$F_{DE} = 195 / 4 \times 0.5 = 97.5$ kN (Compressive)

Force in member CD;

$\Sigma M_B = 0 \Rightarrow F_{DC} \times 4 \sin 60^o = 90 \times F_B = 90 \times 3$

$F_{DC} = 270 / 4 \times 0.866 = 77.9$ kN (Compressive)

Force in member BC;

$\Sigma M_D = 0 \Rightarrow F_{BC} \times (F_B \tan 30^o) = RB \times FB$

$F_{BC} \times 3 \tan 30^o = 71.25 \times 3$

$F_{BC} = 71.25 / \tan 30^o = 123.4$ kN (Tensile)

23.
24.
25.

26.
27.
28.
29.
30.
31.
32. Define point of contra flexure? In which beam it occurs?

Point at which BM changes to zero is point of contra flexure. It occurs in overhanging beam.

23. What is mean by positive or sagging BM?

BM is said to positive if moment on left side of beam is clockwise or right side of the beam is counter clockwise.

24. What is mean by negative or hogging BM?

BM is said to negative if moment on left side of beam is counterclockwise or right side of the beam is clockwise.

25. Define shear force and bending moment?

SF at any cross section is defined as algebraic sum of all the forces acting either side of beam.

BM at any cross section is defined as algebraic sum of the moments of all the forces which are placed either side from that point.

26. What is meant by transverse loading of beam?

If load is acting on the beam which is perpendicular to center line of it is called transverse loading of beam.

27. When will bending moment is maximum?

BM will be maximum when shear force change its sign.

28. What is maximum bending moment in a simply supported beam of span 'L' subjected to UDL of 'w' over entire span?

Max BM $=wL^2/8$

29. In a simply supported beam how will you locate point of maximum bending moment?

The bending moment is max. when SF is zero. Write SF equation at that point and equating to zero we can find out the distances 'x' from one end .then find maximum bending moment at that point by taking all moment on right or left hand side of beam.

30. What is shear force?

The algebric sum of the vertical forces at any section of the beam to the left or right of the section is called shear force.

31. What is shear force and bending moment diagram?

It shows the variation of the shear force and bending moment along the length of the beam.

32. What are the types of beams?

1. Cantilever beam
2. Simply supported beam
3. Fixed beam
4. Continuous beam

33. What are the types of loads?

1. Concentrated load or point load
2. Uniform distributed load
3. Uniform varying load

34. Draw the shear stress distribution diagram for a *I*–section.

35. In which point the bending moment is maximum?

When the shear force change of sign or the shear force is zero

36. Write the assumption in the theory of simple bending?

1. The material of the beam is homogeneous and isotropic.
2. The beam material is stressed within the elastic limit and thus obey hooke's law.
3. The transverse section which was plane before bending remains plains after bending also.

4. Each layer of the beam is free to expand or contract independently about the layer, above or below.

5. The value of E is the same in both compression and tension.

37. Write the theory of simple bending equation?

M/ I = F/Y = E/R

Where

M - Maximum bending moment

I - Moment of inertia

F - Maximum stress induced

Y - Distance from the neutral axis

E - Young's modulus

R - Constant.

38. What types of stresses are caused in a beam subjected to a constant shear force?

Vertical and horizontal shear stress

39. State the main assumptions while deriving the general formula for shear stresses

The material is homogeneous, isotropic and elastic

The modulus of elasticity in tension and compression are same.

The shear stress is constant along the beam width

The presence of shear stress does not affect the distribution of bending stress.

40. Define: Shear stress distribution

The variation of shear stress along the depth of the beam is called shear stress distribution

41. What is the ratio of maximum shear stress to the average shear stress for the rectangular section?

Qmax is 1.5 times the Qave.

42. What is the ratio of maximum shear stress to the average shear stress in the case of solid circular section?

Qmax is 4/3 times the Qave.

43. What is the maximum value of shear stress for triangular section?

$Q_{max}=Fh^2/12I$

Where

h- Height

F-load

44. Draw the shear stress distribution of I-symmetrical section

45. What is the shear stress distribution value of Flange portion of the I-section?

$q = f/2I * (D^2/4 - y)$

Where

D-depth

y- Distance from neutral axis

46. Draw the shear stress distribution in the case of 'T'section

47. What is the value of maximum of minimum shear stress in a rectangular cross section?

Qmax=3/2 * F/ (bd)

48. Define -section modulus

It is the ratio of moment of inertia of the section to the distance of plane from neutral axis.

Section modulus Z = I/Y

UNIT- III

49. Define Torsion

When a pair of forces of equal magnitude but opposite directions acting on body, it tends to twist the body. It is known as twisting moment or torsional moment or simply as torque.

Torque is equal to the product of the force applied and the distance between the point of application of the force and the axis of the shaft.

50. What are the assumptions made in Torsion equation

- The material of the shaft is homogeneous, perfectly elastic and obeys Hooke's law.
- Twist is uniform along the length of the shaft
- The stress does not exceed the limit of proportionality
- The shaft circular in section remains circular after loading
- Strain and deformations are small.

51. Define polar modulus

It is the ratio between polar moment of inertia and radius of the shaft.

$$z = \frac{\text{polar moment of inertia}}{\text{Radius}} = \frac{J}{R}$$

52. Write the polar modulus for solid shaft and circular shaft.

$$z = \frac{\text{polar moment of inertia}}{\text{Radius}} = \frac{J}{R}$$

$$J = \frac{\pi D^4}{32}$$

53. Why hollow circular shafts are preferred when compared to solid circular shafts?

- The torque transmitted by the hollow shaft is greater than the solid shaft.
- For same material, length and given torque, the weight of the hollow shaft will be less compared to solid shaft.

54. Write torsional equation

$$T/J = C\theta/L = q/R$$

Where

T-Torque

J- Polar moment of inertia

C-Modulus of rigidity

L- Length

q- Shear stress

R- Radius

55. Write down the expression for power transmitted by a shaft

P=2πNT/60

N-speed in rpm

T-torque

56. Write down the expression for torque transmitted by hollow shaft

$T= (\pi/16)*Fs*((D^4-d^4)/d^4$

T-torque

q- Shear stress

D-outer diameter

D- inner diameter

57. Write the polar modulus for solid shaft and circular shaft

It is ratio between polar moment of inertia and radius of shaft

58. Write down the equation for maximum shear stress of a solid circular section in diameter 'D' when subjected to torque 'T' in a solid shaft shaft.

$T=\pi/16 * Fs*D^3$

Where

T-torque

q Shear stress

D diameter

59. Define torsional rigidity

Product of rigidity modulus and polar moment of inertia is called torsional rigidity.

60. What is composite shaft?

Some times a shaft is made up of composite section i.e. one type of shaft is sleeved over other types of shaft. At the time of sleeving, the two shafts are joined together, that the composite shaft behaves like a single shaft.

61. What is a spring?

A spring is an elastic member, which deflects, or distorts under the action of load and regains its original shape after the load is removed.

62. State any two functions of springs.

1. To measure forces in spring balance, meters and engine indicators.
2. To store energy.

63. What are the various types of springs?

i. Helical springs
ii. Spiral springs
iii. Leaf springs
iv. Disc spring or Belleville springs

64. Classify the helical springs.

1. Close – coiled or tension helical spring.
2. Open –coiled or compression helical spring.

65. What is spring index (C)?

The ratio of mean or pitch diameter to the diameter of wire for the spring is called the spring index.

8. Define strain energy

Whenever a body is strained, some amount of energy is absorbed in the body. The energy which is absorbed in the body due to straining effect is known as strain energy.

9. Define resilience?

The total strain energy stored in the body is generally known as resilience.

10. Define proof resilience

The maximum strain energy that can be stored in a material within elastic limit is known as proof resilience.

11. Define modulus of resilience

It is the proof resilience of the material per unit volume

$$\text{Modulus of resilience} = \frac{\text{Proof resilience}}{\text{Volume of the body}}$$

66. What is solid length?

The length of a spring under the maximum compression is called its solid length. It is the product of total number of coils and the diameter of wire.

$$Ls = n_t \times d$$

Where, n_t = total number of coils.

67. Define free length.

Free length of the spring is the length of the spring when it is free or unloaded condition. It is equal to the solid length plus the maximum deflection or compression plus clash allowance.

$$L_f = \text{solid length} + Y_{max} + 0.15\, Y_{max}$$

68. Define spring rate (stiffness).

The spring stiffness or spring constant is defined as the load required per unit deflection of the spring.

$K = W/\delta$

Where

W –load,

δ – deflection

69. Define pitch.

Pitch of the spring is defined as the axial distance between the adjacent coils in uncompressed state. Mathematically

$$\text{Pitch} = \frac{\text{free length}}{n-1}$$

70. Define helical springs.

The helical springs are made up of a wire coiled in the form of a helix and is primarily intended for compressive or tensile load

71. What are the stresses induced in the helical compression spring due to axial load?

1. Direct shear stress
2. Torsional shear stress
3. Effect of curvature

72. What are the differences between closed coil & open coil helical springs?

The spring wires are coiled very closely, each turn is nearly at right angles to the axis of helix	The wires are coiled such that there is a gap between the two consecutive turns.

Helix angle is less than 10^{o}	Helix angle is large ($>10^{o}$)

73. What is whal's stress factor?

$$C = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

74. What is buckling of springs?

The helical compression spring behaves like a column and buckles at a comparative small load when the length of the spring is more than 4 times the mean coil diameter.

75. What is surge in springs?

The material is subjected to higher stresses, which may cause early fatigue failure. This effect is called as spring surge.

76. Define active turns.

Active turns of the spring are defined as the number of turns, which impart spring action while loaded. As load increases the no of active coils decreases.

77. Define inactive turns.

An inactive turn of the spring is defined as the number of turns which does not contribute to the spring action while loaded. As load increases number of inactive coils increases from 0.5 to 1 turn.

78. What are the different kinds of end connections for compression helical springs?

The different kinds of end connection for compression helical springs are

a. Plain ends
b. Ground ends
c. Squared ends
d. Ground & square ends

UNIT IV

16. State principle plane.

The planes, which have no shear stress, are known as principal planes. These planes carry only normal stresses.

17. Define principle stresses and principle plane.

Principle stress: The magnitude of normal stress, acting on a principal plane is known as principal stresses.

Principle plane: The planes which have no shear stress are known as principal planes.

18. What is the radius of Mohr's circle?

Radius of Mohr's circle is equal to the maximum shear stress.

19. What is the use of Mohr's circle?

To find out the normal, resultant stresses and principle stress and their planes.

20. List the methods to find the stresses in oblique plane?

1. Analytical method
2. Graphical method

79. Define –column

Column or strut is defined as a member of a structure, which is subjected to axial compressive load. If the member the structure is vertical and both of its ends are rigidly fixed while subjected to axial compressive load.

80. What are the causes to fail the column?

1. Direct compressive stress

2. Buckling stresses

3. Combined of direct and compressive stresses

81. What is buckling or cripping load?

The load at which the column just buckle is known is buckling load

82. What are the causes to fail the long column?

The column fails due to maximum stresses is more than the crushing stresses

83. What are the assumptions made in the Euler theory?

1. The column is initially straight and the load applied axially
2. The cross section of the column is uniformly throughout the length
3. The column material is perfectly elastic, homogeneous and isotropic and obeys hooke's law.

84. List the end conditions of the column?

1. Both the ends of the column is hinged
2. One end is fixed and other end is free
3. Both the end of the column is fixed
4. One end is fixed and other is pinned

85. What is effective length?

The effective length of the given column with given and conditions is the length of an equivalent column of the same material and cross section with hinged ends, and having the value of the cripping load equal to the given column.

86. Define - slenderness ratio

The ratio of the actual length of a column to the least radiation of gyration of the column.

UNIT-V

87. When will you call a cylinder as thin cylinder?

A cylinder is called as a thin cylinder when the ratio of wall thickness to the diameter of cylinder is less 1/20.

88. In a thin cylinder will the radial stress vary over the thickness of wall?

No, in thin cylinders radial stress developed in its wall is assumed to be constant since the wall thickness is very small as compared to the diameter of cylinder.

89. Distinguish between cylindrical shell and spherical shell.

Sl.no	Cylindrical shell	Spherical shell
1.	Circumferential stress is twice the longitudinal stress.	Only hoop stress presents.
2.	It withstands low pressure than spherical shell for the same diameter	It withstands more pressure than cylindrical shell for the same diameter.

90. What is the effect of riveting a thin cylindrical shell?

Riveting reduces the area offering the resistance. Due to this, the circumferential and longitudinal stresses are more. It reduces the pressure carrying capacity of the shell.

In thin spherical shell, volumetric strain is -------- times the circumferential strain.

[Three]

91. What do you understand by the term wire winding of thin cylinder?

In order to increase the tensile strength of a thin cylinder to withstand high internal pressure without excessive increase in wall thickness, they are sometimes pre stressed by winding with a steel wire under tension.

92. What are the types of stresses setup in the thin cylinders?

1. Circumferential stresses (or) hoop stresses
2. Longitudinal stresses

93. Define – hoop stress?

The stress is acting in the circumference of the cylinder wall (or) the stresses induced perpendicular to the axis of cylinder.

94. Define- longitudinal stress?

The stress is acting along the length of the cylinder is called longitudinal stress.

95. A thin cylinder of diameter d is subjected to internal pressure p. Write down the expression for hoop stress and longitudinal stress.

Hoop stress $\sigma h = pd/2t$

Longitudinal stress $\sigma l = pd/4t$

Where

p- Pressure (gauge)

d- Diameter

t- Thickness

96. Expression for hoop stress and longitudinal stress.

Hoop stress

$\sigma h = pd/2t$

Longitudinal stress

$\sigma l = pd/4t$

p- Pressure (gauge)

d- Diameter

t- Thickness

97. Define principle stresses and principle plane.

Principle stress: The magnitude of normal stress, acting on a principal plane is known as principal stresses.

Principle plane: The planes which have no shear stress are known as principal planes.

97. What is the radius of Mohr's circle?

Radius of Mohr's circle is equal to the maximum shear stress.

98. What is the use of Mohr's circle?

To find out the normal, resultant stresses and principle stress and their planes.

99. List the methods to find the stresses in oblique plane?

1. Analytical method
2. Graphical method

100. In case of equal like principle stresses, what is the diameter of the Mohr's circle?

Answer: Zero

101. A bar of cross sectional area 600 mm^2 is subjected to a tensile load of 50 KN applied at each end. Determine the normal stress on a plane inclined at 30° to the direction of loading.

A = 600 mm^2

Load, P = 50KN

θ = 30°

Stress, σ = Load/Area

= 50*102/600

= 83.33 N/mm^2

Normal stress, σn = σ cos2θ

= 83.33*cos230°

= 62.5 N/mm^2

102. Derive an expression for the longitudinal stress in a thin cylinder subjected to a uniform internal fluid pressure.

Force due to fluid pressure = p x π /4 xd^2

Force due to longitudinal stress = f_2 x π d x t

p x π /4 xd^2 = f_2 x π d x t

f_2 = pd/4t

16 MARKS

UNIT – I

1) A bar 30 mm in diameter and 200mm long was subjected to an axial pull of 60 kN. The extension of the bar was found to be 0.1 mm, while decrease in the diameter was found to be 0.004 mm. Find the Young's modulus, Poisson's ratio, rigidity modulus and bulk modulus of the material of the bar.

REFER BANSAL PAGENO: 38

2. An aluminium rod 22 mm diameter passes through a steel tube of 25 mm internal diameter and 3 mm thick. The rod and tube are fixed together at the ends at a temperature of 30°C. Find the stresses in the rod and tube when the temperature is raised to 150°C.

$E_S = 200 \text{ kN/mm}^2$, $E_{al} = 70 \text{ kN/mm}^2$

$\alpha_S = 12 \times 10^{-6} /°\text{C}$, $\alpha_{al} = 23 \times 10^{-6} /°\text{C}$.

REFER RAJPUT PAGENO: 25

3. A reinforced concrete column 500x500 mm in section is reinforced with a steel bar of 25mm diameter, one in each corner, the column is carrying the load of 1000 KN Find the stresses induced in the concrete and steel bar. Take E for steel = 210x10^3 m N/mm^2 and E for concrete = 14x10^3 N/mm^2

REFER KHURMI PAGENO: 27

4. A steel tube 30mm external diameter and 25mm internal diameter encloses a gun metal rod 20mm diameter to which it is rigidly joined at each end. The temperature of the whole assembly is raised to 150^{o}C. Find the intensity of stress in the rod when the common temperature has fallen to 20^{o}C. The value of the Young's modulus for steel and the gun metal is 2.1x10^5N/mm^2 and 1x10^5N/mm^2 respectively. The coefficient of linear expansion for steel is 12x10^{-6}/^{o}C and for gun metal is 20x10^{-6}/^{o}C.

REFER KHURMI PAGENO: 62

5. i) Derive relation for change in length of the bar of uniformly tapering circular section subjected to an axial tensile load W.

ii) Derive relation for change in length of the bar of uniformly tapering rectangular section subjected to an axial tensile load W.

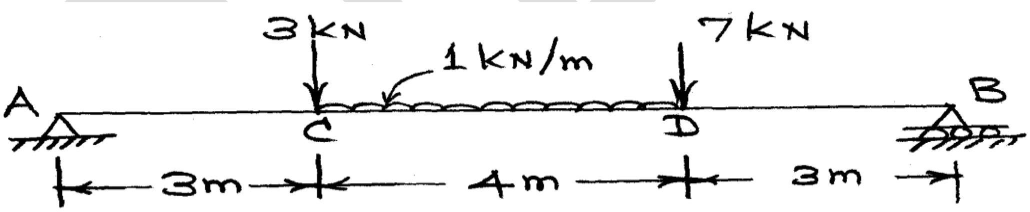
iii) Expression for Young's modulus in terms of bulk modulus.

iv) Derive the relation between modulus of elasticity and rigidity modulus.

REFER RAJPUT PAGENO: 34

UNIT – II

6. Draw the SF and BM diagram for the beam shown in Fig. Find the values at important points and indicate in the diagram.



7. Draw the shear force and bending moment diagram for a simply supported beam of length 9m and carrying the UDL OF 10KN/m for a distance of 6m from the left end and also carrying point load 3KN for a distance of 2m from the left end. Calculate the shear force and bending moment also calculates the maximum bending moment.

REFER KHURMI PAGENO: 300

8. A timber beam 240 mm wide and 360 mm deep is simply supported. It carries a udl of 20 kN/m over the entire span. Find the span length if the allowable bending stress is not to exceed 8 N/mm^2.

REFER BANSAL PAGENO: 321

9) Write the assumption in the theory of simple bending?

Derive M/I = F/Y = E/R

10) A cast iron beam is of T section the dimension of the upper flange is 100 x 20mm and the other dimension is 20 x 80mm(b x d) .The beam carries a UDL of 1.5KN/m length of the entire span. Determine the maximum tensile and compressive stress.

REFER RAJPUT PAGENO: 62

11) The shear force acting on a beam at an I section the dimension of the upper flange is 200 x 50 and the web dimension is 50 x 200mm (b x t) lower flange is 130 x 50. The moment of inertia about the N.A is 2.849 $x10^4 mm^4$. Calculate the shear stress at the neutral axis and also draw the shear stress distribution over the depth of the section.

REFER KHURMI PAGENO: 354

UNIT – III

12). A Composite Shaft Consist of a steel rod 60mm diameter surrounded by a closely fitting tube of brass. Find the outside diameter of the tube so that when a torque of 1000Nm is applied to the composite shaft, it will be shared equally by the two materials.

Take C for steel = 8.4 x 10^4 N/mm^2 and C for brass = 4.2 x $10^4 N/mm^2$.

REFER TEXT BOOK KHURMI PAGENO: 665

13) A hollow shaft is to transmit 300 kW at 80 rpm. The internal diameter is 0.6 of the external diameter. The maximum torque is 40% more than the mean torque. If the shear stress is not to exceed 60 N/mm2, find the external and internal diameters of the shaft.

REFER KHURMI PAGENO: 664

14. Design a helical coil compression spring to carry a load of 1.5 KN with a deflection 40mm. Allowable shear stress is 400N/mm$^{2.}$ Modulus of rigidity is 8 x 10$^{10\ N}$/m^{2} $^{.}$Spring index =5

REFER JALALUDEEN PAGENO: 12.28

15. Derive the shear stress produced in the circular shaft subjected to torsion.

REFER KHURMI PAGENO: 648

16. Determine the diameter of the solid shaft which will transmit 300Kw at 250rpm. The maximum shear stress should not be exceed 30N/mm^{2} and twist should not be more than 1 in a length of 2m.take C = 1 X10^{5}17.

REFER KHURMI PAGENO: 645

17) The solid cylindrical shaft is to transmit 300kw power at 100r.p.m.

a) The shear stress should not 6 exceed 80N/mm2, find its diameter.

b) What percentage saving in weight would be obtained if this shaft replace by a hollow one whose internal diameter equal to0.6mm the external diameter, the length, the material and maximum shear stress being same.

REFER BANSAL PAGENO: 562

UNIT – IV

18) A 1.5m long column has a circular cross section of 5cm diameter. One of the ends of the column is fixed in direction and position and other end is free. Taking factor of safety as 3. Calculate safe load using:

(a) Rankine formula, take yield stress, f_c = 560N/mm^2 and a = 1/1600 for pinned ends.

(b) Euler formula, Young's modulus for C.I = 1.2 x 10^5

REFER RAJPUT PAGENO: 562

19) A beam of length 8m is simply supported at its ends. It carries a uniform distributed load of 40kN/m actingt1m to 5m from the left end. Determine the deflection of the beam at its midpoint and also the position of the maximum deflection and maximum deflection. Take E = 2x10^{5}N/mm^2 and I = 4.3 x 10^{8}mm^4

REFER RAJPUT PAGENO: 762

20) A cast iron hollow column having 8cm external diameter and 6cm internal diameter, is used as a column of 2m length using Rankine formula determine the cripping load ,when both the ends are fixed. Take f_c = 6000Kg/cm^2.

REFER RAJPUT PAGENO: 862

21) Derive the slope and deflection for simply supported beam is subjected to eccentrically point load by using McCauley methods.

REFER RAJPUT PAGENO: 762

22) A thin cylinder 1.5 m internal diameter and 5 m long is subjected to an internal pressure of 2 N/mm^2. If the maximum stress is limited to160 N/mm^2, find the thickness of the cylinder. *E* = 200 kN/mm^2 and Poisson's ratio = 0.3. Also find the changes in diameter, length and volume of the cylinder.

REFER RAJPUT PAGENO: 567

23) At a point in a strained material the horizontal tensile stress is 80 N/mm^2 and the vertical compressive stress is 140 N/mm^2. The shear stress is 40 N/mm^2. Find the principal stresses and the principal planes. Find also the maximum shear stress and its planes.

REFER RAJPUT PAGENO: 102

24) Derive the Mohr's circle for stresses on an oblique section of a body subjected to direct stresses in mutually perpendicular direction accompanied with shear stresses.

REFER RAJPUT PAGENO: 106

25) At a point in a strained material the principal stresses are 80 N/mm^2 and 60 N/mm^2(compressive). Determine the direct normal stress shear stress and resultant stress on a plane inclined at 50to the axis of major principal stress. Also determine the maximum shear stress at that point

REFER RAJPUT PAGENO: 103